Blob Detection with Wavelet Maxima Lines

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Abstract- In this paper, we propose a novel approach to blob detection based on wavelet transform modulus maxima. We use maxima lines in scale-space to build a new blob detector. The algorithm we propose enables automatic blob detection and blob size determination. The robustness to noise of the blob detector we propose is also shown

Keywords- Scale-space representation, maxima lines of wavelet decomposition, blob detection.
EDICS Category:MDSP

I. INTRODUCTION

Blob detection is an important low-level operation in computer vision. It is usually a first step towards more complicated tasks such as the determination of local deformations in images [3][4] or the extraction of scale invariant interest points [7][5]. Blob detection is often carried out through the computation of local extrema of some normalized derivatives of the linear scale-space image representation [3]. First, we show that modulus maxima of such image representations make up connected curves in scale-space called maxima lines. We also highlight the relation that exists with wavelet decomposition. The demonstration of the existence of such maxima lines is an extension to the bidimensional case of the theorem proved by Yuille and Poggio [9]. Then, we show that such maxima lines still exist when the image undergoes affine transformation of its coordinates. Once the existence of maxima lines is proved, we investigate how to use them for blob detection. As we will see, blob detection can be carried out studying the normalized Laplacian of the linear-scale scale representation. The novelty of our approach is that instead of considering modulus maxima of the continuous wavelet transform (CWT) in scale-space, we consider modulus maxima of the CWT along maxima lines.With this approach, blob detection is related to the largest modulus maximum along some maxima lines of interest. To compute the blob size, we propose a new approach that consists in following the maxima lines of interest from the scale that corresponds to the largest modulus maximum to the finest scale. The distance between the location of the modulus maxima at these two scales provides a good approximation of the radius of the blob. In that framework, blobs are characterized by their center and their radius. Finally, we show the robustness to noise of the detected blobs.

II. LINEAR SCALE-SPACE AND SCALE SELECTION

A. The One-dimensional Case

First, we recall the definition of the linear scale-space in the one-dimensional case. Linear scale-space representation consists of convolutions with Gaussian kernels at different scales. If we put $g(x) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})$ and $g_t(x) = \frac{1}{\sqrt{t}}g(\frac{x}{\sqrt{t}})$, we define, as in [4], the linear scale-space as:

$$L(x,t) = g_t * f(x) = \int_{\mathbb{R}} \frac{1}{\sqrt{t}} g(\frac{x-u}{\sqrt{t}}) f(u) du.$$

Scale selection is then carried out through the study of normalized derivatives of L. Lindeberg [3] suggests to normalize the derivatives as follows:

$$\partial^m_{x,\gamma norm}L(x,t) = t^{\frac{\gamma m}{2}}\partial^m_x L(x,t)$$
$$= \int_{\mathbb{R}} t^{\frac{\gamma m}{2}}\frac{1}{\sqrt{t}}g\left(\frac{u}{\sqrt{t}}\right)f^{(m)}(x-u)du = t^{\frac{\gamma m}{2}}f^{(m)} * g_t(x). \tag{1}$$

It is worth noting that the solution of the heat equation:

$$\begin{cases} \frac{\partial h(x,\tau)}{\partial \tau} &= \frac{1}{2}\frac{\partial^2 h(x,\tau)}{\partial^2 x} \\ h(x,0) &= h_0(x) \end{cases}$$

is $h(x,\tau) = h_0 * g_\tau(x)$. This implies that $\partial^m_{x,\gamma norm}L(x,t) = t^{\frac{\gamma m}{2}}h(x,t)$, taking $h_0(x) = f^{(m)}(x)$. In other words, the normalized derivative is proportional to the solution of the heat equation with initial condition $f^{(m)}(x)$. The maximum principle for the heat equation [1] proves that a global maximum of $|h(x,t)|$ for $(x,t) \in [a,b] \times [t_0,t_1]$ is necessarily either on the boundary $x = a, b$ or at $t = t_0$. A modulus maximum of $\partial^m_{x,\gamma norm}L(x,t)$ at (x_1,t_1) is a local maximum of $|h(x,t)|$ for fixed t and x varying. Suppose that a line of modulus maxima is interrupted at (x_1,t_1), with $t_1 > 0$. One can then verify that there exists $\epsilon > 0$ such that a global maximum of $|h(x,t)|$ over $[x_1-\epsilon, x_1+\epsilon] \times [t_1-\epsilon, t_1+\epsilon]$ is at (x_1,t_1). This contradicts the maximum principle, and thus proves that all modulus maxima propagate towards finer scales. It is also interesting to draw a parallel between the normalized derivatives of the linear scale-space and wavelet decomposition. Indeed, when $\gamma = 1$, equation (1) rewrites, using m integrations by parts and as the Gaussian function and its derivatives are with fast decay:

