Fluid mechanics and granular matter

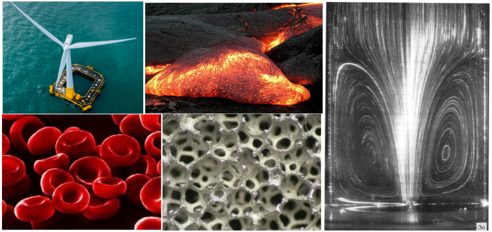
At ENSIMAG, bat. H, room H102
Monday 14h-17h
Lectures in two parts:

A. **Fluid mechanics**
   Pierre Saramito, 6×3h
   - 23, 30 September 2024
   - 7, 14, 21 October 2024
   - 4 November 2024

B. **Granular matter**
   Didier Bresch, 6×3h
   - 18, 25 November 2024
   - 2, 9, 16 December 2024
   - 6 January 2025

Aim: computer science $\Longrightarrow$ natural hazards, health, industry

A. Fluid mechanics

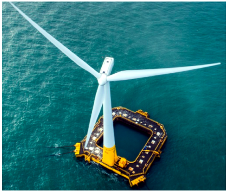


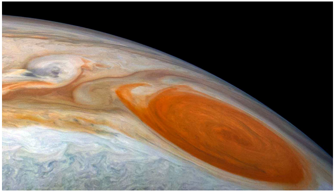
Fives chapters:

1. Navier-Stokes equations : 3h
2. Quasi-Newtonian fluids : 3h
3. Visco-plasticity : 4h30
4. Visco-elasticity : 6h
5. Elasto-visco-plasticity : 1h30

1. Navier-Stokes equations

for simple fluids only : air & water





(P): find $\boldsymbol{v}$ and p such that

$$\begin{cases} \rho\left(\partial_t \boldsymbol{v} + (\boldsymbol{v}.\nabla)\boldsymbol{v}\right) - \eta\Delta\boldsymbol{v} + \nabla p = \rho\boldsymbol{g} \\ \mathrm{div}\ \boldsymbol{v} = 0 \end{cases}$$

2. Quasi-Newtonian fluids: η non-constant

3. Visco-plasticity

= for mushy fluids : paste, mud



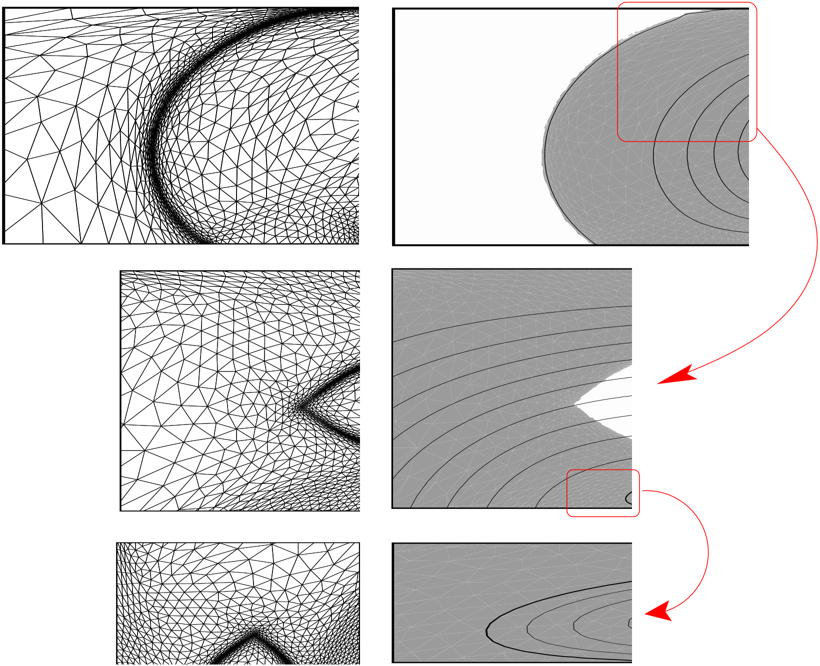


$$(P): \min_{v \in H_0^1(\Omega} J(v)$$

$$J(v) = \int_\Omega |\nabla v|^2 \, \mathrm{d}x + \sigma_0 \int_\Omega |\nabla v| \, \mathrm{d}x - \int_\Omega fv \, \mathrm{d}x$$

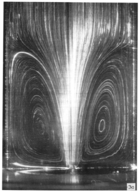
min : J is non-differentiable : $j(x) = x^2 + \sigma_0 |x| - fx$

- optimization & convex analysis
- automatic adaptive mesh



4. Visco-elasticity

= for suspensions of long elastic molecules : polymers, biology



(P): find $(\boldsymbol{\tau}, \mathbf{v}, p)$ such that :

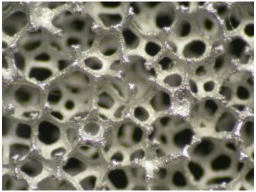
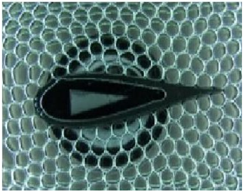
$$\left\{ \begin{array}{rcccccl} {\color{red}\lambda}\dot{\boldsymbol{\tau}} + \boldsymbol{\tau} & - & \nabla \mathbf{v} - (\nabla \mathbf{v})^T & & & = & 0 \\ \mathbf{div}\, \boldsymbol{\tau} & + & \varepsilon\, \Delta \mathbf{v} & - & \nabla p & = & f \\ & & \mathrm{div}\, \mathbf{v} & & & = & 0 \end{array} \right.$$

$$\dot{\boldsymbol{\tau}} = \partial_t \tau + (\mathbf{v}.\nabla)\boldsymbol{\tau} + \nabla \mathbf{v}\ \boldsymbol{\tau} - \boldsymbol{\tau}\, \nabla \mathbf{v}$$

- upwind schemes : discontinuous Galerkin methods
- mixed finite elements : inf-sup condition

5. Elasto-visco-plasticity

= for soft grains : biology, liquid foam

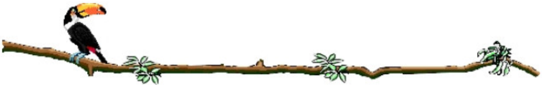
$$\lambda \dot{\boldsymbol{\tau}} + \max\left(0, 1 - \frac{\sigma_0}{|\boldsymbol{\tau}|}\right) \boldsymbol{\tau} = \nabla \mathbf{v} + (\nabla \mathbf{v})^T$$

$\Longrightarrow$ combination of previous problems with (λ, σ_0)

