Fluid mechanics and granular matter

Monday 14h-17h, at amphi Néel

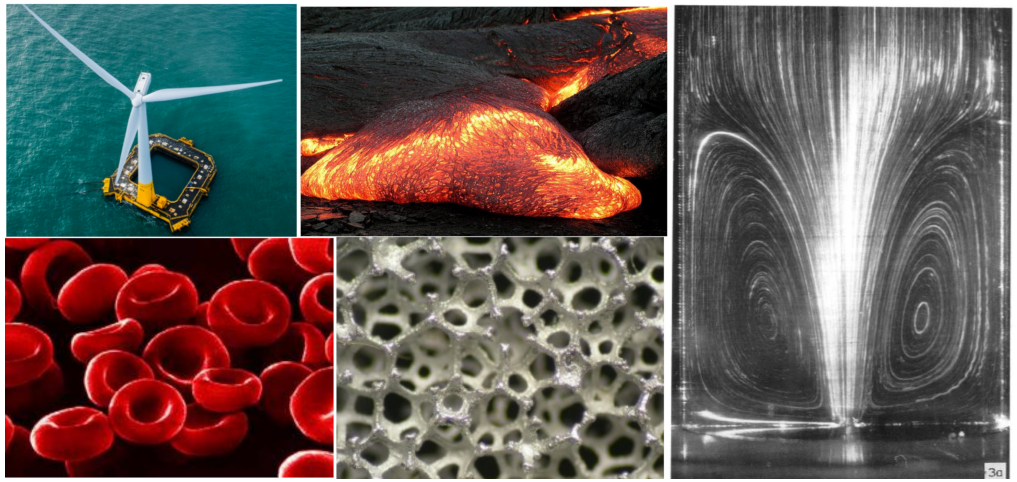
Lectures in two parts:

A. **Fluid mechanics**
   Pierre Saramito, 6 × 3h
   - 26 September 2022
   - 3, 10, 17, 24 October 2022
   - 7 November 2022

B. **Granular matter**
   Didier Bresch, 6 × 3h
   - 14, 21, 28 November 2022
   - 5, 12 December 2022
   - 9 January 2023
     at amphi D ENSIMAG

Aim: computer science $\Longrightarrow$ natural hazards, health, industry

A. Fluid mechanics

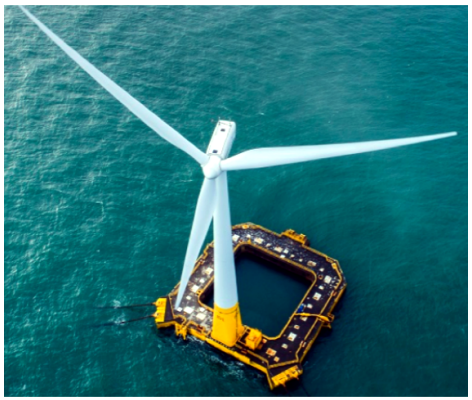


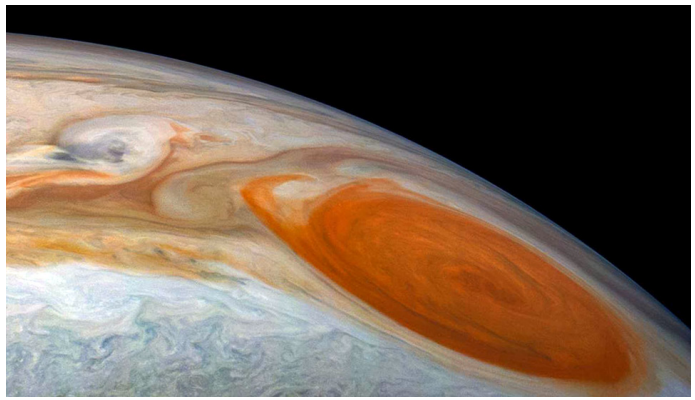
Fives chapters:

1. Navier-Stokes equations : 3h
2. Quasi-Newtonian fluids : 3h
3. Visco-plasticity : 4h30
4. Visco-elasticity : 6h
5. Elasto-visco-plasticity : 1h30

1. Navier-Stokes equations

for simple fluids only : air & water





(P): find $\boldsymbol{v}$ and p such that

$$\begin{cases} \rho\left(\partial_t \boldsymbol{v} + (\boldsymbol{v}.\nabla)\boldsymbol{v}\right) - \eta\Delta\boldsymbol{v} + \nabla p = \rho\boldsymbol{g} \\ \operatorname{div}\boldsymbol{v} = 0 \end{cases}$$

