Case 22

The minimal zero supports are given by (1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 6), (2, 4, 6), (3, 5, 6)

$$A = \begin{pmatrix} 1 & -cos(\phi_2) & -cos(\phi_1) & cos(\phi_2+\phi_3) & cos(\phi_2+\phi_4) & cos(\phi_1+\phi_5) \\ -cos(\phi_2) & 1 & cos(\phi_1+\phi_2) & -cos(\phi_3) & -cos(\phi_4) & cos(\phi_3+\phi_6) \\ -cos(\phi_1) & cos(\phi_1+\phi_2) & 1 & b_1 & cos(\phi_5+\phi_7) & -cos(\phi_5) \\ cos(\phi_2+\phi_3) & -cos(\phi_3) & b_1 & 1 & b_2 & -cos(\phi_6) \\ cos(\phi_2+\phi_4) & -cos(\phi_4) & cos(\phi_5+\phi_7) & b_2 & 1 & -cos(\phi_7) \\ cos(\phi_1+\phi_5) & cos(\phi_3+\phi_6) & -cos(\phi_5) & -cos(\phi_6) & -cos(\phi_7) & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1,u_2,u_3,u_4,u_5,u_6,u_7) = \begin{pmatrix} sin(\phi_1+\phi_2) \\ sin(\phi_1) \\ sin(\phi_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_3) \\ sin(\phi_2+\phi_3) \\ 0 \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_4) \\ sin(\phi_2+\phi_4) \\ 0 \\ 0 \\ sin(\phi_2) \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_5) \\ 0 \\ sin(\phi_1+\phi_5) \\ 0 \\ 0 \\ sin(\phi_1) \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_6) \\ 0 \\ sin(\phi_3+\phi_6) \\ 0 \\ sin(\phi_3) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_7) \\ 0 \\ sin(\phi_5) \\ sin(\phi_5+\phi_7) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ b_1 sin(\phi_2) + sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_2)cos(\phi_5+\phi_7) \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) + sin(\phi_1)cos(\phi_3+\phi_6) \end{pmatrix} => \begin{cases} b_1 \geq -cos(\phi_1+\phi_2+\phi_3) \\ cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_5+\phi_7) \\ cos(\phi_1+\phi_2+\phi_5) \geq -cos(\phi_3+\phi_6) \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ 0 \\ b_1 sin(\phi_2) + sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) \\ 0 \\ b_2 sin(\phi_2) - sin(\phi_2)cos(\phi_3-\phi_4) \\ sin(\phi_3)cos(\phi_1+\phi_5) + sin(\phi_3)cos(\phi_2+\phi_3+\phi_6) \end{pmatrix} => \begin{cases} b_1 \geq -cos(\phi_1+\phi_2+\phi_3) \\ b_2 \geq cos(\phi_3-\phi_4) \\ cos(\phi_2+\phi_3+\phi_6) \geq -cos(\phi_1+\phi_5) \end{cases}$$

3. $$Au_3 = \begin{pmatrix} 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_2)cos(\phi_5+\phi_7) \\ b_2 sin(\phi_2) - sin(\phi_2)cos(\phi_3-\phi_4) \\ 0 \\ sin(\phi_4)cos(\phi_1+\phi_5) + sin(\phi_2+\phi_4)cos(\phi_3+\phi_6) - sin(\phi_2)cos(\phi_7) \end{pmatrix}$$
$$=> \begin{cases} cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_5+\phi_7) \\ b_2 \geq cos(\phi_3-\phi_4) \\ sin(\phi_4)cos(\phi_1+\phi_5) + sin(\phi_2+\phi_4)cos(\phi_3+\phi_6) - sin(\phi_2)cos(\phi_7) \geq 0 \end{cases}$$

4. $$Au_4 = \begin{pmatrix} 0 \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) + sin(\phi_1)cos(\phi_3+\phi_6) \\ 0 \\ b_1 sin(\phi_1+\phi_5) + sin(\phi_5)cos(\phi_2+\phi_3) - sin(\phi_1)cos(\phi_6) \\ sin(\phi_5)cos(\phi_2+\phi_4) + sin(\phi_5)cos(\phi_1+\phi_5+\phi_7) \\ 0 \end{pmatrix} => \begin{cases} cos(\phi_1+\phi_2+\phi_5) \geq -cos(\phi_3+\phi_6) \\ b_1 sin(\phi_1+\phi_5) + sin(\phi_5)cos(\phi_2+\phi_3) - sin(\phi_1)cos(\phi \\ cos(\phi_1+\phi_5+\phi_7) \geq -cos(\phi_2+\phi_4) \end{cases}$$

