Case 1

Roland Hildebrand

December 27, 2013

The supports of the minimal zeros are given by $\{1,2\},\{1,3\},\{1,4\},\{2,5\},\{3,6\},\{5,6\}$. This determines a number of off-diagonal elements, leading to matrices of the form

$$A = \begin{pmatrix} 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & b_1 & b_2 \\ 1 & -1 & 1 & b_1 & 1 & -1 \\ 1 & 1 & -1 & b_2 & -1 & 1 \end{pmatrix},$$

where b_i denote still unknown elements. For the pairs $(i,j) = (4,5)$ and $(i,j) = (4,6)$ there does not exist a minimal zero such that $u_i u_j > 0$. Hence there must exist a minimal zero such that $u_i + u_j > 0$ and $(Au)_i = (Au)_j = 0$. In both cases this minimal zero has to be the one with support $\{5,6\}$, otherwise $A_{ij} = -1$ and we would get a minimal zero with support $\{i,j\}$ which is not on the list. Hence $A_{45} + A_{46} = 0$ and $A_{45} = -A_{46} = \lambda \in (-1,1)$.

Now note that by setting $\lambda = \pm 1$ we get matrices with ± 1 entries which are copositive by the criterion of Haynsworth and Hoffman. Hence their convex conic combinations are also copositive, and $A \in \mathcal{C}_6$ for all $\lambda \in (-1,1)$.

However, setting $\Delta = \frac{dA}{d\lambda}$, we also have $A \pm \varepsilon\Delta \in \mathcal{C}_6$ whenever $\lambda \pm \varepsilon \in [-1,1]$, and A is not extremal.