$$\partial^m_{x,\gamma norm}L(x,t) = \int_{\mathbb{R}} f(u)\frac{1}{\sqrt{t}}g^{(m)}\left(\frac{x-u}{\sqrt{t}}\right)du.$$

Note that, as $g^{(m)}$ is either odd or even, the normalized derivative corresponds to the CWT (or to its opposite) at scale $\sqrt{t}$ using the $g^{(m)}$ wavelet and a L^1 normalization. Due to the equivalence of the two approaches when $\gamma = 1$, scale selection, as explained later, is carried out studying the modulus maxima of the CWT along maxima lines of interest.

B. The Bidimensional Case

The study is similar to the one-dimensional case: we now put $g(x_1,x_2) = \frac{1}{2\pi}\exp(-\frac{x_1^2+x_2^2}{2})$ and $g_t(x_1,x_2) = \frac{1}{t}g(\frac{x_1}{\sqrt{t}}, \frac{x_2}{\sqrt{t}})$. The bidimensional linear scale-space definition is identical to its one dimensional counterpart. Let us define $\partial_{x^\alpha}L = L_{x^\alpha} = \partial_{x_1^{\alpha_1}x_2^{\alpha_2}}L = L_{x_1^{\alpha_1}x_2^{\alpha_2}}$, with $\alpha = (\alpha_1,\alpha_2)$ (i.e. α_i is the order of differentiation in the direction x_i). We, then, consider differentiations of the linear scale-space of the form:

$$\mathcal{D}L = \sum_{j=1}^{I} c_j L_{x^{\alpha^j}}$$

where $|\alpha^j| = \alpha_1^j + \alpha_2^j = M$ is independent on j. For such differentiations of the linear scale-space, the appropriate normalization of the operator $\mathcal{D}$ is given by [3]:

$$\mathcal{D}_{x,\gamma norm} L = t^{\frac{M\gamma}{2}} \mathcal{D}L.$$

The Hessian matrix of L defined as:

$$\mathcal{H} = \begin{pmatrix} L_{x_1x_1} L_{x_1x_2} \\ L_{x_2x_1} L_{x_2x_2} \end{pmatrix}$$

is a basic tool for the analysis of the characteristic scale of objects in images [7][3]. A simple normalized feature usually extracted from this matrix is [5] [4]:

$$\text{trace } (\mathcal{H}_{\gamma norm}) = t^{\gamma} \Delta L.$$

Scale selection is often carried out through the determination of the scale-space modulus maxima of this quantity. We now prove that these modulus maxima propagate towards finer scales, thus making connected curves called maxima lines. Let us first write:

$$\text{trace}(\mathcal{H}_{\gamma norm}) = t^{\gamma} \Delta f * g_t.$$

Now, as in one dimension, the solution of the bidimensional heat equation:

$$\begin{cases} \frac{\partial h(x,\tau)}{\partial \tau} = \frac{1}{2} \Delta h(x,\tau) \\ h(x,0) = h_0(x) \end{cases}$$

is $h(x,\tau) = h_0 * g_\tau(x)$, where $x = (x_1, x_2)$. From this, we deduce that trace($\mathcal{H}_{\gamma norm}$) is proportional to the solution of the heat equation at $\tau = t$ with initial condition Δf.

Once again, applying the maximum principle for the heat equation [1], we deduce that the modulus maxima of trace($\mathcal{H}_{\gamma norm}$) propagate towards finer scales.

