1 Case 32

The minimal zero supports are given by (1,2,3),(1,2,4),(1,3,5),(2,4,5),(1,5,6),(4,5,6).

$$A = \begin{pmatrix} 1 & -cos(\phi_2) & -cos(\phi_1) & cos(\phi_2+\phi_3) & -cos(\phi_4) & cos(\phi_4+\phi_6) \\ -cos(\phi_2) & 1 & cos(\phi_1+\phi_2) & -cos(\phi_3) & cos(\phi_3+\phi_5) & b_1 \\ -cos(\phi_1) & cos(\phi_1+\phi_2) & 1 & b_2 & cos(\phi_1+\phi_4) & b_3 \\ cos(\phi_2+\phi_3) & -cos(\phi_3) & b_2 & 1 & -cos(\phi_5) & cos(\phi_5+\phi_6) \\ -cos(\phi_4) & cos(\phi_3+\phi_5) & cos(\phi_1+\phi_4) & -cos(\phi_5) & 1 & -cos(\phi_6) \\ cos(\phi_4+\phi_6) & b_1 & b_3 & cos(\phi_5+\phi_6) & -cos(\phi_6) & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1,u_2,u_3,u_4,u_5,u_6) = \begin{pmatrix} sin(\phi_1+\phi_2) \\ sin(\phi_1) \\ sin(\phi_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_3) \\ sin(\phi_2+\phi_3) \\ 0 \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_1+\phi_4) \\ 0 \\ sin(\phi_4) \\ 0 \\ sin(\phi_1) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_5) \\ 0 \\ sin(\phi_3+\phi_5) \\ sin(\phi_3) \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_6) \\ 0 \\ 0 \\ 0 \\ sin(\phi_4+\phi_6) \\ sin(\phi_4) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ sin(\phi_6) \\ sin(\phi_5+\phi_6) \\ sin(\phi_5) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ b_2sin(\phi_2)+sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) \\ -sin(\phi_1)cos(\phi_2-\phi_4)+sin(\phi_1)cos(\phi_3+\phi_5) \\ b_1sin(\phi_1)+b_3sin(\phi_2)+sin(\phi_1+\phi_2)cos(\phi_4+\phi_6) \end{pmatrix} => \begin{cases} b_2 \geq -cos(\phi_1+\phi_2+\phi_3) \\ cos(\phi_3+\phi_5) \geq cos(\phi_2-\phi_4) \\ b_1sin(\phi_1)+b_3sin(\phi_2)+sin(\phi_1+\phi_2)cos(\phi_4+\phi_6) \geq 0 \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ 0 \\ b_2sin(\phi_2)+sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) \\ 0 \\ -sin(\phi_3)cos(\phi_4)+sin(\phi_3)cos(\phi_2+\phi_3+\phi_5) \\ b_1sin(\phi_2+\phi_3)+sin(\phi_3)cos(\phi_4+\phi_6)+sin(\phi_2)cos(\phi_5+\phi_6) \end{pmatrix} => \begin{cases} b_2 \geq -cos(\phi_1+\phi_2+\phi_3) \\ cos(\phi_2+\phi_3+\phi_5) \geq cos(\phi_4) \\ b_1sin(\phi_2+\phi_3)+sin(\phi_3)cos(\phi_4+\phi_6)+sin(\phi_2 \end{cases}$$

3. $$Au_3 = \begin{pmatrix} 0 \\ -sin(\phi_1)cos(\phi_2-\phi_4)+sin(\phi_1)cos(\phi_3+\phi_5) \\ 0 \\ b_2sin(\phi_4)+sin(\phi_1+\phi_4)cos(\phi_2+\phi_3)-sin(\phi_1)cos(\phi_5) \\ 0 \\ b_3sin(\phi_4)+sin(\phi_4)cos(\phi_1+\phi_4+\phi_6) \end{pmatrix} => \begin{cases} cos(\phi_3+\phi_5) \geq cos(\phi_2-\phi_4) \\ b_2sin(\phi_4)+sin(\phi_1+\phi_4)cos(\phi_2+\phi_3)-sin(\phi_1)cos(\\ b_3 \geq -cos(\phi_1+\phi_4+\phi_6) \end{cases}$$

4. $$Au_4 = \begin{pmatrix} -sin(\phi_3)cos(\phi_4)+sin(\phi_3)cos(\phi_2+\phi_3+\phi_5) \\ 0 \\ sin(\phi_3)cos(\phi_1+\phi_4)+sin(\phi_5)cos(\phi_1+\phi_2)+b_2sin(\phi_3+\phi_5) \\ 0 \\ 0 \\ b_1sin(\phi_5)+sin(\phi_5)cos(\phi_3+\phi_5+\phi_6) \end{pmatrix} => \begin{cases} cos(\phi_2+\phi_3+\phi_5) \geq cos(\phi_4) \\ sin(\phi_3)cos(\phi_1+\phi_4)+sin(\phi_5)cos(\phi_1+\phi_2)+b_2 \\ b_1 \geq -cos(\phi_3+\phi_5+\phi_6) \end{cases}$$

5. $$Au_5 = \begin{pmatrix} 0 \\ b_1sin(\phi_4)-sin(\phi_6)cos(\phi_2)+sin(\phi_4+\phi_6)cos(\phi_3+\phi_5) \\ b_3sin(\phi_4)+sin(\phi_4)cos(\phi_1+\phi_4+\phi_6) \\ sin(\phi_6)cos(\phi_2+\phi_3)-sin(\phi_6)cos(\phi_4-\phi_5) \\ 0 \\ 0 \end{pmatrix} => \begin{cases} b_1sin(\phi_4)-sin(\phi_6)cos(\phi_2)+sin(\phi_4+\phi_6)cos(\phi_3+\phi \\ b_3 \geq -cos(\phi_1+\phi_4+\phi_6) \\ cos(\phi_2+\phi_3) \geq cos(\phi_4-\phi_5) \end{cases}$$