Teaching material: on my web page

https://membres-ljk.imag.fr/Pierre.Saramito/ens 110 %

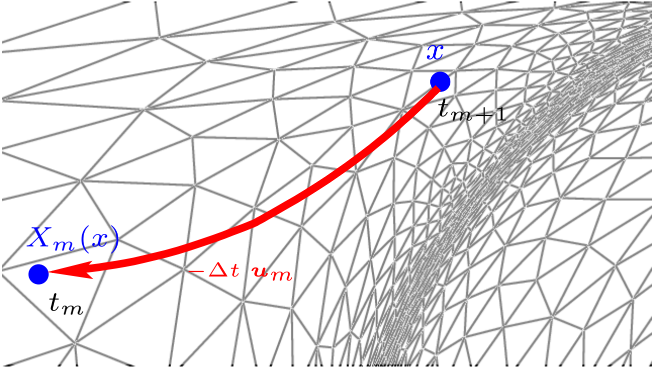
Pierre Saramito : teaching



- **2011, 2012, 2013, 2018, 2022, 2023** : ED MSTII and M2 MSIAM, Grenoble
  Fluid mechanics and granular matter.
  - NEW **2023**: room Ensimag-H102 each Monday 14h00-17h15: from September 25 to December 18.
    building Ensimag-H: access map.
  - **slides** : short presentation of the lectures (7 pages): slides (9 mb).
  - **book**: P. Saramito, *Complex fluids: modeling and algorithms*, Springer, 2016.
    - electronic version : pdf (12 mo), epub (7 mo)
    - paper version: buy online (276 pages, 65 euros)
  - video of the lectures in 2013 (French, 13 x 1h30).

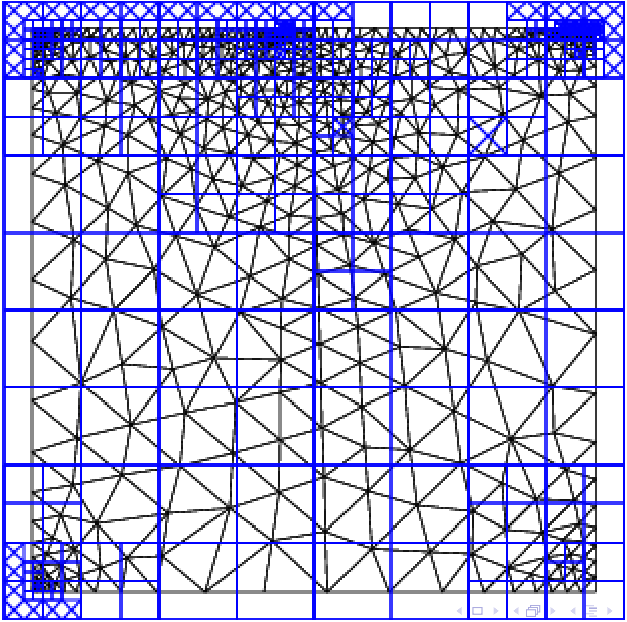
Evaluation of characteristics

$$X_m(x) \approx x - \Delta t \; \boldsymbol{u}_m(x)$$

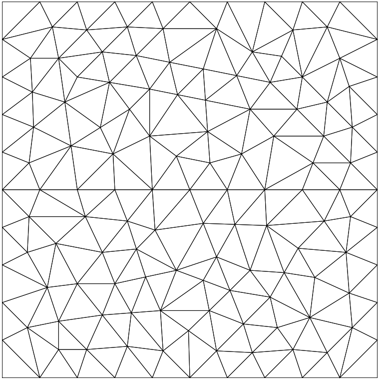


$\Longrightarrow$ interpolation : $\boldsymbol{u}_m(X_m(x))$

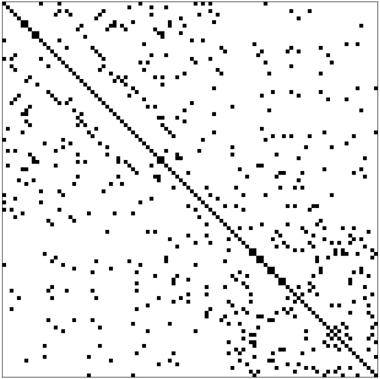
Searching: quadtree



Mesh & sparse matrix

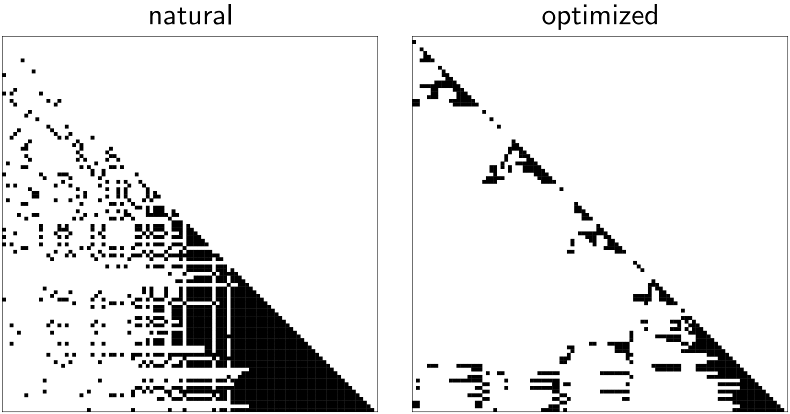


$\mathscr{T}_h$



A

Sparse factorization $A = LDL^T$: renumbering



Implementation with Rheolef (C++ library)

```
int main (int argc, char** argv) {
  geo omega (argv[1]);
  space Qh (omega, "P1");
  space Xh (omega, "P2", "vector");
  Xh.block ("boundary");
  trial u(Xh), p(Qh); test v(Xh), q(Qh);
  form a = integrate (2*ddot(D(u),D(v)));
  form b = integrate (-q*div(u));
  form m = integrate (p*q);
  field uh (Xh, 0), ph (Qh, 0);
  uh[1]["top"] = 0;
  solver_abtb stokes (a.uu(), b.uu(), m.uu());
  stokes.solve(-a.ub()*uh.b(), -b.ub()*uh.b(),
      uh.set_u(), ph.set_u());
  cout ≪ catchmark("u") ≪ uh
       ≪ catchmark("p") ≪ ph;
}
```

$\Longrightarrow$ 15 lines of code

soit $\Omega \subset \mathbb{R}^d$, $d = 1, 2, 3$

$Q_h = \{q \in L^2(\Omega);\ q_{|K} \in P_1,\ \forall K \in \mathcal{T}_h\}$

$X_h = \{\mathbf{v} \in H^1(\Omega)^d;\ \mathbf{v}_{|K} \in (P_2)^d,\ \forall K \in \mathcal{T}_h\}$