5. $$Au_5 = \begin{pmatrix} sin(\phi_3)cos(\phi_1+\phi_5) + sin(\phi_3)cos(\phi_2+\phi_3+\phi_6) \\ 0 \\ -sin(\phi_3)cos(\phi_5) + sin(\phi_6)cos(\phi_1+\phi_2) + b_1 sin(\phi_3+\phi_6) \\ 0 \\ b_2 sin(\phi_3+\phi_6) - sin(\phi_6)cos(\phi_4) - sin(\phi_3)cos(\phi_7) \\ 0 \end{pmatrix} =>$$
$$\begin{cases} cos(\phi_2+\phi_3+\phi_6) \geq -cos(\phi_1+\phi_5) \\ -sin(\phi_3)cos(\phi_5) + sin(\phi_6)cos(\phi_1+\phi_2) + b_1 sin(\phi_3+\phi_6) \geq 0 \\ b_2 sin(\phi_3+\phi_6) - sin(\phi_6)cos(\phi_4) - sin(\phi_3)cos(\phi_7) \geq 0 \end{cases}$$

6. $Au_6 = \begin{pmatrix} sin(\phi_5)cos(\phi_2+\phi_4)+sin(\phi_5)cos(\phi_1+\phi_5+\phi_7) \\ -sin(\phi_5)cos(\phi_4)+sin(\phi_7)cos(\phi_1+\phi_2)+sin(\phi_5+\phi_7)cos(\phi_3+\phi_6) \\ 0 \\ b_2sin(\phi_5)+b_1sin(\phi_7)-sin(\phi_5+\phi_7)cos(\phi_6) \\ 0 \\ 0 \end{pmatrix} =>$

$$\begin{cases} cos(\phi_1+\phi_5+\phi_7) \geq -cos(\phi_2+\phi_4) \\ -sin(\phi_5)cos(\phi_4)+sin(\phi_7)cos(\phi_1+\phi_2)+sin(\phi_5+\phi_7)cos(\phi_3+\phi_6) \geq 0 \\ b_2sin(\phi_5)+b_1sin(\phi_7)-sin(\phi_5+\phi_7)cos(\phi_6) \geq 0 \end{cases}$$

System of inequalities on ϕ_i: $\begin{cases} \phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \leq \pi \\ \phi_1+\phi_2+\phi_4+\phi_5+\phi_7 \leq \pi \end{cases}$

Copositivity:

Sets $(1,3,4),(2,3,4),(1,3,5),(2,3,5),(1,4,5),(2,4,5),(3,4,5),(1,3,4,5),(2,3,4,5),(1,2,6),$ $(2,3,6),(1,4,6),(3,4,6),(1,5,6),(2,5,6),(4,5,6),(1,4,5,6)$ have to be considered.

