Case 39

The minimal zero supports are given by (1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 6), (1, 4, 6), (2, 5, 6), (3 , 5, 6)

$$A = \begin{pmatrix} 1 & -cos(\phi_2) & -cos(\phi_1) & -cos(\phi_3) & cos(\phi_2+\phi_4) & cos(\phi_1+\phi_5) \\ -cos(\phi_2) & 1 & cos(\phi_1+\phi_2) & cos(\phi_2+\phi_3) & -cos(\phi_4) & cos(\phi_4+\phi_6) \\ -cos(\phi_1) & cos(\phi_1+\phi_2) & 1 & b_1 & cos(\phi_5+\phi_6) & -cos(\phi_5) \\ -cos(\phi_3) & cos(\phi_2+\phi_3) & b_1 & 1 & b_2 & -cos(\phi_1+\phi_5-\phi_3) \\ cos(\phi_2+\phi_4) & -cos(\phi_4) & cos(\phi_5+\phi_6) & b_2 & 1 & -cos(\phi_6) \\ cos(\phi_1+\phi_5) & cos(\phi_4+\phi_6) & -cos(\phi_5) & -cos(\phi_1+\phi_5-\phi_3) & -cos(\phi_6) & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1,u_2,u_3,u_4,u_5,u_6,u_7) = \begin{pmatrix} sin(\phi_1+\phi_2) \\ sin(\phi_1) \\ sin(\phi_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_2+\phi_3) \\ sin(\phi_3) \\ 0 \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_4) \\ sin(\phi_2+\phi_4) \\ 0 \\ 0 \\ sin(\phi_2) \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_5) \\ 0 \\ sin(\phi_1+\phi_5) \\ 0 \\ 0 \\ sin(\phi_1) \end{pmatrix} \begin{pmatrix} sin(\phi_5+\phi_1-\phi_3) \\ 0 \\ 0 \\ sin(\phi_1+\phi_5) \\ 0 \\ sin(\phi_3) \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ sin(\phi_6) \\ 0 \\ 0 \\ sin(\phi_4+\phi_6) \\ sin(\phi_4) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_6) \\ 0 \\ sin(\phi_5) \\ sin(\phi_5+\phi_6) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ b_1 sin(\phi_2) - sin(\phi_2)cos(\phi_1-\phi_3) \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_2)cos(\phi_5+\phi_6) \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) + sin(\phi_1)cos(\phi_4+\phi_6) \end{pmatrix} => \begin{cases} b_1 \geq cos(\phi_1-\phi_3) \\ cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_5+\phi_6) \\ cos(\phi_1+\phi_2+\phi_5) \geq -cos(\phi_4+\phi_6) \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ 0 \\ b_1 sin(\phi_2) - sin(\phi_2)cos(\phi_1-\phi_3) \\ 0 \\ b_2 sin(\phi_2) + sin(\phi_2)cos(\phi_2+\phi_3+\phi_4) \\ sin(\phi_3)cos(\phi_1+\phi_2+\phi_5) + sin(\phi_3)cos(\phi_4+\phi_6) \end{pmatrix} => \begin{cases} b_1 \geq cos(\phi_1-\phi_3) \\ b_2 \geq -cos(\phi_2+\phi_3+\phi_4) \\ cos(\phi_1+\phi_2+\phi_5) \geq -cos(\phi_4+\phi_6) \end{cases}$$

3. $$Au_3 = \begin{pmatrix} 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_2)cos(\phi_5+\phi_6) \\ b_2 sin(\phi_2) + sin(\phi_2)cos(\phi_2+\phi_3+\phi_4) \\ 0 \\ sin(\phi_4)cos(\phi_1+\phi_5) + sin(\phi_4)cos(\phi_2+\phi_4+\phi_6) \end{pmatrix} => \begin{cases} cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_5+\phi_6) \\ b_2 \geq -cos(\phi_2+\phi_3+\phi_4) \\ cos(\phi_1+\phi_5) \geq -cos(\phi_2+\phi_4+\phi_6) \end{cases}$$

4. $$Au_4 = \begin{pmatrix} 0 \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) + sin(\phi_1)cos(\phi_4+\phi_6) \\ 0 \\ b_1 sin(\phi_1+\phi_5) - sin(\phi_1+\phi_5)cos(\phi_1-\phi_3) \\ sin(\phi_5)cos(\phi_2+\phi_4) + sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ 0 \end{pmatrix} => \begin{cases} cos(\phi_1+\phi_2+\phi_5) \geq -cos(\phi_4+\phi_6) \\ b_1 \geq cos(\phi_1-\phi_3) \\ cos(\phi_2+\phi_4) \geq -cos(\phi_1+\phi_5+\phi_6) \end{cases}$$