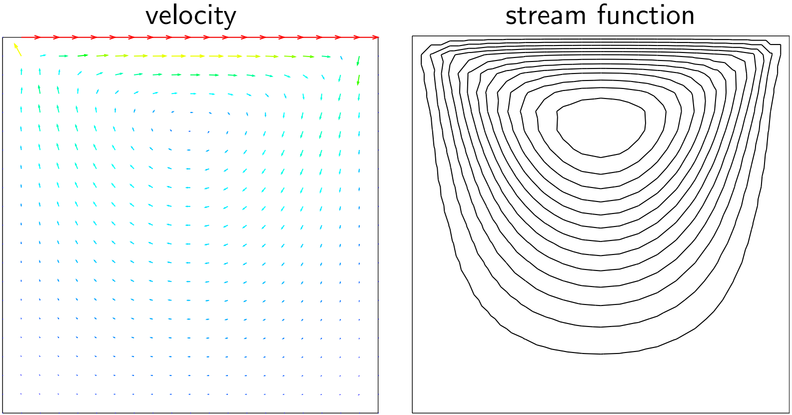
$\forall$ $\mathbf{u}$, $\mathbf{v}$, et p, q, définissons :

$$a(\mathbf{u},\mathbf{v}) = \int_\Omega 2D(\mathbf{u}):D(\mathbf{v})$$
$$b(\mathbf{u},q) = -\int_\Omega q\,\mathrm{div}\,\mathbf{u}$$
$$m(p,q) = \int_\Omega p\,q$$

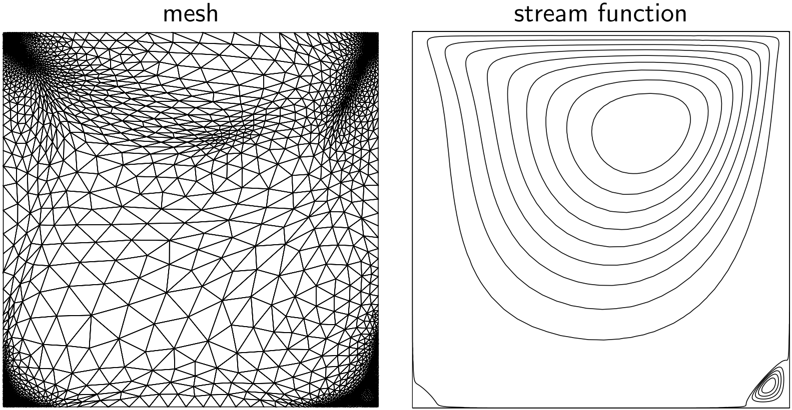
trouver $\mathbf{u} \in X_h$, $\mathbf{u}_h = 0$ sur $\partial\Omega$, et $p_h \in Q_h$ tels que

$$\begin{aligned} a(\mathbf{u}_h,\mathbf{v}_h) + b(\mathbf{v}_h,p) &= 0, \quad \forall \mathbf{v}_h \in X_h, \mathbf{v}_h = 0 \text{ sur } \partial\Omega \\ b(\mathbf{u}_h,q) &= 0, \quad \forall q_h \in Q_h \end{aligned}$$

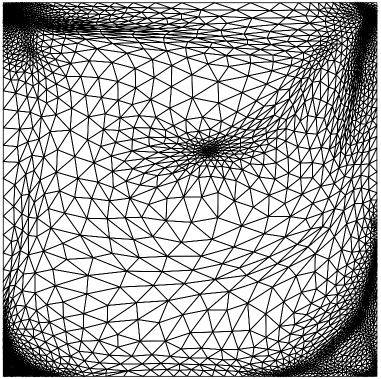
Stokes in the driven cavity

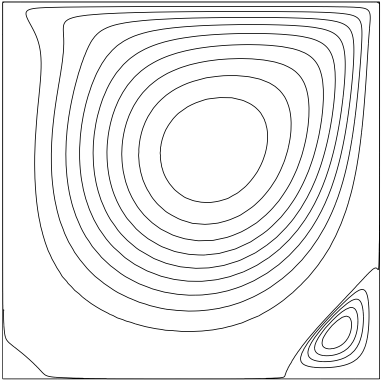


Navier-Stokes $Re = 100$: 4804 elements

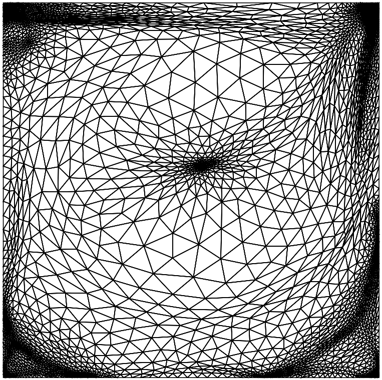


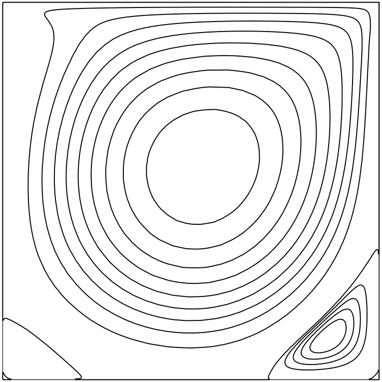
Navier-Stokes $Re = 400$: 5233 elements





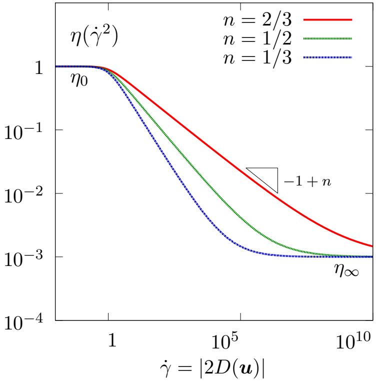
Navier-Stokes $Re = 1000$: 5873 elements