1. $I=(1,3,5): \phi_1+\pi-\phi_2-\phi_4-\phi_5-\phi_7 \geq 0$ holds true
2. $I=(2,3,5): \phi_4+\pi-\phi_2-\phi_1-\phi_5-\phi_7 \geq 0$ holds true
3. $I=(1,2,6): \phi_2+\pi-\phi_1-\phi_5-\phi_3-\phi_6 \geq 0$ holds true
4. $I=(2,3,6): \phi_5+\pi-\phi_2-\phi_1-\phi_3-\phi_6 \geq 0$ holds true
5. $I=(1,4,6): \phi_7+\pi-\phi_2-\phi_6-\phi_5-\phi_1 \geq 0$ holds true
6. $I=(1,5,6): \phi_7+\pi-\phi_2-\phi_6-\phi_5-\phi_1 \geq 0$ holds true
7. $I=(2,5,6): \phi_4+\phi_7 \geq \phi_3+\phi_6$
8. $I=(1,3,4): b_1 \geq -cos(\phi_1+\phi_2+\phi_3), \phi_1-\phi_2-\phi_3+\phi_1+\phi_2+\phi_3 \geq 0$ holds true
9. $I=(2,3,4): b_1 \geq -cos(\phi_1+\phi_2+\phi_3), \phi_3-\phi_2-\phi_1+\phi_1+\phi_2+\phi_3 \geq 0$ holds true
10. $I=(1,4,5): b_2 \geq cos(\phi_3-\phi_4), \pi-\phi_3-\phi_2-|\phi_3-\phi_4|+\pi-\phi_2-\phi_4 \geq 0$ holds true
11. $I=(2,4,5): b_2 \geq cos(\phi_3-\phi_4), \phi_3+\phi_4-|\phi_3-\phi_4| \geq 0$ holds true
12. $I=(3,4,5): b_1 \geq -cos(\phi_1+\phi_2+\phi_3), b_2 \geq cos(\phi_3-\phi_4), \pi-\phi_5-\phi_7-|\phi_3-\phi_4|+\phi_1+\phi_2+\phi_3 \geq 0$
    if $\phi_3 \geq \phi_4 => \pi-\phi_5-\phi_7+\phi_4+\phi_1+\phi_2 \geq 0$ holds true
    if $\phi_4 \geq \phi_3 => \pi-\phi_5-\phi_7+\phi_3-\phi_4+\phi_1+\phi_2+\phi_3 \geq 0, \pi-\phi_5-\phi_7-\phi_4 > 0$
    holds true
13. $I=(3,4,6): b_1 = cos(\phi), \phi_5+\phi_6 \geq \phi => b_1 = cos(\phi) \geq cos(\phi_5+\phi_6)$
14. $I=(4,5,6): b_2 = cos(\phi), \phi_7+\phi_6 \geq \phi =>$ if $\phi_7+\phi_6 \geq \pi, \phi_7+\phi_6 \geq \phi$ - holds true
    if $\phi_7+\phi_6 \leq \pi => b_2 = cos(\phi) \geq cos(\phi_7+\phi_6)$

1. If $b_1 = -cos(\phi_1+\phi_2+\phi_3) \geq cos(\phi_5+\phi_6) => \phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \geq \pi$ - holds only if it's an equation
2. If $b_1 = \frac{sin(\phi_1)cos(\phi_6)-sin(\phi_5)cos(\phi_2+\phi_3)}{sin(\phi_1+\phi_5)} \geq cos(\phi_5+\phi_6)$ from inequality 4.2
    $-cos(\phi_2+\phi_3) \geq cos(\phi_1+\phi_5+\phi_6) => \phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \geq \pi$ - holds only if it's an equation
3. If $b_1 = \frac{sin(\phi_3)cos(\phi_5)-sin(\phi_6)sin(\phi_1+\phi_2)}{sin(\phi_3+\phi_6)} \geq cos(\phi_5+\phi_6)$ from inequality 5.2
    $-cos(\phi_3+\phi_5+\phi_6) \geq cos(\phi_1+\phi_2) => \phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \geq \pi$ - holds only if it's an equation
4. If $b_1 = \frac{sin(\phi_5+\phi_7)cos(\phi_6)-b_2sin(\phi_5)}{sin(\phi_7)} \geq cos(\phi_5+\phi_6)$ from inequality 6.3
    $cos(\phi_6-\phi_7) \geq b_2$

$b_2 = cos(\phi_3 - \phi_4), \phi_7 + \phi_3 + \phi_6 \geq \phi_4$
$b_2 = \frac{sin(\phi_3)cos(\phi_7)+cos(\phi_4)sin(\phi_6)}{sin(\phi_3+\phi_6)}, \phi_4 \geq \phi_7 + \phi_3 + \phi_6 => \phi_6 + \phi_7 \leq \pi, b_2 \geq cos(\phi_6 + \phi_7) => cos(\phi_4) > cos(\phi_3 + \phi_6 + \phi_7) => \phi_4 \leq \phi_3 + \phi_6 + \phi_7$-contradiction
$= b_2 = cos(\phi_3 - \phi_4), \phi_7 + \phi_3 + \phi_6 > \phi_4$ $cos(\phi_6 - \phi_7) \geq b_2 \geq cos(\phi_3 - \phi_4) => |\phi_3 - \phi_4| \geq |\phi_6 - \phi_7|$
$cos(\phi_6 - \phi_7) \geq b_2 \geq \frac{sin(\phi_3)cos(\phi_7)+cos(\phi_4)sin(\phi_6)}{sin(\phi_3+\phi_6)} => \phi_4 \geq |\phi_3 + \phi_6 - \phi_7|$
Consider domain of ϕ_i under gotten conditions:

1. $\phi_3 \geq \phi_4, \phi_6 \geq \phi_7 => \begin{cases} \phi_4 + \phi_7 \geq \phi_3 + \phi_6 \\ \phi_3 + \phi_7 \geq \phi_4 + \phi_6 \end{cases} => \phi_7 \geq \phi_6$ - contradiction
2. $\phi_4 \geq \phi_3, \phi_6 \geq \phi_7 => \phi_4 + \phi_7 \geq \phi_3 + \phi_6$
3. $\phi_3 \geq \phi_4, \phi_7 \geq \phi_6 => \begin{cases} \phi_4 + \phi_7 \geq \phi_3 + \phi_6 \\ \phi_3 + \phi_6 \geq \phi_7 + \phi_4 \end{cases} => \phi_4 + \phi_7 = \phi_3 + \phi_6$ - additional minimal zero (2, 5, 6)
4. $\phi_4 \geq \phi_3, \phi_7 \geq \phi_6 => \begin{cases} \phi_4 + \phi_6 \geq \phi_3 + \phi_7 \\ \phi_7 + \phi_4 \geq \phi_3 + \phi_6 \\ \phi_4 \geq |\phi_3 + \phi_6 - \phi_7| \end{cases} => \phi_3 + \phi_6 \geq \phi_7 : \begin{cases} \phi_4 + \phi_6 \geq \phi_3 + \phi_7 \\ \phi_7 + \phi_4 \geq \phi_3 + \phi_6 \\ \phi_3 + \phi_6 \geq \phi_7 \end{cases}$

   $\phi_7 \geq \phi_3 + \phi_6 : \begin{cases} \phi_4 + \phi_6 \geq \phi_3 + \phi_7 \\ \phi_7 \geq \phi_3 + \phi_6 \end{cases}$

But, if $\phi_4 + \phi_6 = \phi_3 + \phi_7 => b_1 = cos(\phi_5 + \phi_6) =>$ additional minimal zero (3, 4, 6) Finally, consider 4-elem sets:

1. $I = (1, 3, 4, 5) : u = e_1 + e_3$
2. $I = (2, 3, 4, 5) : u = e_2 + e_3$
3. $I = (1, 4, 5, 6) : u = u = sin(\phi_4 - \phi_3)e_1 - sin(\phi_2 + \phi_4)e_4 + sin(\phi_2 + \phi_3)e_5 =>$

   $$Au = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a \end{pmatrix},$$

   $a = sin(\phi_4 - \phi_3)cos(\phi_1 + \phi_5) + sin(\phi_2 + \phi_4)cos(\phi_6) - cos(\phi_7)sin(\phi_2 + \phi_3),$
   $\phi_7 \geq |\phi_3 + \phi_6 - \phi_4| => -cos(\phi_7) \geq -cos(\phi_3 + \phi_6 - \phi_4) => a \geq sin(\phi_4 - \phi_3)cos(\phi_1 + \phi_5) + sin(\phi_2 + \phi_4)cos(\phi_6) - cos(\phi_3 + \phi_6 - \phi_4)sin(\phi_2 + \phi_3) = sin(\phi_4 - \phi_3)[cos(\phi_1 + \phi_5) + cos(\phi_2 + \phi_3 + \phi_6)] \geq 0,$
   $\phi_1 + \phi_2 + \phi_3 + \phi_5 + \phi_6 \leq \pi$

$b_1 = \frac{sin(\phi_5+\phi_7)cos(\phi_6)-cos(\phi_3-\phi_4)sin(\phi_5)}{sin(\phi_7)}, b_2 = cos(\phi_3 - \phi_4)$

