Case 24

The minimal zero supports are given by (1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 6), (3, 4, 6), (3, 5, 6)

$$A = \begin{pmatrix} 1 & -cos(\phi_2) & -cos(\phi_1) & cos(\phi_2+\phi_3) & cos(\phi_2+\phi_4) & cos(\phi_1+\phi_5) \\ -cos(\phi_2) & 1 & cos(\phi_1+\phi_2) & -cos(\phi_3) & -cos(\phi_4) & b_1 \\ -cos(\phi_1) & cos(\phi_1+\phi_2) & 1 & cos(\phi_5+\phi_6) & cos(\phi_5+\phi_7) & -cos(\phi_5) \\ cos(\phi_2+\phi_3) & -cos(\phi_3) & cos(\phi_5+\phi_6) & 1 & b_2 & -cos(\phi_6) \\ cos(\phi_2+\phi_4) & -cos(\phi_4) & cos(\phi_5+\phi_7) & b_2 & 1 & -cos(\phi_7) \\ cos(\phi_1+\phi_5) & b_1 & -cos(\phi_5) & -cos(\phi_6) & -cos(\phi_7) & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1,u_2,u_3,u_4,u_5,u_6,u_7) = \begin{pmatrix} sin(\phi_1+\phi_2) \\ sin(\phi_1) \\ sin(\phi_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_3) \\ sin(\phi_2+\phi_3) \\ 0 \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_4) \\ sin(\phi_2+\phi_4) \\ 0 \\ 0 \\ sin(\phi_2) \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_5) \\ 0 \\ sin(\phi_1+\phi_5) \\ 0 \\ 0 \\ sin(\phi_1) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_6) \\ sin(\phi_5) \\ 0 \\ sin(\phi_5+\phi_6) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_7) \\ 0 \\ sin(\phi_5) \\ sin(\phi_5+\phi_7) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) + sin(\phi_2)cos(\phi_5+\phi_6) \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_2)cos(\phi_5+\phi_7) \\ b_1 sin(\phi_1) + sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) \end{pmatrix} => \begin{cases} cos(\phi_1+\phi_2+\phi_3) \geq -cos(\phi_5+\phi_6) \\ cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_5+\phi_7) \\ b_1 \geq -cos(\phi_1+\phi_2+\phi_5) \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) + sin(\phi_2)cos(\phi_5+\phi_6) \\ 0 \\ b_2 sin(\phi_2) - sin(\phi_2)cos(\phi_3-\phi_4) \\ b_1 sin(\phi_2+\phi_3) + sin(\phi_3)cos(\phi_1+\phi_5) - sin(\phi_2)cos(\phi_6) \end{pmatrix}$$
$$=> \begin{cases} cos(\phi_1+\phi_2+\phi_3) \geq -cos(\phi_5+\phi_6) \\ b_2 \geq cos(\phi_3-\phi_4) \\ b_1 sin(\phi_2+\phi_3) + sin(\phi_3)cos(\phi_1+\phi_5) - sin(\phi_2)cos(\phi_6) \geq 0 \end{cases}$$

3. $$Au_3 = \begin{pmatrix} 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_2)cos(\phi_5+\phi_7) \\ b_2 sin(\phi_2) - sin(\phi_2)cos(\phi_3-\phi_4) \\ 0 \\ b_1 sin(\phi_2+\phi_4) + sin(\phi_4)cos(\phi_1+\phi_5) - sin(\phi_2)cos(\phi_7) \end{pmatrix}$$
$$=> \begin{cases} cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_5+\phi_7) \\ b_2 \geq cos(\phi_3-\phi_4) \\ b_1 sin(\phi_2+\phi_4) + sin(\phi_4)cos(\phi_1+\phi_5) - sin(\phi_2)cos(\phi_7) \geq 0 \end{cases}$$

4. $$Au_4 = \begin{pmatrix} 0 \\ b_1 sin(\phi_1) + sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) \\ 0 \\ sin(\phi_5)cos(\phi_2+\phi_3) + sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ sin(\phi_5)cos(\phi_2+\phi_4) + sin(\phi_5)cos(\phi_1+\phi_5+\phi_7) \\ 0 \end{pmatrix} => \begin{cases} b_1 \geq -cos(\phi_1+\phi_2+\phi_5) \\ cos(\phi_1+\phi_5+\phi_6) \geq -cos(\phi_2+\phi_3) \\ cos(\phi_1+\phi_5+\phi_7) \geq -cos(\phi_2+\phi_4) \end{cases}$$

5. $$Au_5 = \begin{pmatrix} sin(\phi_5)cos(\phi_2+\phi_3) + sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ -sin(\phi_5)cos(\phi_3) + sin(\phi_6)cos(\phi_1+\phi_2) + b_1 sin(\phi_5+\phi_6) \\ 0 \\ 0 \\ b_2 sin(\phi_5) - sin(\phi_5)cos(\phi_6-\phi_7) \\ 0 \end{pmatrix} =>$$

$$\begin{cases} cos(\phi_1+\phi_5+\phi_6) \geq -cos(\phi_2+\phi_3) \\ -sin(\phi_5)cos(\phi_3)+sin(\phi_6)cos(\phi_1+\phi_2)+b_1 sin(\phi_5+\phi_6) \geq 0 \\ b_2 \geq cos(\phi_6-\phi_7) \end{cases}$$