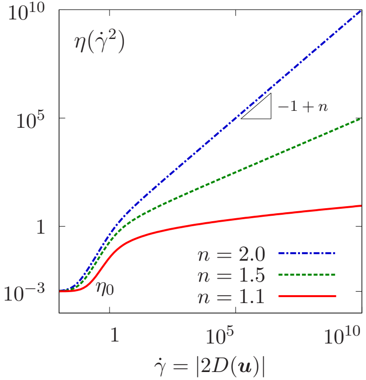


Viscosity : Carreau's law

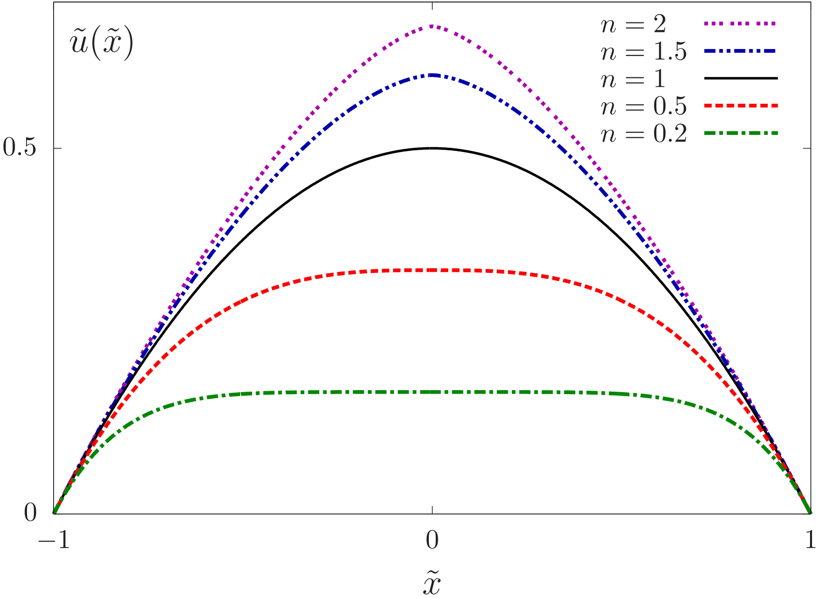
$0 < n < 1$
shear-thinning

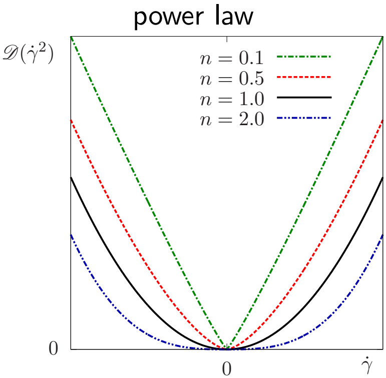


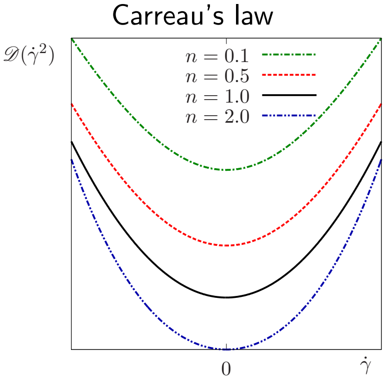
$n > 1$
shear-thickening



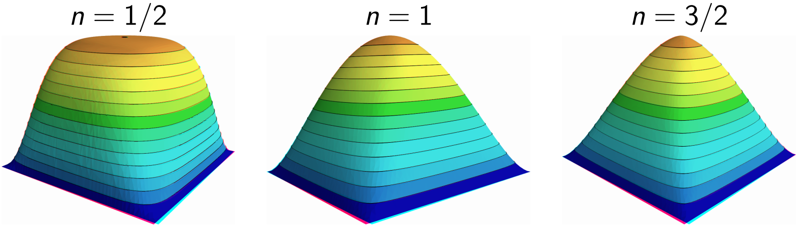
Poiseuille flow with power law

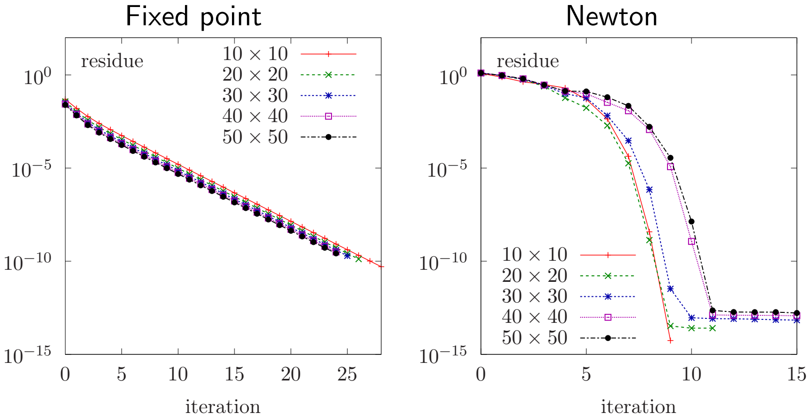



power law
$\mathscr{D}(\dot\gamma^2)$
$n = 0.1$
$n = 0.5$
$n = 1.0$
$n = 2.0$
0
0
$\dot\gamma$

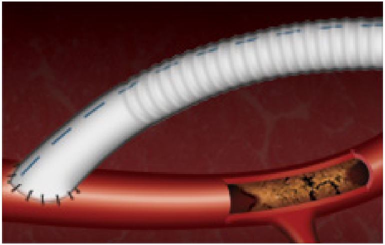

Carreau's law
$\mathscr{D}(\dot\gamma^2)$
$n = 0.1$
$n = 0.5$
$n = 1.0$
$n = 2.0$
0
$\dot\gamma$

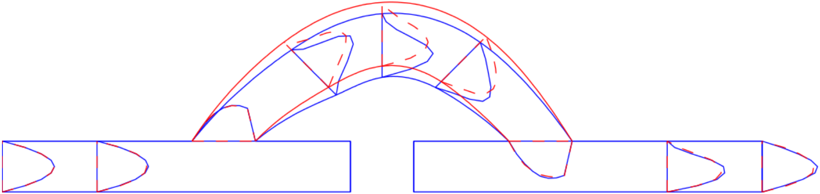
Poiseuille flow in a square pipe section




Fixed point
residue
10 × 10
20 × 20
30 × 30
40 × 40
50 × 50
10^0
10^{-5}
10^{-10}
10^{-15}
0
5
10
15
20
25
iteration
Newton
residue
10 × 10
20 × 20
30 × 30
40 × 40
50 × 50
10^0
10^{-5}
10^{-10}
10^{-15}
0
5
10
15
iteration

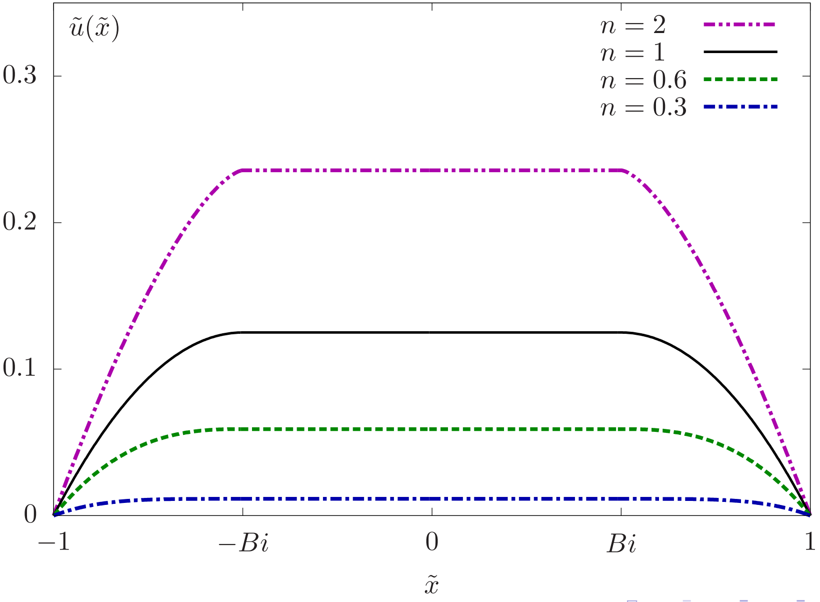
Example 3: arterial graft shape optimization