6. $$Au_6 = \begin{pmatrix} sin(\phi_6)cos(\phi_2+\phi_3)-sin(\phi_6)cos(\phi_4-\phi_5) \\ b_1sin(\phi_5)+sin(\phi_5)cos(\phi_3+\phi_5+\phi_6) \\ b_3sin(\phi_5)+b_2sin(\phi_6)+sin(\phi_5+\phi_6)cos(\phi_1+\phi_4) \\ 0 \\ 0 \\ 0 \end{pmatrix} => \begin{cases} cos(\phi_2+\phi_3) \geq cos(\phi_4-\phi_5) \\ b_1 \geq -cos(\phi_3+\phi_5+\phi_6) \\ b_3sin(\phi_5)+b_2sin(\phi_6)+sin(\phi_5+\phi_6)cos(\phi_1+\phi_4) \geq 0 \end{cases}$$

Considering inequalities on ϕ_i we get: $\begin{cases} \phi_3+\phi_5 \leq |\phi_2-\phi_4| \\ \phi_2+\phi_3 \leq |\phi_4-\phi_5| \end{cases}$
Consider inequality 2.2: if $\phi_2+\phi_3+\phi_5 \geq \pi => 2\pi-(\phi_2+\phi_3+\phi_5) \leq \phi_4 => 2\pi \leq \phi_2+\phi_3+\phi_5+\phi_4$ doesn't hold, because of above inequalities.

As result, we get $\phi_2+\phi_3+\phi_5 \leq \phi_4$

Copositivity:

It's necessery to consider next sets for I:
$(1,3,4),(2,3,4),(1,2,5),(2,3,5),(1,4,5),(3,4,5),(1,2,6),(1,3,6),(2,3,6),(2,4,6),(3,4,6),(1,3,4,6),(2,3,4,6),(2,5,6),$
$(3,5,6),(2,3,5,6)$

1. $I=(1,2,5): \phi_2+\phi_4 \geq \phi_3+\phi_5$ Performs strictly
2. $I=(2,3,5): 2\pi \geq 2\phi_1+\phi_4+\phi_2+\phi_3+\phi_5$, but $\phi_2+\phi_3+\phi_5 \leq \phi_4 =>$ Performs strictly
3. $I=(1,4,5): \phi_5+\phi_4 \geq \phi_3+\phi_2$ Performs strictly
4. $I=(1,4,6): 2\pi \geq 2\phi_6+\phi_4+\phi_2+\phi_3+\phi_5$, but $\phi_2+\phi_3+\phi_5 \leq \phi_4 =>$ Performs strictly

Now consider set (1, 2, 6): $b_1 = cos(\phi) => \phi_2+\pi-\phi_4-\phi_6 \geq \phi => cos(\phi) \geq -cos(\phi_4+\phi_6-\phi_2)$
If $b_1 = -cos(\phi_3+\phi_5+\phi_6) =>$ then substitute it in copositive inequality and get $\phi_3+\phi_5+\phi_2 \geq \phi_4$ - doesn't hold

If $b_1 = \frac{-sin(\phi_2)cos(\phi_5+\phi_6)-sin(\phi_3)cos(\phi_4+\phi_6)}{sin(\phi_2+\phi_3)} \geq -cos(\phi_4+\phi_6-\phi_2)$
$cos(\phi_4+\phi_6-\phi_2)sin(\phi_2+\phi_3)-sin(\phi_2)cos(\phi_5+\phi_6)-sin(\phi_3)cos(\phi_4+\phi_6) \geq 0$
$cos(\phi_2+\phi_3-\phi_4-\phi_6) \geq cos(\phi_5+\phi_6) => -\phi_2-\phi_3+\phi_4+\phi_6 \leq \phi_5+\phi_6 => \phi_4 \leq \phi_5+\phi_2+\phi_3$ - doesn't hold

If $b_1 = \frac{sin(\phi_6)cos(\phi_2)-sin(\phi_4+\phi_6)cos(\phi_3+\phi_5)}{sin(\phi_4)} \geq -cos(\phi_4+\phi_6-\phi_2)$
$cos(\phi_4+\phi_6-\phi_2)sin(\phi_4)+sin(\phi_6)cos(\phi_2)-sin(\phi_4+\phi_6)cos(\phi_3+\phi_5) \geq 0$
$cos(\phi_2-\phi_4) \geq cos(\phi_3+\phi_5) => \phi_4-\phi_2 \leq \phi_3+\phi_5 => \phi_4 \leq \phi_5+\phi_2+\phi_3$ - doesn't hold

If $b_1 = \frac{-b_3 sin(\phi_2)-sin(\phi_1+\phi_2)cos(\phi_4+\phi_6)}{sin(\phi_1)} \geq -cos(\phi_4+\phi_6-\phi_2)$
$cos(\phi_4+\phi_6-\phi_2)sin(\phi_1)-b_3 sin(\phi_2)-sin(\phi_1+\phi_2)cos(\phi_4+\phi_6) \geq 0$
$b_3 \leq -cos(\phi_1+\phi_4+\phi_6)$, but from above inequality 5.2 $b_3 \geq -cos(\phi_1+\phi_4+\phi_6) =>$
holds only if $b_3 = -cos(\phi_1+\phi_4+\phi_6)$ and,as result, there is additional minimal zero (1, 2, 6)

Result:

There isn't any copositive matrix with such minimal zeros set.