5. $$Au_5 = \begin{pmatrix} 0 \\ sin(\phi_3)cos(\phi_1+\phi_2+\phi_5) + sin(\phi_3)cos(\phi_4+\phi_6) \\ b_1 sin(\phi_1+\phi_5) - sin(\phi_1+\phi_5)cos(\phi_1-\phi_3) \\ 0 \\ b_2 sin(\phi_1+\phi_5) + sin(\phi_1-\phi_3+\phi_5)cos(\phi_2+\phi_4) - sin(\phi_3)cos(\phi_6) \\ 0 \end{pmatrix} =>$$

$$\begin{cases} cos(\phi_1+\phi_2+\phi_5) \geq -cos(\phi_4+\phi_6) \\ b_1 \geq cos(\phi_1-\phi_3) \\ b_2 sin(\phi_1+\phi_5) + sin(\phi_1-\phi_3+\phi_5)cos(\phi_2+\phi_4) - sin(\phi_3)cos(\phi_6) \geq 0 \end{cases}$$

6. $Au_6 = \begin{pmatrix} sin(\phi_4)cos(\phi_1+\phi_5)+sin(\phi_4)cos(\phi_2+\phi_4+\phi_6) \\ 0 \\ sin(\phi_6)cos(\phi_1+\phi_2)+sin(\phi_6)cos(\phi_4+\phi_5+\phi_6) \\ -sin(\phi_4)cos(\phi_1-\phi_3+\phi_5)+sin(\phi_6)cos(\phi_2+\phi_3)+b_2sin(\phi_4+\phi_6) \\ 0 \\ 0 \end{pmatrix} =>$

$$\begin{cases} cos(\phi_1+\phi_5) \geq -cos(\phi_2+\phi_4+\phi_6) \\ cos(\phi_1+\phi_2) \geq -cos(\phi_4+\phi_5+\phi_6) \\ -sin(\phi_4)cos(\phi_1-\phi_3+\phi_5)+sin(\phi_6)cos(\phi_2+\phi_3)+b_2sin(\phi_4+\phi_6) \geq 0 \end{cases}$$

7. $Au_7 = \begin{pmatrix} sin(\phi_5)cos(\phi_2+\phi_4)+sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ sin(\phi_6)cos(\phi_1+\phi_2)+sin(\phi_6)cos(\phi_4+\phi_5+\phi_6) \\ 0 \\ b_2sin(\phi_5)+b_1sin(\phi_6)-sin(\phi_5+\phi_6)cos(\phi_1-\phi_3+\phi_5) \\ 0 \\ 0 \end{pmatrix} =>$

$$\begin{cases} cos(\phi_2+\phi_4) \geq -cos(\phi_1+\phi_5+\phi_6) \\ cos(\phi_1+\phi_2) \geq -cos(\phi_4+\phi_5+\phi_6) \\ b_2sin(\phi_5)+b_1sin(\phi_6)-sin(\phi_5+\phi_6)cos(\phi_1-\phi_3+\phi_5) \geq 0 \end{cases}$$

From inequalities on ϕ_i we get:

$$\begin{cases} \phi_1+\phi_2+\phi_4+\phi_5+\phi_6 \leq \pi \\ \phi_3 \leq \phi_1+\phi_5 \end{cases} =>$$

Consider inequality 7.3, if $b_1 = cos(\phi_1-\phi_3)$, then $b_2 \geq cos(\phi_1-\phi_3+\phi_5+\phi_6)$
All inequalities for b_2 are not fulfilled, if $b_2 = -cos(\phi_2+\phi_3+\phi_4)$

Copositivity:

Sets $(1,3,4),(2,3,4),(1,3,5),(2,3,5),(1,4,5),(2,4,5),(3,4,5),(1,3,4,5),(2,3,4,5),(1,2,6),$ $(2,3,6),(2,4,6),(3,4,6),(2,3,4,6),(1,5,6),(4,5,6)$ have to be considered.

Consider set (4,5,6): let $b_2 = cos(\phi) => \phi \leq \phi_1+\phi_5-\phi_3+\phi_6 => b_2 = cos(\phi) \geq cos(\phi_1+\phi_5-\phi_3+\phi_6)$, For b_2 defined from 2.3, 5.3, 6.3 such inequality hold only if $\phi_1+\phi_2+\phi_4+\phi_5+\phi_6 = \pi$ and in this case there is additional zero (4, 5, 6).

But if in 7.3 inequality let $b_1 = cos(\phi_1-\phi_3)$, we get $b_2 \geq cos(\phi_1-\phi_3+\phi_5+\phi_6)$.
And if we are increasing b_1 in this inequality, the boundary magnitude of b_2 is decreasing and it follows, that there is no copositivity in this case. As result we have $b_1 = cos(\phi_1-\phi_3), b_2 = cos(\phi_1-\phi_3+\phi_5+\phi_6)$ and there is additional minimal zero (4, 5, 6).

Result

There aren't any copositive matrices with such minimal zeros set.

References

[1] Peter J.C. Dickinson. *A new certificate for copositivity. Optimization Online.*

[2] Peter J.C. Dickinson and Roland Hildebrand. *Considering copositivity locally.* J.Math. Anal.Appl., 437(2):1184–1195, 2016

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