Case 29

The minimal zero supports are given by (1, 2, 3), (1, 2, 4), (1, 3, 5), (2, 4, 5), (2, 3, 6), (2, 5, 6)

$$A = \begin{pmatrix} 1 & -cos(\phi_2) & -cos(\phi_1) & cos(\phi_2+\phi_3) & cos(\phi_1+\phi_4) & b_1 \\ -cos(\phi_2) & 1 & cos(\phi_1+\phi_2) & -cos(\phi_3) & -cos(\phi_5) & -cos(\phi_6) \\ -cos(\phi_1) & cos(\phi_1+\phi_2) & 1 & b_2 & -cos(\phi_4) & -cos(\phi_1+\phi_2-\phi_6) \\ cos(\phi_2+\phi_3) & -cos(\phi_3) & b_2 & 1 & cos(\phi_3+\phi_5) & b_3 \\ cos(\phi_1+\phi_4) & -cos(\phi_5) & -cos(\phi_4) & cos(\phi_3+\phi_5) & 1 & cos(\phi_5+\phi_6) \\ b_1 & -cos(\phi_6) & -cos(\phi_1+\phi_2-\phi_6) & b_3 & cos(\phi_5+\phi_6) & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1,u_2,u_3,u_4,u_5,u_6,u_7) = \begin{pmatrix} sin(\phi_1+\phi_2) \\ sin(\phi_1) \\ sin(\phi_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_3) \\ sin(\phi_2+\phi_3) \\ 0 \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_4) \\ 0 \\ sin(\phi_1+\phi_4) \\ 0 \\ sin(\phi_1) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_3+\phi_5) \\ 0 \\ sin(\phi_5) \\ sin(\phi_3) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_1+\phi_2-\phi_6) \\ sin(\phi_6) \\ 0 \\ 0 \\ sin(\phi_1+\phi_2) \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_5+\phi \\ 0 \\ 0 \\ sin(\phi_6) \\ sin(\phi_5) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ b_2 sin(\phi_2) + sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_4) - sin(\phi_1)cos(\phi_5) \\ b_1 sin(\phi_1+\phi_2) - sin(\phi_1+\phi_2)cos(\phi_2-\phi_6) \end{pmatrix}$$
   $$=> \begin{cases} b_2 \geq -cos(\phi_1+\phi_2+\phi_3) \\ cos(\phi_1+\phi_2+\phi_4) \geq cos(\phi_5) \\ b_1 \geq cos(\phi_2-\phi_6) \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ 0 \\ b_2 sin(\phi_2) + sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) \\ 0 \\ sin(\phi_3)cos(\phi_1+\phi_4) - sin(\phi_3)cos(\phi_2-\phi_5) \\ b_3 sin(\phi_2) + b_1 sin(\phi_3) - sin(\phi_2+\phi_3)cos(\phi_6) \end{pmatrix}$$
   $$=> \begin{cases} b_2 \geq -cos(\phi_1+\phi_2+\phi_3) \\ cos(\phi_1+\phi_4) \geq cos(\phi_2-\phi_5) \\ b_3 sin(\phi_2) + b_1 sin(\phi_3) - sin(\phi_2+\phi_3)cos(\phi_6) \geq 0 \end{cases}$$

3. $$Au_3 = \begin{pmatrix} 0 \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_4) - sin(\phi_1)cos(\phi_5) \\ 0 \\ b_2 sin(\phi_1+\phi_4) + sin(\phi_4)cos(\phi_2+\phi_3) + sin(\phi_1)cos(\phi_3+\phi_5) \\ 0 \\ b_1 sin(\phi_4) + sin(\phi_1)cos(\phi_5+\phi_6) - sin(\phi_1+\phi_4)cos(\phi_1+\phi_2-\phi_6) \end{pmatrix}$$
   $$=> \begin{cases} cos(\phi_1+\phi_2+\phi_4) \geq cos(\phi_5) \\ b_2 sin(\phi_1+\phi_4) + sin(\phi_4)cos(\phi_2+\phi_3) + sin(\phi_1)cos(\phi_3+\phi_5) \geq 0 \\ b_1 sin(\phi_4) + sin(\phi_1)cos(\phi_5+\phi_6) - sin(\phi_1+\phi_4)cos(\phi_1+\phi_2-\phi_6) \geq 0 \end{cases}$$

