Case 17

The minimal zero supports are given by (1, 2), (1, 3, 4), (2, 3, 5), (3, 4, 5), (2, 4, 6), (3, 4, 6).

$$A = \begin{pmatrix} 1 & -1 & -cos(\phi_2) & cos(\phi_1+\phi_2) & b_1 & b_2 \\ -1 & 1 & cos(\phi_3+\phi_4) & cos(\phi_5+\phi_6) & -cos(\phi_4) & -cos(\phi_6) \\ -cos(\phi_2) & cos(\phi_3+\phi_4) & 1 & -cos(\phi_1) & -cos(\phi_3) & cos(\phi_1+\phi_5) \\ cos(\phi_1+\phi_2) & cos(\phi_5+\phi_6) & -cos(\phi_1) & 1 & cos(\phi_1+\phi_3) & -cos(\phi_5) \\ b_1 & -cos(\phi_4) & -cos(\phi_3) & cos(\phi_1+\phi_3) & 1 & b_3 \\ b_2 & -cos(\phi_6) & cos(\phi_1+\phi_5) & -cos(\phi_5) & b_3 & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1,u_2,u_3,u_4,u_5,u_6) = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_1) \\ 0 \\ sin(\phi_1+\phi_2) \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_3) \\ sin(\phi_4) \\ 0 \\ sin(\phi_3+\phi_4) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_1+\phi_3) \\ sin(\phi_3) \\ sin(\phi_1) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_5) \\ 0 \\ sin(\phi_6) \\ 0 \\ sin(\phi_5+\phi_6) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_5) \\ sin(\phi_1+\phi_5) \\ 0 \\ sin(\phi_1) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ -cos(\phi_2)+cos(\phi_3+\phi_4) \\ cos(\phi_1+\phi_2)+cos(\phi_5+\phi_6) \\ b_1 - cos(\phi_4) \\ b_2 - cos(\phi_6) \end{pmatrix} => \begin{cases} cos(\phi_3+\phi_4) \geq cos(\phi_2) \\ cos(\phi_1+\phi_2)+cos(\phi_5+\phi_6) \geq 0 \\ b_1 \geq cos(\phi_4) \\ b_2 \geq cos(\phi_6) \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ -sin(\phi_1)+sin(\phi_1+\phi_2)cos(\phi_3+\phi_4)+sin(\phi_2)cos(\phi_5+\phi_6) \\ 0 \\ 0 \\ b_1 sin(\phi_1) - sin(\phi_1)cos(\phi_2-\phi_3) \\ b_2 sin(\phi_1)+sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) \end{pmatrix} => \begin{cases} -sin(\phi_1)+sin(\phi_1+\phi_2)cos(\phi_3+\phi_4)+sin(\phi_2 \\ b_1 \geq cos(\phi_2-\phi_3) \\ b_2 \geq -cos(\phi_1+\phi_2+\phi_5) \end{cases}$$

3. $$Au_3 = \begin{pmatrix} -sin(\phi_3)-sin(\phi_4)cos(\phi_2)+b_1 sin(\phi_3+\phi_4) \\ 0 \\ 0 \\ sin(\phi_3)cos(\phi_1+\phi_3+\phi_4)+sin(\phi_3)cos(\phi_5+\phi_6) \\ 0 \\ b_3 sin(\phi_3+\phi_4)+sin(\phi_4)cos(\phi_1+\phi_5)-sin(\phi_3)cos(\phi_6) \end{pmatrix} => \begin{cases} -sin(\phi_3)-sin(\phi_4)cos(\phi_2)+b_1 sin(\phi_3+\phi_4) \geq 0 \\ cos(\phi_1+\phi_3+\phi_4) \geq -cos(\phi_5+\phi_6) \\ b_3 sin(\phi_3+\phi_4)+sin(\phi_4)cos(\phi_1+\phi_5)-sin(\phi_3)cos(\end{cases}$$

4. $$Au_4 = \begin{pmatrix} b_1 sin(\phi_1)-sin(\phi_1)cos(\phi_2-\phi_3) \\ sin(\phi_3)cos(\phi_1+\phi_3+\phi_4)+sin(\phi_3)cos(\phi_5+\phi_6) \\ 0 \\ 0 \\ 0 \\ b_3 sin(\phi_1)+sin(\phi_1)cos(\phi_1+\phi_3+\phi_5) \end{pmatrix} => \begin{cases} b_1 \geq cos(\phi_2-\phi_3) \\ cos(\phi_1+\phi_3+\phi_4) \geq -cos(\phi_5+\phi_6) \\ b_3 \geq -cos(\phi_1+\phi_3+\phi_5) \end{cases}$$