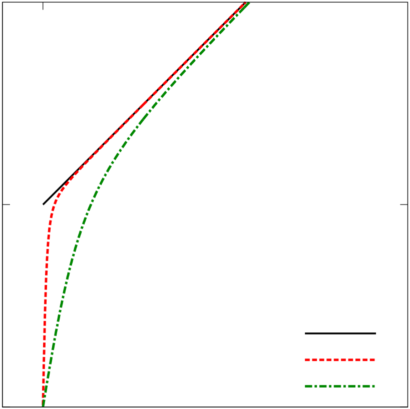


comparison: Newtonian and Carreau's law (Abraham *et al.* 2005)

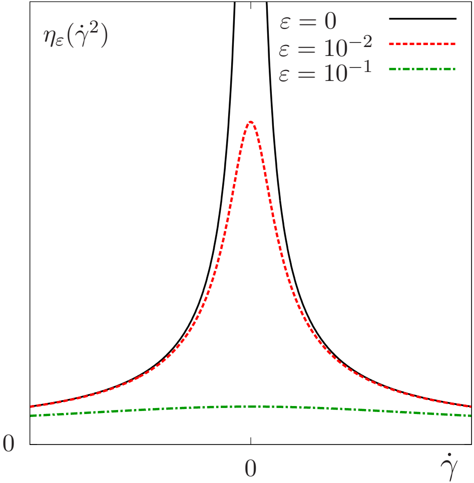
Poiseuille flow with viscoplastic fluid



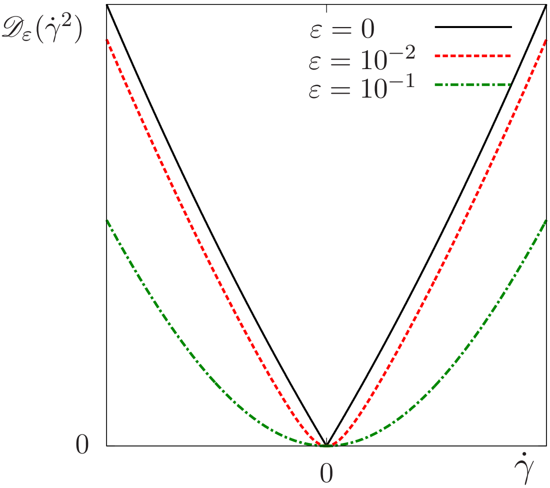
Regularization: stress vs shear



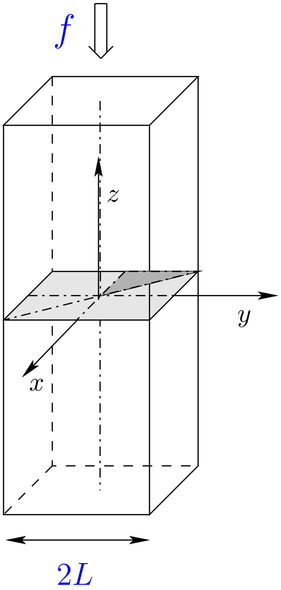
Regularization: viscosity



Dissipation: convexity



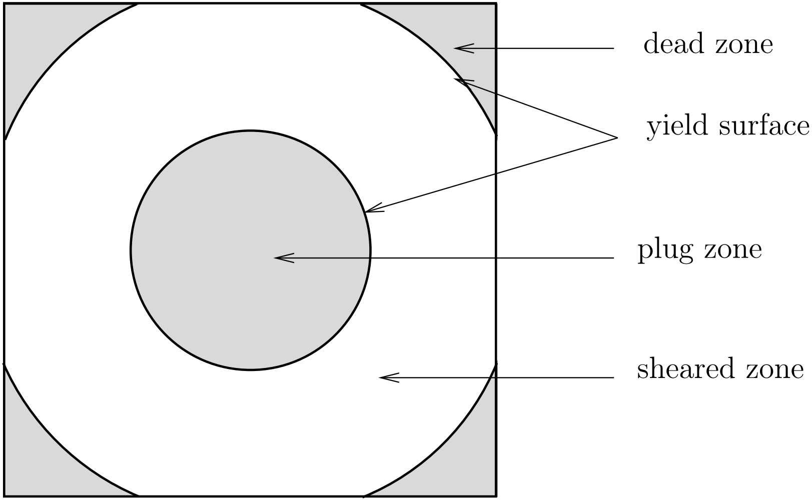
Poiseuille in a square section



dimensionless number

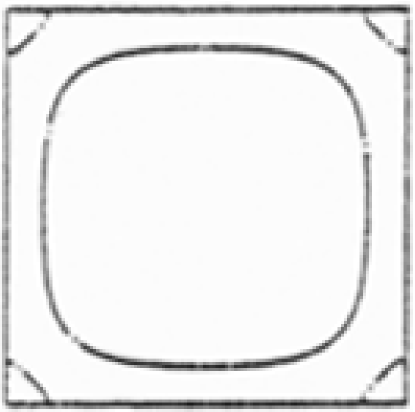
$$Bi = \frac{\sigma_0}{L\,f}$$

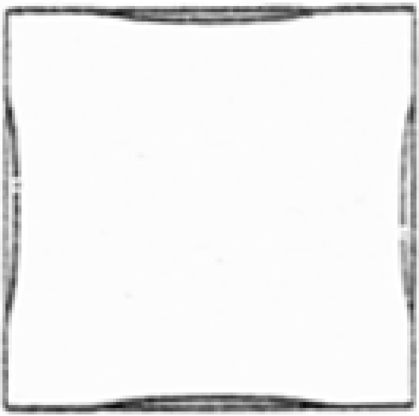
Terminology



Regularization: Taylor & Wilson (1997)

$Bi = 0.8$ | $Bi = 1.0$

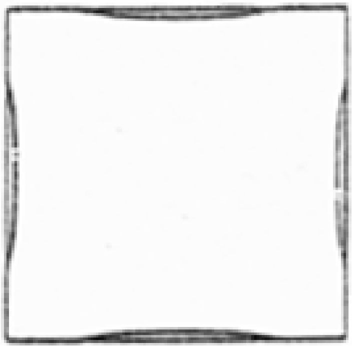




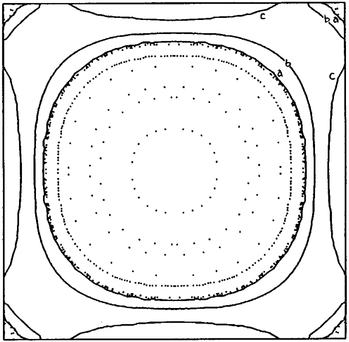
$\Rightarrow$ badly shaped rigid zones...

