Cases 7,10,14,18,19,20,26,27,38,40,44

Roland Hildebrand, Andrei Afonin

January 13, 2019

Suppose the minimal support set has a subset of the form

$$\{a,b,c\}, \{a,b,d\}, \{a,c,e\}, \{a,d,e\},$$

where $a,\dots,e$ are mutually distinct indices. Then the corresponding 5×5 submatrix of A has the form

$$\begin{pmatrix} 1 & -\cos\phi_1 & -\cos\phi_2 & -\cos\phi_3 & -\cos\phi_4 \\ -\cos\phi_1 & 1 & \cos(\phi_1+\phi_2) & \cos(\phi_1+\phi_3) & \star \\ -\cos\phi_2 & \cos(\phi_1+\phi_2) & 1 & \star & \cos(\phi_2+\phi_4) \\ -\cos\phi_3 & \cos(\phi_1+\phi_3) & \star & 1 & \cos(\phi_3+\phi_4) \\ -\cos\phi_4 & \star & \cos(\phi_2+\phi_4) & \cos(\phi_3+\phi_4) & 1 \end{pmatrix},$$

and the corresponding sub-vectors of the zeros are given by

$$\begin{pmatrix} \sin(\phi_1+\phi_2) \\ \sin\phi_2 \\ \sin\phi_1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sin(\phi_1+\phi_3) \\ \sin\phi_3 \\ 0 \\ \sin\phi_1 \\ 0 \end{pmatrix}, \begin{pmatrix} \sin(\phi_2+\phi_4) \\ 0 \\ \sin\phi_4 \\ 0 \\ \sin\phi_2 \end{pmatrix}, \begin{pmatrix} \sin(\phi_3+\phi_4) \\ 0 \\ 0 \\ \sin\phi_4 \\ \sin\phi_3 \end{pmatrix}.$$

The other components of the zeros all vanish.

These 4 zeros are linearly dependent, namely we have

$$\sin\phi_3\sin\phi_4 u_1 - \sin\phi_2\sin\phi_4 u_2 - \sin\phi_3\sin\phi_1 u_3 + \sin\phi_2\sin\phi_1 u_4 = 0.$$

All coefficients are non-zero, hence every one of the 4 zeros can be represented as a linear combination of the other 3.

In this way we may establish linear dependencies of the minimal zeros of a copositive matrix just by examining its minimal support set. By removing a zero which is dependent on other zeros we do not change the span of the zeros. If after successive removing of zeros which are dependent on zeros which are still present we obtain a total number of zeros strictly smaller than the order of the matrix, then all minimal zeros must be contained in a proper subspace. In this case the matrix is not reduced with respect to the cone of positive semi-definite matrices and cannot be exceptional extremal.