5. $$Au_5 = \begin{pmatrix} -sin(\phi_5)+sin(\phi_6)cos(\phi_1+\phi_2)+b_2 sin(\phi_5+\phi_6) \\ 0 \\ sin(\phi_5)cos(\phi_3+\phi_4)+sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ 0 \\ -sin(\phi_5)cos(\phi_4)+sin(\phi_6)cos(\phi_1+\phi_3)+b_3 sin(\phi_5+\phi_6) \\ 0 \end{pmatrix} => \begin{cases} -sin(\phi_5)+sin(\phi_6)cos(\phi_1+\phi_2)+b_2 sin(\phi_5+\phi_6) \geq \\ cos(\phi_3+\phi_4) \geq -cos(\phi_1+\phi_5+\phi_6) \\ -sin(\phi_5)cos(\phi_4)+sin(\phi_6)cos(\phi_1+\phi_3)+b_3 sin(\phi_5 \end{cases}$$

6. $$Au_6 = \begin{pmatrix} b_2 sin(\phi_1)+sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) \\ sin(\phi_5)cos(\phi_3+\phi_4)+sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ 0 \\ 0 \\ b_3 sin(\phi_1)+sin(\phi_1)cos(\phi_1+\phi_3+\phi_5) \\ 0 \end{pmatrix} => \begin{cases} b_2 \geq -cos(\phi_1+\phi_2+\phi_5) \\ cos(\phi_3+\phi_4) \geq -cos(\phi_1+\phi_5+\phi_6) \\ b_3 \geq -cos(\phi_1+\phi_3+\phi_5) \end{cases}$$

Consider inequalities on phi_i and get:

$$\begin{cases}\phi_3+\phi_4 \leq \phi_2\\ \phi_1+\phi_2+\phi_5+\phi_6 \leq \pi\end{cases}$$

From inequalities on b_i:

$$\begin{cases}b_1 = cos(\phi_4)\\ b_2 = cos(\phi_6)\end{cases}$$

$b_3 = \max(-cos(\phi_1+\phi_3+\phi_5), \frac{sin(\phi_5)cos(\phi_4)-sin(\phi_6)cos(\phi_1+\phi_3)}{sin(\phi_5+\phi_6)}, \frac{-sin(\phi_4)cos(\phi_1+\phi_5)+sin(\phi_3)cos(\phi_6)}{sin(\phi_3+\phi_4)})$

Copositivity: It's necessary to consider next sets for I:

$(2,3,4),(1,3,5),(1,4,5),(2,4,6),(1,3,6),(2,3,6),(1,4,6),(1,5,6),(2,5,6),(3,5,6),(1,3,5,6),(4,5,6),(1,4,5,6)$

Let's consider set (2,5,6): $b_3 = cos(\phi) => \phi_4+\phi_6+\pi-\phi \geq \pi => \phi \leq \phi_4+\phi_6$,

If $b_3 = -cos(\phi_1+\phi_3+\phi_5) \geq cos(\phi_4+\phi_6) => \pi \leq \phi_1+\phi_3+\phi_5+\phi_4+\phi_6$

if $b_3 = \frac{sin(\phi_5)cos(\phi_4)-sin(\phi_6)cos(\phi_1+\phi_3)}{sin(\phi_5+\phi_6)} \geq cos(\phi_4+\phi_6) => sin(\phi_5)cos(\phi_4) - sin(\phi_6)cos(\phi_1+\phi_3) - sin(\phi_5+\phi_6)cos(\phi_4+\phi_6) => -cos(\phi_4+\phi_5+\phi_6) - cos(\phi_1+\phi_3) \geq 0 => \pi \leq \phi_1+\phi_3+\phi_5+\phi_4+\phi_6$

if $b_3 = \frac{-sin(\phi_4)cos(\phi_1+\phi_5)+sin(\phi_3)cos(\phi_6)}{sin(\phi_3+\phi_4)} \geq cos(\phi_4+\phi_6) => -sin(\phi_4)cos(\phi_1+\phi_5) + sin(\phi_3)cos(\phi_6) - sin(\phi_3+\phi_4)cos(\phi_4+\phi_6) \geq 0 => -cos(\phi_3+\phi_4+\phi_6) \geq cos(\phi_1+\phi_5) => \phi_1+\phi_5+\phi_3+\phi_4+\phi_6 \geq \pi$

All inequalities perform only if $\phi_1+\phi_5+\phi_3+\phi_4+\phi_6 = \pi$ and as result we have additional zero (2,5,6)