Extremality

$X = FPF^T$

$$X = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & * \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & * & * & * \\ b_{41} & b_{42} & * & b_{44} & * & b_{46} \\ b_{51} & b_{52} & * & * & b_{55} & * \\ * & b_{62} & * & b_{64} & * & b_{66} \end{pmatrix}, F = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{pmatrix} F = \begin{pmatrix} sin(\phi_1) & 0 \\ -sin(\phi_1 + \phi_2) & -sin(\phi_2) \\ 0 & sin(\phi_1) \\ sin(\phi_1 + \phi_2 + \phi_3) & sin(\phi_2 + \phi_3) \\ sin(\phi_1 + \phi_2 + \phi_4) & sin(\phi_2 + \phi_4) \\ -sin(\phi_1 + \phi_2 + \phi_3 + \phi_6) & -sin(\phi_2 + \phi_3 + \phi_6) \end{pmatrix}$$

$Y = GQG^T$

$$Y = \begin{pmatrix} b_{11} & * & b_{13} & * & * & b_{16} \\ * & * & * & * & * & * \\ b_{31} & * & b_{33} & * & b_{35} & b_{36} \\ * & * & * & * & * & * \\ * & * & b_{53} & * & b_{55} & b_{56} \\ b_{61} & * & b_{63} & * & b_{65} & b_{66} \end{pmatrix}, G = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \end{pmatrix} G = \begin{pmatrix} sin(\phi_1) & 0 \\ 0 & 0 \\ 0 & sin(\phi_1) \\ 0 & 0 \\ sin(\phi_5 + \phi_7) & sin(\phi_1 + \phi_5 + \phi_7) \\ -sin(\phi_5) & -sin(\phi_1 + \phi_5) \end{pmatrix}$$

Necessary equalities, that are constrains on elements of matrices P and Q:

$$\begin{cases} x_{11} = y_{11} \\ x_{31} = y_{31} \\ x_{33} = y_{33} \\ x_{55} = y_{55} \\ x_{66} = y_{66} \end{cases}$$

$$=> \begin{cases} p_{11} = q_{11} \\ p_{12} = q_{12} \\ p_{22} = q_{22} \\ [sin^2(\phi_2 + \phi_{24}) - sin^2(\phi_{57})]p_{11} + 2[sin(\phi_1 + \phi_{24})sin(\phi_{24}) - sin(\phi_1 + \phi_{57})sin(\phi_{57})]p_{12} + [sin^2(\phi_{24}) - sin^2(\phi_1 + \phi_{57})]p_{22} = 0 \\ [sin^2(\phi_1 + \phi_{236}) - sin^2(\phi_5)]p_{11} + 2[sin(\phi_1 + \phi_{236})sin(\phi_{236}) - sin(\phi_1 + \phi_5)sin(\phi_5)]p_{12} + [sin^2(\phi_{236}) - sin^2(\phi_1 + \phi_5)]p_{22} = 0 \end{cases}$$

Consider 2 last equations of above system: $\begin{cases} sin(\phi_1 + \phi_{24} + \phi_{57}) = 0 \\ sin(\phi_1 + \phi_{236} + \phi_5) = 0 \end{cases}$ or

$$\begin{cases} sin(\phi_1 + \phi_{236} - \phi_5)p_{11} + 2sin(\phi_{236} - \phi_5)p_{12} + sin(-\phi_1 + \phi_{236} - \phi_5)p_{22} = 0| * sin(\phi_1 + \phi_{24} - \phi_{57}) \\ sin(\phi_1 + \phi_{24} - \phi_{57})p_{11} + 2sin(\phi_{24} - \phi_{57})p_{12} + sin(-\phi_1 + \phi_{24} - \phi_{57})p_{22} = 0| * sin(\phi_1 + \phi_{236} - \phi_5) \end{cases}$$

$$\begin{cases} sin(\phi_{236} - \phi_5)sin(\phi_1 + \phi_{24} - \phi_{57}) = sin(\phi_{24} - \phi_{57})sin(\phi_1 + \phi_{236} - \phi_5) \\ sin(\phi_1 + \phi_{236} - \phi_5)sin(-\phi_1 + \phi_{24} - \phi_{57}) = sin(-\phi_1 + \phi_{236} - \phi_5)sin(\phi_1 + \phi_{24} - \phi_{57}) \end{cases} =>$$

$$\begin{cases} sin(\phi_1)sin(\phi_5 - \phi_{57} - \phi_{236} + \phi_{24}) = 0 \\ sin(2\phi_1)sin(\phi_5 - \phi_{57} - \phi_{236} + \phi_{24}) = 0 \end{cases} => sin(\phi_5 - \phi_{57} - \phi_{236} + \phi_{24}) = 0$$