Now, when $\gamma = 1$, it is interesting to draw a parallel with wavelet decompositions. Using two integrations by part and because of the fast decay of the Gaussian function and of its derivatives, we get

$$\text{trace}(\mathcal{H}_{\gamma norm}) = f * (\Delta g)_t.$$

So, as Δg is even, this quantity is exactly the wavelet decomposition of f using the Δg wavelet with a L^1 normalization at scale $\sqrt{t}$. In this case, scale selection will consist in studying the modulus maxima of the CWT, built using the Δg wavelet, along some maxima lines of interest.

Note also that the Δg wavelet is not exactly the bidimensional Mexican hat wavelet since the latter is defined as:

$$\Psi_{hat}(x_1, x_2) = \frac{1}{2\pi}\left(1 - (x_1^2 + x_2^2)\right)\exp(-\frac{x_1^2 + x_2^2}{2}),$$

while the Δg wavelet is:

$$\Delta g(x_1, x_2) = \frac{\partial^2 g}{\partial^2 x_1} + \frac{\partial^2 g}{\partial^2 x_2} = \frac{1}{2\pi}\left(-2 + (x_1^2 + x_2^2)\right)\exp(-\frac{x_1^2 + x_2^2}{2}).$$

III. Evolution of Maxima Lines under Affine Transformation

A. The One-Dimensional Case

We now consider the signal $f_1(u) = f(\lambda u)$, $u \in \mathbb{R}$, and we recall the relation that links the linear scale-space representation of f_1 to that of f. If we put:

$$L_1(x,t) \quad = \quad \int_{\mathbb{R}} \frac{1}{\sqrt{t}} g(\frac{x-u}{\sqrt{t}}) f(\lambda u) du,$$

we get:

$$\partial^m_{x,\gamma norm} L_1(x,t) = \lambda^m \partial^m_{\gamma norm} L(\lambda x, \lambda^2 t). \tag{2}$$

This tells us that when the maxima lines associated with the derivatives of the linear scale-space representation of f is known, one can deduce those associated with the derivatives of the linear scale-space representation of f_1. Note that we assume the deformation s is global, if the deformation were only local the existence of maxima lines would be a lot more complicated because relation (2) would not hold.

B. The Bidimensional Case

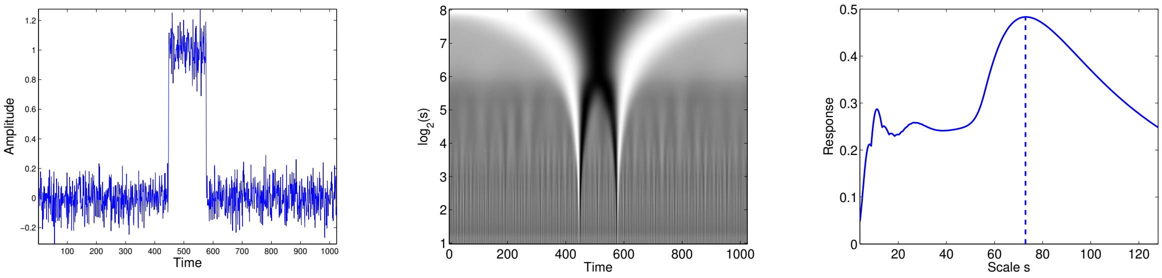
In this section, we first recall the concept of affine Gaussian scale-space representation of images introduced by Lindeberg [3], we then explain why maxima lines are well defined in that context.

Given a symmetric definite positive matrix Σ_t, the non-uniform Gaussian kernel is defined as:

$$g(x, \Sigma_t) = \frac{1}{2\pi\sqrt{det(\Sigma_t)}} e^{-\frac{x^T \Sigma_t^{-1} x}{2}} \tag{3}$$

where $x = (x_1, x_2)$. This enables us to define the affine Gaussian scale-space of f as:

$$L(x, \Sigma_t) = g(., \Sigma_t) * f(x) \tag{4}$$



(A) (B) (C)

Fig. 1. (A): crenel-shaped signal with additional Gaussian white noise (SNR: 20dB) (the steps are at $x_1 = 448$ and at $x_2 = 576$, centered around $x_0 = 512$) ; (B): CWT using the Δg wavelet ; (C): evolution of the modulus maxima of the CWT along the maxima line, starting at $x_2 = 576$ (finest scale) and ending at $x_0 = 512$