Non-convergence $\varepsilon \to 0$? Wang (1999)

Taylor & Wilson
$\varepsilon = 10^{-3}$



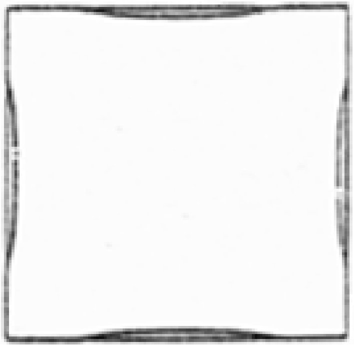
Wang
$\varepsilon = 10^{-13}$



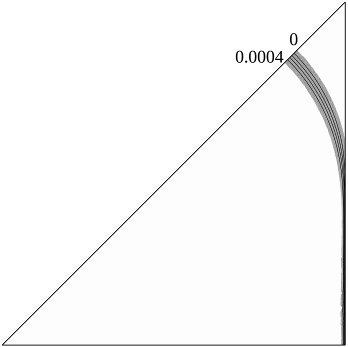
	$\|2D(u_\varepsilon)\|$
(c)	10^{-2}
(b)	10^{-3}
(a)	8×10^{-7}

Augmented Lagrangian algorithm

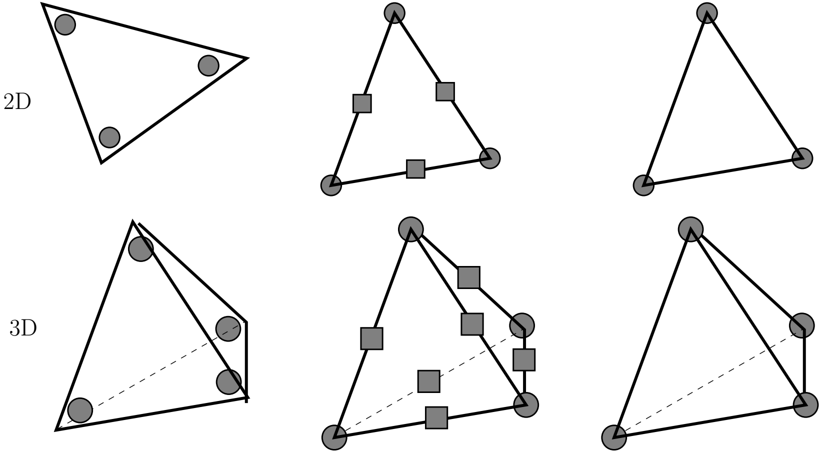
Taylor & Wilson (1997)



Roquet & Saramito (2001)