6. $Au_6 = \begin{pmatrix} sin(\phi_5)cos(\phi_2+\phi_4)+sin(\phi_5)cos(\phi_1+\phi_5+\phi_7) \\ -sin(\phi_5)cos(\phi_4)+sin(\phi_7)cos(\phi_1+\phi_2)+b_1 sin(\phi_5+\phi_7) \\ 0 \\ b_2 sin(\phi_5)-sin(\phi_5)cos(\phi_6-\phi_7) \\ 0 \\ 0 \end{pmatrix} =>$

$$\begin{cases} cos(\phi_1+\phi_5+\phi_7) \geq -cos(\phi_2+\phi_4) \\ -sin(\phi_5)cos(\phi_4)+sin(\phi_7)cos(\phi_1+\phi_2)+b_1 sin(\phi_5+\phi_7) \geq 0 \\ b_2 \geq cos(\phi_6-\phi_7) \end{cases}$$

System of inequalities on ϕ_i: $\begin{cases} \phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \leq \pi \\ \phi_1+\phi_2+\phi_4+\phi_5+\phi_7 \leq \pi \end{cases}$

Copositivity:

Sets $(1,3,4),(2,3,4),(1,3,5),(2,3,5),(1,4,5),(2,4,5),(3,4,5),(1,3,4,5),(2,3,4,5),(1,2,6),$ $(2,3,6),(1,4,6),(2,4,6),(1,5,6),(2,5,6),(4,5,6),(1,4,5,6),(2,4,5,6)$ have to be considered.

1. $I=(1,3,4): \pi \geq -\phi_1+\phi_2+\phi_3+\phi_5+\phi_6$ performs strictly
2. $I=(2,3,4): \pi \geq \phi_1+\phi_2-\phi_3+\phi_5+\phi_6$ performs strictly
3. $I=(1,3,5): \pi \geq -\phi_1+\phi_2+\phi_4+\phi_5+\phi_7$ performs strictly
4. $I=(2,3,5): \pi \geq \phi_1+\phi_2-\phi_4+\phi_5+\phi_7$ performs strictly
5. $I=(1,4,6): \pi \geq \phi_1+\phi_2+\phi_3+\phi_5-\phi_6$ performs strictly
6. $I=(1,5,6): \pi \geq \phi_1+\phi_2+\phi_4+\phi_5-\phi_7$ performs strictly

Consider sets, that relates to b_1 :

1. $I=(2,4,6): b_1 = cos(\phi) => b_1 \geq cos(\phi_3+\phi_6)$
2. $I=(2,5,6): b_1 = cos(\phi) => b_1 \geq cos(\phi_4+\phi_7)$

If $b_1 = -cos(\phi_1+\phi_2+\phi_5) \geq cos(\phi_3+\phi_6)$ from inequality 1.3 and 4.1 $\phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \geq \pi$ - hold only if equation.

If $b_1 = \frac{sin(\phi_2)cos(\phi_6)-sin(\phi_3)cos(\phi_1+\phi_5)}{sin(\phi_2+\phi_3)} \geq cos(\phi_3+\phi_6)$ from inequality 2.3 $-cos(\phi_2+\phi_3+\phi_6) \geq cos(\phi_1+\phi_5) => \phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \geq \pi$ - hold only if equation.

If $b_1 = \frac{sin(\phi_2)cos(\phi_7)-sin(\phi_4)cos(\phi_1+\phi_5)}{sin(\phi_2+\phi_4)} \geq cos(\phi_4+\phi_7)$ from inequality 3.3 $-cos(\phi_2+\phi_4+\phi_7) \geq cos(\phi_1+\phi_5) => \phi_1+\phi_2+\phi_4+\phi_5+\phi_7 \geq \pi$ - hold only if equation.

If $b_1 = \frac{sin(\phi_5)cos(\phi_3)-sin(\phi_6)cos(\phi_1+\phi_2)}{sin(\phi_5+\phi_6)} \geq cos(\phi_3+\phi_6)$ from inequality 5.2 $-cos(\phi_3+\phi_5+\phi_6) \geq cos(\phi_1+\phi_2) => \phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \geq \pi$ - hold only if equation.

If $b_1 = \frac{sin(\phi_5)cos(\phi_4)-sin(\phi_7)cos(\phi_1+\phi_2)}{sin(\phi_5+\phi_7)} \geq cos(\phi_4+\phi_7)$ from inequality 6.2 $-cos(\phi_4+\phi_5+\phi_7) \geq cos(\phi_1+\phi_2) => \phi_1+\phi_2+\phi_4+\phi_5+\phi_7 \geq \pi$ - hold only if equation.

As result, we get, that there is no copositivity.

Result

There aren't any copositive matrices with such minimal zeros set.

References

[1] Peter J.C. Dickinson. *A new certificate for copositivity. Optimization Online.*

[2] Peter J.C. Dickinson and Roland Hildebrand. *Considering copositivity locally.* J.Math. Anal.Appl., 437(2):1184–1195, 2016

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