2. Quasi-Newtonian fluids: η non-constant

3. Visco-plasticity

= for mushy fluids : paste, mud



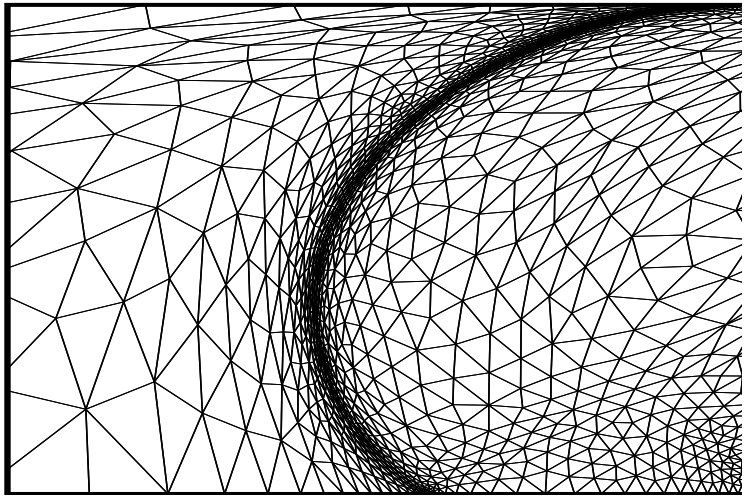


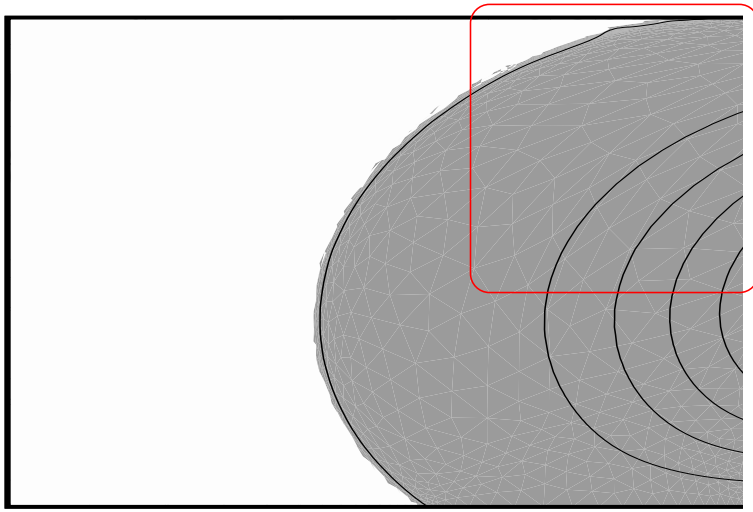
$$(P): \quad \min_{v \in H_0^1(\Omega} J(v)$$

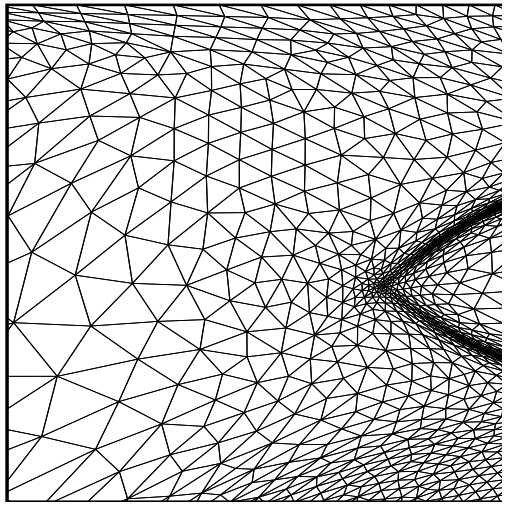
$$J(v) = \int_\Omega |\nabla v|^2 \, \mathrm{d}x + \sigma_0 \int_\Omega |\nabla v| \, \mathrm{d}x - \int_\Omega f v \, \mathrm{d}x$$

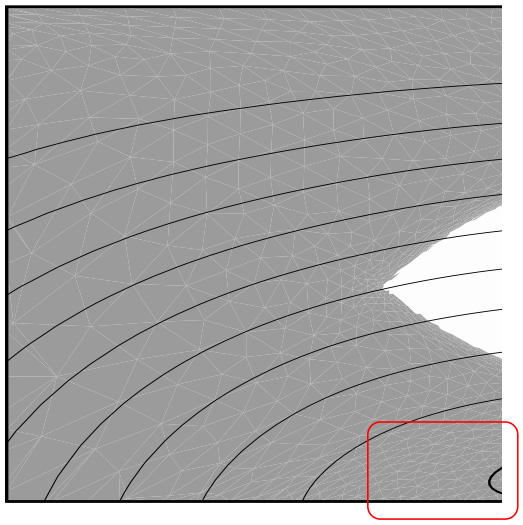
min : J is non-differentiable : $j(x) = x^2 + \sigma_0 |x| - fx$