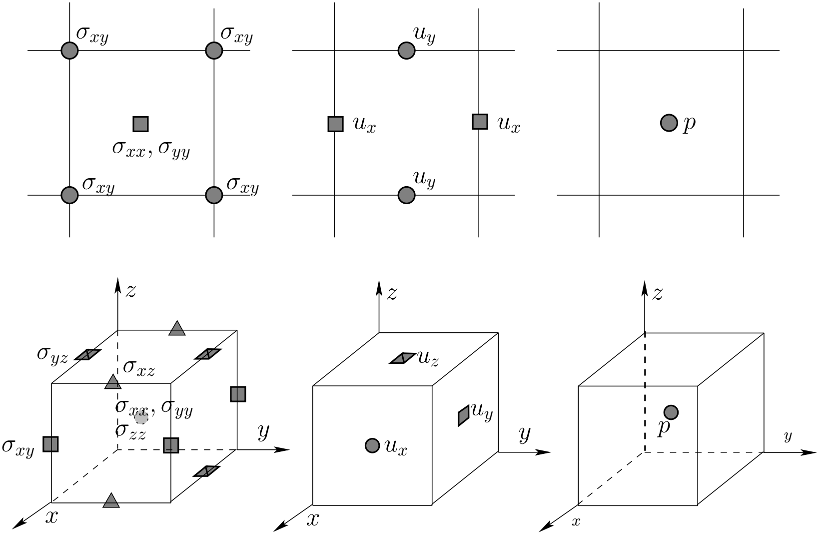


Viscoplastic fluids: space approximation

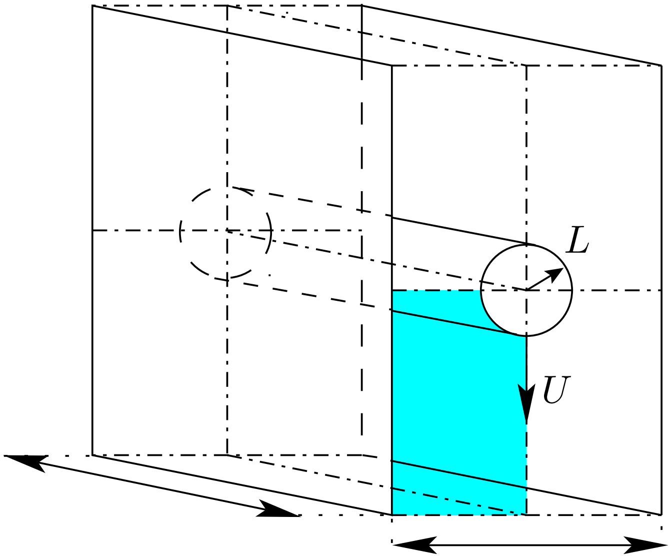


σ and γ: $P_1 - C^{-1}$ $\quad$ $\mathbf{u}$: $P_2 - C^0$ $\quad$ p: $P_1 - C^0$

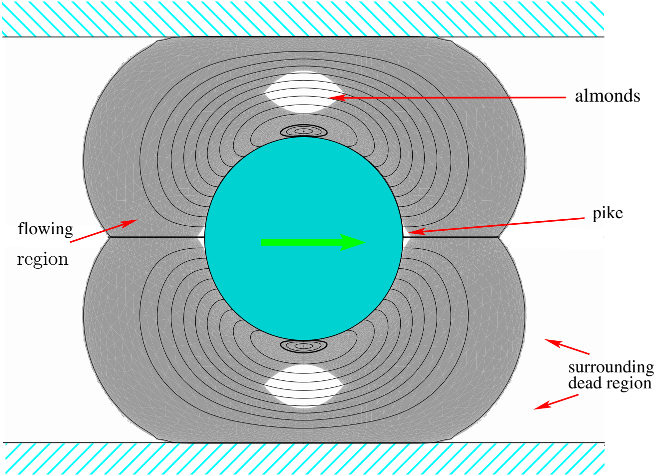
Finite differences



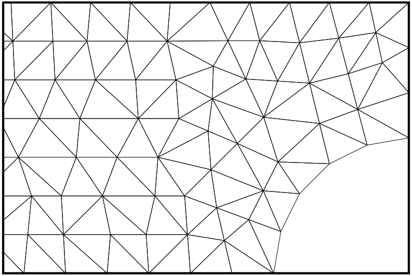
3.9 Example: flow arround an obstacle

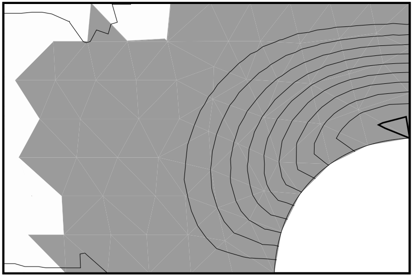


Terminology

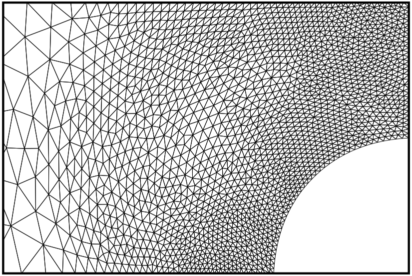


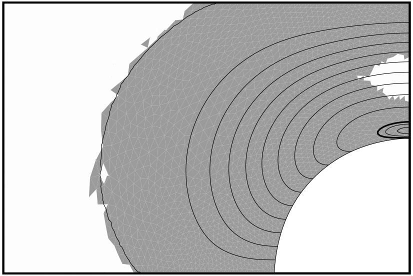
$\mathrm{card}(\mathscr{T}_h) = 539$





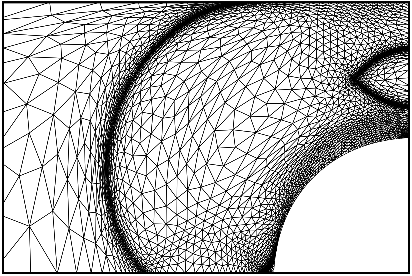
$\mathrm{card}(\mathscr{T}_h) = 15\,466$

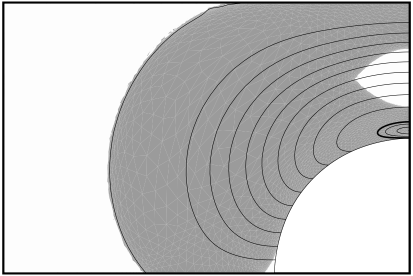




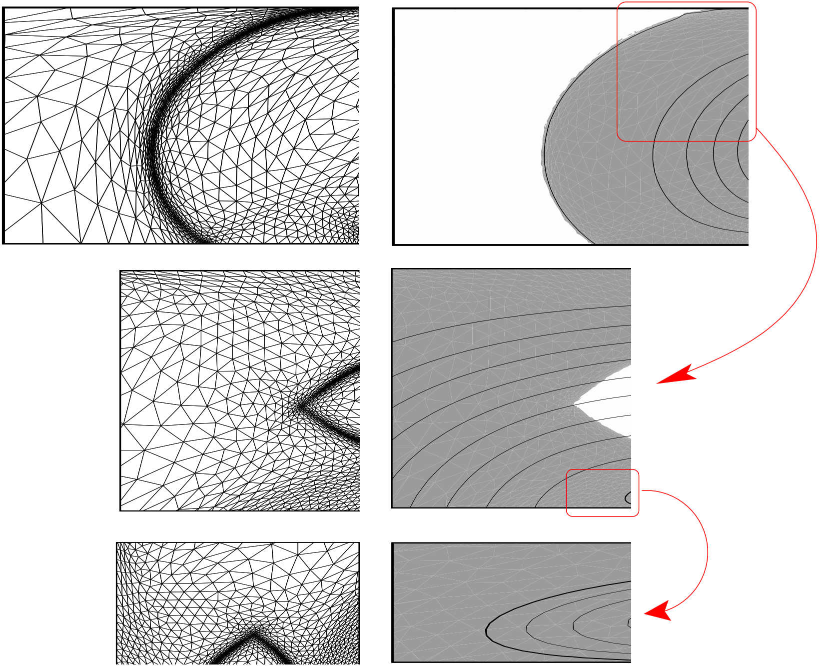
After mesh adaptation

$\mathrm{card}(\mathscr{T}_h) = 41\,955$

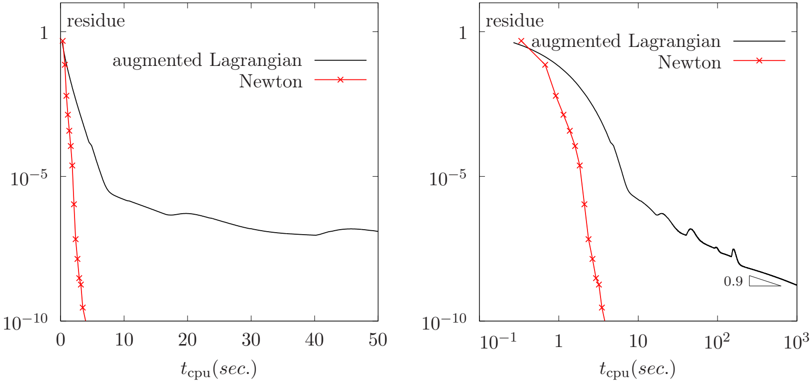


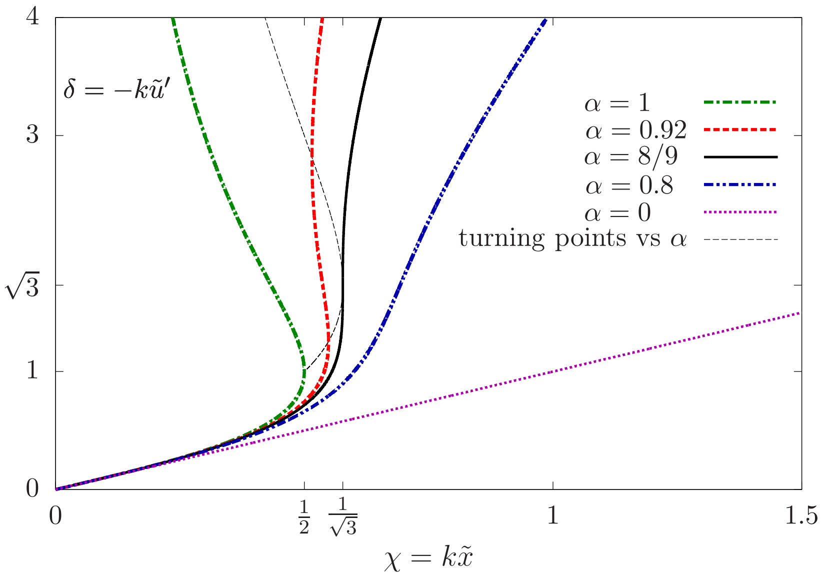


After 9 mesh adaptations



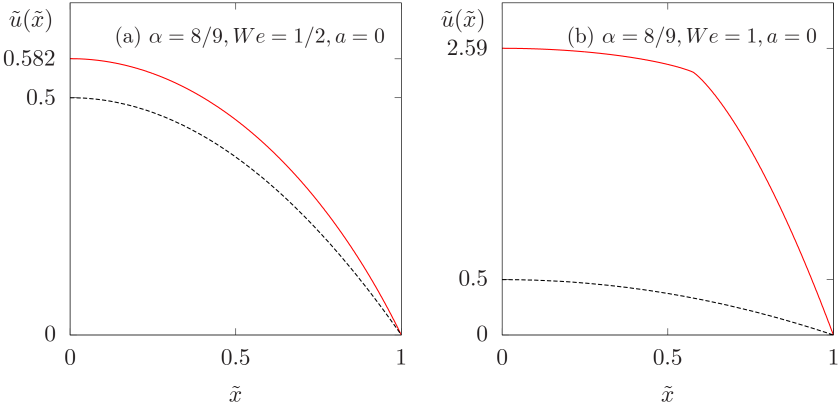
Viscoplastic fluid: augmented Lagrangian vs Newton



