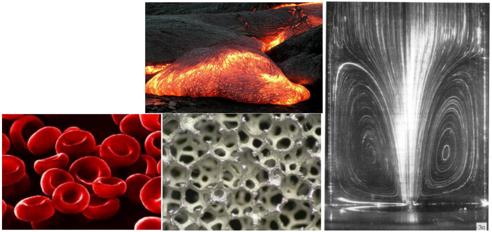
Complex fluids



computer science $\Longrightarrow$ natural hazards, health, industry

Lecture in three main parts:

1. Visco-plasticity
2. Visco-elasticity
3. Elasto-visco-plasticity

1. Visco-plasticity

$$\min_{u\in H_0^1(\Omega} J(u)$$

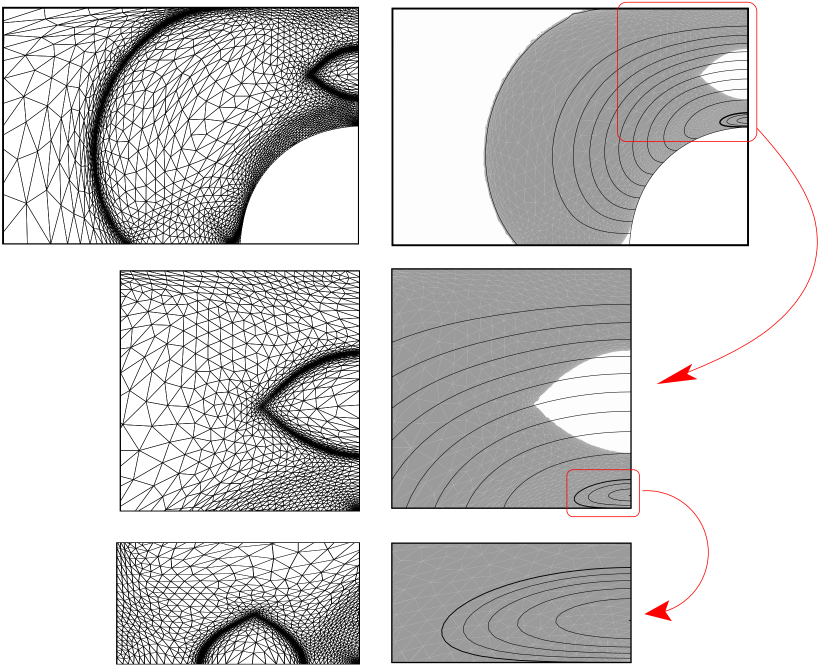
$$J(u) = \int_\Omega |\nabla u|^2 \,\mathrm{d}x + \sigma_0 \int_\Omega |\nabla u| \,\mathrm{d}x - \int_\Omega fu \,\mathrm{d}x$$

min : J is non-differentiable : $j(x) = x^2 + \sigma_0 |x| - fx$

- operator splitting
- automatic adaptive mesh







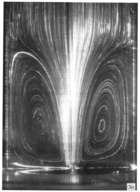
2. Visco-elasticity

Find $(\boldsymbol{\tau}, \mathbf{u}, p)$ such that :

$$\left\{ \begin{array}{rcccccl} \lambda\dot{\boldsymbol{\tau}} + & \boldsymbol{\tau} & - & \nabla\mathbf{u} - (\nabla\mathbf{u})^T & & = & 0 \\ & \mathbf{div}\,\boldsymbol{\tau} & + & \varepsilon\,\Delta\mathbf{u} & - \nabla p & = & f \\ & & & \mathrm{div}\,\mathbf{u} & & = & 0 \end{array} \right.$$

$$\dot{\boldsymbol{\tau}} = \partial_t\tau + (\mathbf{u}.\nabla)\boldsymbol{\tau} + \nabla\mathbf{u}\;\boldsymbol{\tau} - \boldsymbol{\tau}\,\nabla\mathbf{u}$$

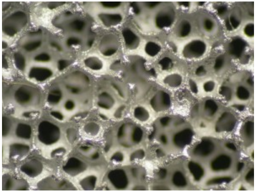
- upwind schemes : discontinuous Galerkin methods
- mixted finite elements : inf-sup condition

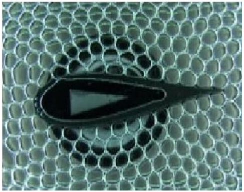


3. Elasto-visco-plasticity

$$\lambda \dot{\boldsymbol{\tau}} + \max\left(0, 1 - \frac{\sigma_0}{|\boldsymbol{\tau}|}\right)\boldsymbol{\tau} = \nabla \mathbf{u} + (\nabla \mathbf{u})^T$$

- theory : convex analysis, hyperbolic systems
- applied : combination of problems (1) and (2) with (σ_0, λ)





$\Longrightarrow$ PDE, numerical resolution, applications