As result, there is no extremality, when : $\phi_1 + \phi_2 + \phi_3 + \phi_5 + \phi_6 = \pi$ or $\phi_1 + \phi_2 + \phi_4 + \phi_5 + \phi_7 = \pi$ or $\phi_7 + \phi_3 + \phi_6 = \phi_4$

But, it is already known, that $\phi_7 + \phi_3 + \phi_6 > \phi_4$ and $\phi_4 + \phi_7 > \phi_3 + \phi_6 => \phi_1 + \phi_2 + \phi_3 + \phi_5 + \phi_6 < \pi$

It follows, that one remaining case should be considered : $\phi_1 + \phi_2 + \phi_4 + \phi_5 + \phi_7 = \pi$

$\phi_1 + \phi_2 + \phi_4 + \phi_5 + \phi_7 = \pi$:

Additional equalities :

$$\begin{cases} (Au_1)_5 = 0 \\ (Au_3)_3 = 0 \\ (Au_4)_5 = 0 \\ (Au_6)_1 = 0 \end{cases} => \begin{cases} sin(\phi_1 + \phi_2)b_{51} + sin(\phi_1)b_{52} + sin(\phi_2)b_{53} = 0 \\ sin(\phi_4)b_{31} + sin(\phi_2 + \phi_4)b_{32} + sin(\phi_2)b_{35} = 0 \\ sin(\phi_1 + \phi_2 + \phi_4 + \phi_7)b_{51} + sin(\phi_2 + \phi_4 + \phi_7)b_{53} + sin(\phi_1)b_{56} = 0 \\ sin(\phi_7)b_{13} + sin(\phi_1 + \phi_2 + \phi_4 + \phi_7)b_{15} + sin(\phi_1 + \phi_2 + \phi_4)b_{16} = 0 \end{cases}$$

$$F = \begin{pmatrix} sin(\phi_1) & 0 \\ -sin(\phi_1 + \phi_2) & -sin(\phi_2) \\ 0 & sin(\phi_1) \\ sin(\phi_1 + \phi_2 + \phi_3) & sin(\phi_2 + \phi_3) \\ sin(\phi_1 + \phi_2 + \phi_4) & sin(\phi_2 + \phi_4) \\ -sin(\phi_1 + \phi_2 + \phi_3 + \phi_6) & -sin(\phi_2 + \phi_3 + \phi_6) \end{pmatrix} G = \begin{pmatrix} sin(\phi_1) & 0 \\ 0 & 0 \\ 0 & sin(\phi_1) \\ 0 & 0 \\ sin(\phi_1 + \phi_2 + \phi_4) & sin(\phi_2 + \phi_4) \\ -sin(\phi_1 + \phi_2 + \phi_4 + \phi_7) & -sin(\phi_2 + \phi_4 + \phi_7) \end{pmatrix}$$

It's not difficult to show, that substituting into the above system b_{ij} using F, G factors every equation becomes identity. As result, there aren't any additional non-dependent equalities and there is no extremality.

Result

There are extremal copositive matrices under conditions on parameters $\phi_i : \phi_1 + \phi_2 + \phi_3 + \phi_5 + \phi_6 < \pi$, $\phi_1 + \phi_2 + \phi_4 + \phi_5 + \phi_7 < \pi, \phi_4 + \phi_7 > \phi_3 + \phi_6, \phi_4 + \phi_6 > \phi_3 + \phi_7, \phi_7 + \phi_3 + \phi_6 > \phi_4$

References

[1] Peter J.C. Dickinson. *A new certificate for copositivity. Optimization Online.*

[2] Peter J.C. Dickinson and Roland Hildebrand. *Considering copositivity locally.* J.Math. Anal.Appl., 437(2):1184–1195, 2016

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