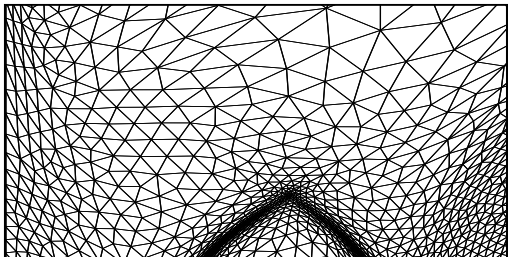
- optimization & convex analysis
- automatic adaptive mesh

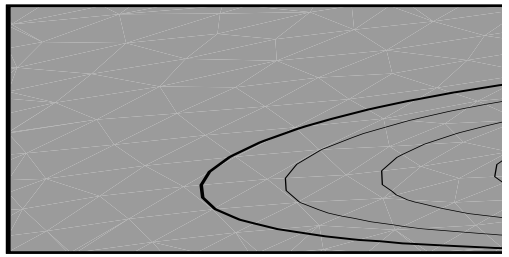






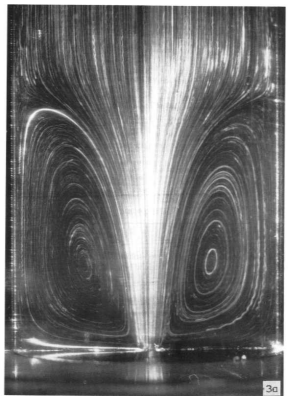






4. Visco-elasticity

= for suspensions of long elastic molecules : polymers, biology



(P): find $(\boldsymbol{\tau}, \mathbf{v}, p)$ such that :

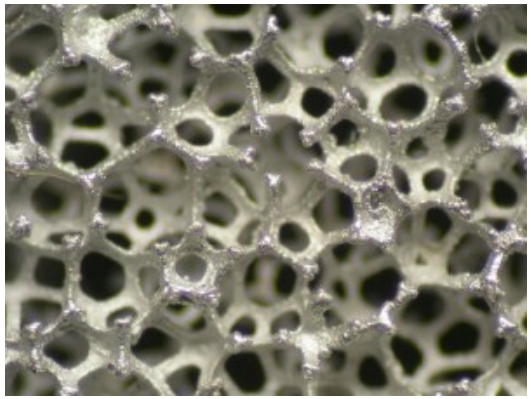
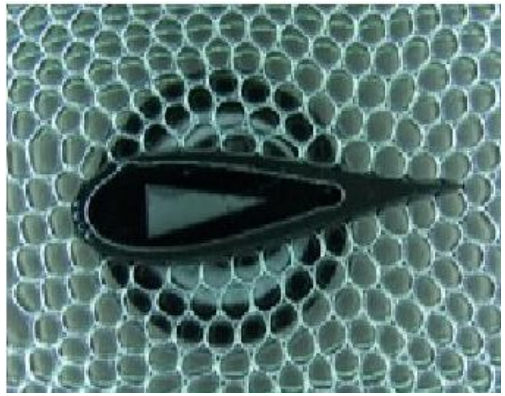
$$\begin{cases} \lambda\dot{\boldsymbol{\tau}} + \boldsymbol{\tau} - \boldsymbol{\nabla}\mathbf{v} - (\boldsymbol{\nabla}\mathbf{v})^T = 0 \\ \mathbf{div}\,\boldsymbol{\tau} + \varepsilon\,\Delta\mathbf{v} - \nabla p = f \\ \mathrm{div}\,\mathbf{v} = 0 \end{cases}$$

$$\dot{\boldsymbol{\tau}} = \partial_t \tau + (\mathbf{v}.\boldsymbol{\nabla})\boldsymbol{\tau} + \boldsymbol{\nabla}\mathbf{v}\,\boldsymbol{\tau} - \boldsymbol{\tau}\,\boldsymbol{\nabla}\mathbf{v}$$

- upwind schemes : discontinuous Galerkin methods
- mixed finite elements : inf-sup condition

5. Elasto-visco-plasticity

= for soft grains : biology, liquid foam

$$\lambda \dot{\boldsymbol{\tau}} + \max\left(0, 1 - \frac{\sigma_0}{|\boldsymbol{\tau}|}\right) \boldsymbol{\tau} = \boldsymbol{\nabla} \mathbf{v} + (\boldsymbol{\nabla} \mathbf{v})^T$$

$\Longrightarrow$ combination of previous problems with (λ, σ_0)