In this general form, it is not possible to associate with L a partial differential equation. However, when $\Sigma_t = t\Sigma_0$, $L(x, \Sigma_t)$ is the solution of the following partial differential equation [3]:

$$\begin{cases} \frac{\partial L}{\partial t} = \frac{1}{2} div(\Sigma_0 \nabla L) \\ L(x, 0) = f(x). \end{cases} \tag{5}$$

This equation is elliptic since Σ_0 is symmetric definite positive so the maximum principle holds and the modulus maxima of $L(x, \Sigma_t)$ propagate towards finer scales.

Now, we recall the relation that links the scale-space representation of the signal f to that of the signal f_B satisfying $f(x) = f_B(Bx)$ with B invertible. This relation is [3]:

$$L(x, \Sigma_t) = g(., B\Sigma_t B^T) * f_B(x).$$

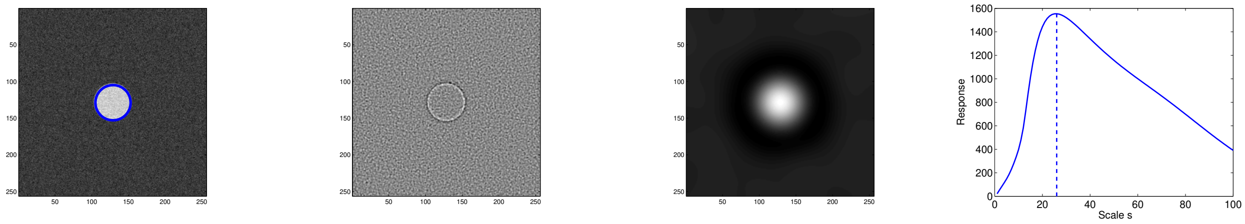
This means that trace$(\mathcal{H}_{\gamma norm})$, where $\mathcal{H}_{\gamma norm}$ is defined using $L(x, \Sigma_t)$, is proportional to the solution of the following partial differential equation:

$$\begin{cases} \frac{\partial L}{\partial t} = \frac{1}{2} div(B\Sigma_0 B^T \nabla L) \\ L(x, 0) = \Delta f_B(x). \end{cases} \tag{6}$$

So, applying (5) with the initial condition Δf leads to the same maxima lines as those given by the application of (6).

IV. Maxima Line Method for Blob Detection

Blob detection in scale-space involves its localization and the computation of a characteristic scale. It is related, in a CWT framework, to the determination of scale-space modulus maxima. Formally, Lindeberg



(A) (B) (C) (D)

Fig. 2. (A):Image of a disk with additional Gaussian white noise ; (B):the corresponding bidimensional wavelet decomposition at the finest scale; (C):idem at $s = s_{max} = 100$ (D):evolution of the modulus maxima of the CWT along a maxima line corresponding to the blob-like structure

pointed out in [4] that these modulus maxima should be searched along *extremum paths* of differentiations of the linear scale-space. However, in practice, when dealing with discrete data, he suggests to search scale-space modulus maxima within a three-dimensional discrete space. The same idea was used in [5] but with the difference of Gaussians as analysis filter (which is an approximation of the Δg wavelet).

Our approach relies on the effective computation of such *extremum paths* also called maxima lines. From the theoretical study of section II, any modulus maximum belongs to a maxima line. This property is difficult to obtain in practice and we use the following ad hoc algorithm to compute the maxima lines [6]: a modulus maximum propagates from scale s to scale $s+1$ if its position is close at scale $s+1$ and if it has the same sign at scales s and $s+1$. Note that it means we use integer scales. These maxima lines enable us to follow the drift of modulus maxima of the CWT with scales. The computational cost of the construction of the maxima lines is the following. As we know an analytical expression of the Fourier transform of the Δg wavelet, it can be computed at each scale with $O(N \log_2(N))$ operations, using the fast Fourier transform (N is the size of the data). Then, the computation of the local modulus maxima is done with $O(N)$ operations while the construction of the maxima lines is achieved with $O(N_s)$ operations, where N_s is the number of local modulus maxima at scale s. Finally, as the computation is carried out at integer scales, the global computation cost is $O(N \log_2(N))$, provided $s_{max} << N$, where s_{max} is the largest scale we consider.