4. $$Au_4 = \begin{pmatrix} sin(\phi_3)cos(\phi_1+\phi_4) - sin(\phi_3)cos(\phi_2-\phi_5) \\ 0 \\ -sin(\phi_3)cos(\phi_4) + b_2 sin(\phi_5) + sin(\phi_3+\phi_5)cos(\phi_1+\phi_2) \\ 0 \\ 0 \\ b_3 sin(\phi_5) - sin(\phi_5)cos(\phi_3-\phi_6) \end{pmatrix}$$
   $$=> \begin{cases} cos(\phi_1+\phi_4) \geq cos(\phi_2-\phi_5) \\ -sin(\phi_3)cos(\phi_4) + b_2 sin(\phi_5) + sin(\phi_3+\phi_5)cos(\phi_1+\phi_2) \geq 0 \\ b_3 \geq cos(\phi_3-\phi_6) \end{cases}$$

5. $Au_5 = \begin{pmatrix} b_1 sin(\phi_1+\phi_2) - sin(\phi_1+\phi_2)cos(\phi_2-\phi_6) \\ 0 \\ 0 \\ b_3 sin(\phi_1+\phi_2) + b_2 sin(\phi_6) - sin(\phi_3)cos(\phi_1+\phi_2-\phi_6) \\ -sin(\phi_6)cos(\phi_4) + sin(\phi_6)cos(\phi_1+\phi_2+\phi_5) \\ 0 \end{pmatrix} =>$

$$\begin{cases} b_1 \geq cos(\phi_2-\phi_6) \\ b_3 sin(\phi_1+\phi_2) + b_2 sin(\phi_6) - sin(\phi_1+\phi_2-\phi_6)cos(\phi_3) \geq 0 \\ cos(\phi_1+\phi_2+\phi_5) \geq cos(\phi_4) \end{cases}$$

6. $Au_6 = \begin{pmatrix} b_1 sin(\phi_5) + sin(\phi_6)cos(\phi_1+\phi_4) - sin(\phi_5+\phi_6)cos(\phi_2) \\ 0 \\ -sin(\phi_6)cos(\phi_4) + sin(\phi_6)cos(\phi_1+\phi_2+\phi_5) \\ b_3 sin(\phi_5) - sin(\phi_5)cos(\phi_3-\phi_6) \\ 0 \\ 0 \end{pmatrix} =>$

$$\begin{cases} b_1 sin(\phi_5) + sin(\phi_6)cos(\phi_1+\phi_4) - sin(\phi_5+\phi_6)cos(\phi_2) \geq 0 \\ cos(\phi_1+\phi_2+\phi_5) \geq cos(\phi_4) \\ b_3 \geq cos(\phi_3-\phi_6) \end{cases}$$

Consider inequalities on ϕ_i :
From the minimal zeros : $\phi_1 + \phi_2 \geq \phi_6$
inequality 4.1: $\phi_1 + \phi_4 \leq |\phi_2 - \phi_5|$
inequality 1.2: if $\phi_1 + \phi_2 + \phi_4 > \pi => 2\pi \leq \phi_1 + \phi_2 + \phi_4 + \phi_5$ - doesn't hold because of previous one if $\phi_1 + \phi_2 + \phi_4 \leq \pi => \phi_1 + \phi_2 + \phi_4 \leq \phi_5$
inequality 6.2: if $\phi_1 + \phi_2 + \phi_5 > \pi => 2\pi \leq \phi_1 + \phi_2 + \phi_4 + \phi_5$ - doesn't hold because of inequality 4.1 if $\phi_1 + \phi_2 + \phi_5 \leq \pi => \phi_1 + \phi_2 + \phi_5 \leq \phi_4$
As result, we get from 1.2 $\phi_5 > \phi_4$ and from 6.2 $\phi_4 > \phi_5$.

Result

There aren't any copositive matrices with such minimal zeros set.

References

[1] Peter J.C. Dickinson. *A new certificate for copositivity. Optimization Online.*

[2] Peter J.C. Dickinson and Roland Hildebrand. *Considering copositivity locally.* J.Math. Anal.Appl., 437(2):1184–1195, 2016

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