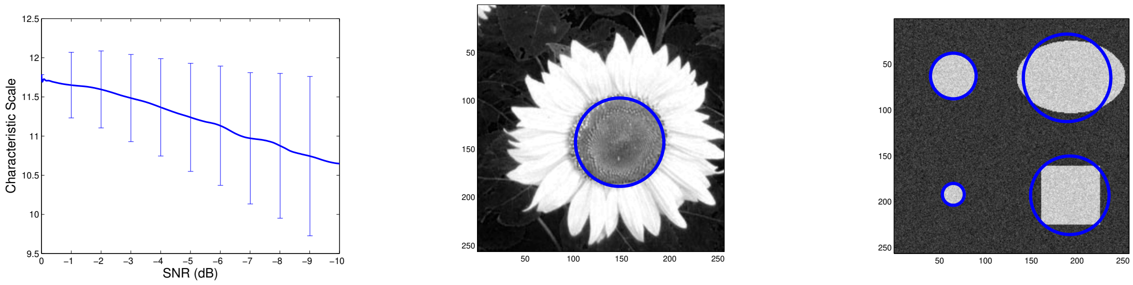
A. Selection of the Maxima Lines of Interest

In our framework, blob detection is then achieved by selecting a certain local maximum along some maxima lines of interest. This way, we will see that we get simultaneously the characteristic scale and the localization of the blob-like structure. Moreover, since we use an appropriate normalization, this approach leads to a scale-invariant interest point detector.

As already pointed out by Mallat [6], the coefficients on maxima lines associated to noise fluctuations increase on average when the scale decreases. Such maxima lines are removed from the set of maxima lines of interest. Furthermore, it is proved in [6] that for a white noise, on average, the number of modulus maxima decreases by a factor of 2 when the scale increases by 2. Therefore, we reject the maxima lines that do not cross at least 5 integer scales. Next, we remark that a blob-like structure generates many maxima lines among which only some are characteristic of the blob size. More precisely, the maxima lines we are interested in are those that are inside the boundary of the blob and that are not associated to noise fluctuations (see Figure 1 (B), for a one-dimensional example). These maxima lines join, as we will see, at a scale which is related to the blob size. The junction of such maxima lines is very close in space to the location of the global maximum of the modulus maxima of the CWT on each maxima line that is involved in the junction. In the case of an isolated blob, the resulting maxima line is spatially stable at scales larger than the scale of junction. Consequently, if we denote by (x^*, y^*, s^*) the global maximum of the modulus maxima on a maxima line of interest, the maxima line will be spatially stable for $s \geq s^*$. In practice, as blobs with different size and location may interfere, the criterion we use to select relevant maxima lines is based on the computation of the local spatial stability of the maxima lines between scales s^* and $s^* + 2$.

On the contrary, other maxima lines associated to the blob structure (i.e. that are outside the blob boundary) will always deviate when the scale increases (see Figure 1 (B)), for a one-dimensional example).

Let us now give some numerical examples. In a 1D context, we consider a noisy crenel signal, on which we perform a CWT at integer scales using the Δg wavelet ; among the different maxima lines, we focus on a significant one, corresponding to the step singularity and we then draw the amplitude of the modulus maxima of the CWT with respect to the scale. Based on the results shown in Figure 1, it appears that the largest modulus maximum of the CWT along this maxima line corresponds to the maximum we are interested in, whereas the first local maximum starting from the finest scale is irrelevant. In a 2D context, we consider a simple image of a disk (centered at (x, y)) with additive Gaussian white noise (see Figure 2 (A)). We display in Figure 2 (B) and (C), the bidimensional wavelet decomposition at $s = 1$ and $s = s_{max} = 100$ respectively. A maxima line of interest is displayed in Figure 2 (D). Analysis evidences one main interest point given by (x, y, s^*). The computation of s^* enables to find the radius of the blob (see the circle displayed on Figure 2 (A)) ; we are going to see how s^* is related to the radius of the blob.



(A) (B) (C)

Fig. 3. (A): Average characteristic scale S^* and its standard deviation computed over noisy images of a disk as a function of the SNR ratio, the radius of the disk in absence of noise is 11.78; (B):Blob detection for an image of sunflower; (C):Blob detection in an image that contains blobs with different sizes

B. Computation of Interest Points

Now, let us explain how the maxima lines method provides a new way to estimate the blob characteristic scale. Classically, it is computed through the selection of a particular scale within a predetermined range. As explained earlier (section IV. A), a maxima line of interest usually joins several maxima lines which share the following property: they have their origin at the boundary of the blob and have their global maximum at the ”center” of the blob (see Figure 1 (B), for a one-dimensional example). Consequently, for each maxima line of interest, we compute the distance between the localization of the modulus maximum of the CWT at the finest scale and the localization of the largest modulus maximum. This distance is called the maxima line characteristic scale. If the contour of the blob is a circle, this distance corresponds to the radius, and hence can be viewed as the circle characteristic scale. This suggests to compute the blob characteristic scale as the median of these maxima lines characteristic scales as follows:

1) Compute the 2D CWT of the image using the wavelet Δg (and L^1 normalization for scale invariance)
2) At every scale, compute modulus maxima in space
3) Connect these modulus maxima across scales and select the maxima lines of interest (see section IV. A).
4) Along each maxima line of interest, determine the global maximum. Suppose a maxima line of interest starts at (x_0, y_0, s_0) and has its global maximum at (x^*, y^*, s^*), the localization is given by (x^*, y^*) and the maxima line characteristic scale is $S^* = \sqrt{(x^* - x_0)^2 + (y^* - y_0)^2}$

5) Determination of interest points: group the maxima lines $\{L_i, i \in I\}$ that join at (x, y, s). The interest point is finally given by
   - its location: (x, y) and
   - its characteristic scale: $S^* = median\{S_i^*, i \in I\}$.

   If the set I contains only one element, the interest point is (x^*, y^*, S^*).

C. Robustness to Noise of the Maxima Line Method

Now, we study the performance of the maxima lines method in terms of robustness to noise. To do so, we consider an image representing a disk (image size 128×128) with additive Gaussian white noise with standard deviation σ ; our approach enables us to detect the main blob and to compute its characteristic scale. For each value of σ ($\sigma = 1, \cdots, 50$, gray level of the disk = 100, gray level of the background = 0), we generate 100 noisy images with which we compute the mean SNR ratio, the average characteristic scale S^* and its standard deviation. We display in Figure 3 (A), the average characteristic scale S^* and its standard deviation (the error bars) as a function of the SNR ratio. Results show that the computation of the characteristic scale with the maxima lines method is stable even for low SNR ratio: the variation of the average characteristic scale with noise does not exceed one pixel. We also display the blob detection with the maxima lines method for a natural image of sunflower in Figure 3 (B). The example of Figure 3 (C) illustrates the fact that our method enables the detection of blobs with different size in a noisy context.

These examples show that the maxima line method is robust to noise and that it enables automatic detection of blobs with different characteristic scales. The main advantage of the proposed method is that it is based on intrinsic properties of the maxima lines of interest rather than on the strength of the CWT modulus maxima. This differs from other methods which are highly dependent on a thresholding step. A comparative study of the maxima lines method to other scale invariant points detector such as those proposed in [3][5] [8][2] will be the subject for future work.

V. CONCLUSION

In this paper, we have presented a new method for blob detection based on maxima lines of the continuous wavelet transform. We have shown that blob detection could be carried out considering the global maximum of modulus maxima along some maxima lines. We have also put forward that the use of maxima lines allows to compute the size of the blob in a efficient way. The model we propose is robust to the presence of noise in the image. The advantage of the proposed method over existing ones is that, in our framework, blob detection is not based on a thresholding step but uses intrinsic properties of some maxima lines of interest. Future work will involve a complete comparative study with other existing scale-invariant methods in terms of robustness to noise and also in terms of accuracy of the blob detection. In another direction, potential use of this new blob detector for higher level computer vision operation such as the determination of local deformations in images or the computation of affine invariant interest points should be investigated. This is motivated by the results of section III where we put forward the relation that link the maxima lines of a deformed image to these of the original image.

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