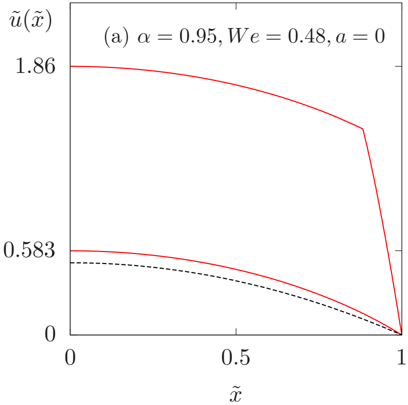


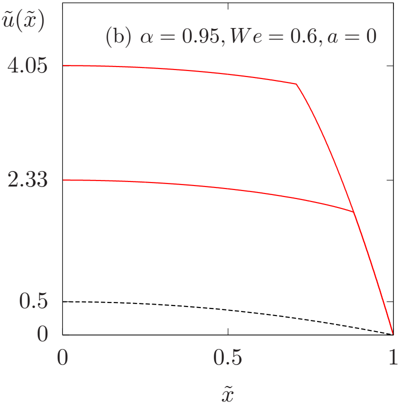
4
3
√3
1
0
0
1/2
1/√3
1
1.5
δ = −kũ′
χ = kx̃
α = 1
α = 0.92
α = 8/9
α = 0.8
α = 0
turning points vs α

Poiseuille: Oldroyd $\alpha = 8/9$

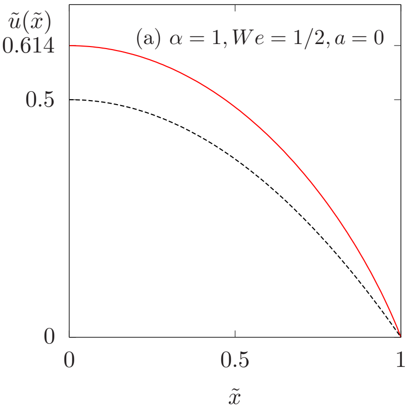


Poiseuille: Oldroyd $\alpha = 0.95$





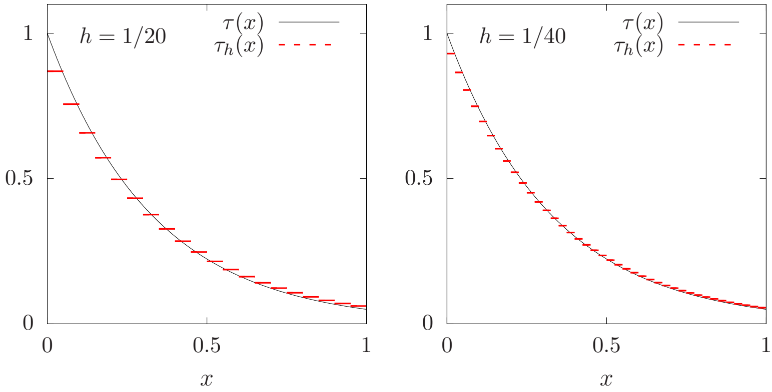
Poiseuille: Maxwell $\alpha = 1$



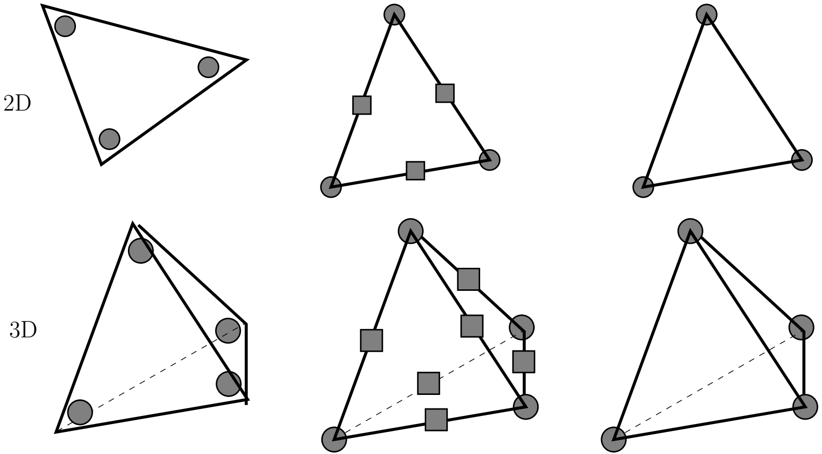
C++ code with Rheolef: scalar transport

```
#include "rheolef.h"
using namespace rheolef;
using namespace std;
int main(int argc, char**argv) {
  environment rheolef (argc, argv);
  geo omega (argv[1]);
  space Xh (omega, argv[2]);
  Float alpha   = (argc > 3) ? atof(argv[3]) : 1;
  Float sigma   = (argc > 4) ? atof(argv[4]) : 3;
  point u (1,0,0);
  trial phi (Xh); test psi (Xh);
  form ah = integrate (dot(u,grad_h(phi))*psi + sigma*phi*psi)
          + integrate ("boundary", max(0, -dot(u,normal()))*phi*psi)
          + integrate ("internal_sides",
                 - dot(u,normal())*jump(phi)*average(psi)
                 + 0.5*alpha*abs(dot(u,normal()))*jump(phi)*jump(psi));
  field lh = integrate ("boundary", max(0, -dot(u,normal()))*psi);
  solver sah (ah.uu());
  field phi_h(Xh);
  phi_h.set_u() = sah.solve(lh.u());
  dout << catchmark("sigma") << sigma << endl
       << catchmark("phi")   << phi_h;
}
```

Example: scalar transport

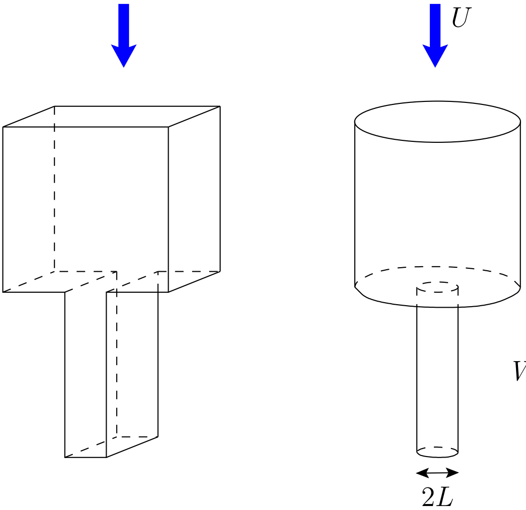


Viscoelastic fluids: space approximation



σ and γ: $P_1 - C^{-1}$ $\quad$ $\mathbf{u}$: $P_2 - C^0$ $\quad$ p: $P_1 - C^0$

Example: abrupt contraction

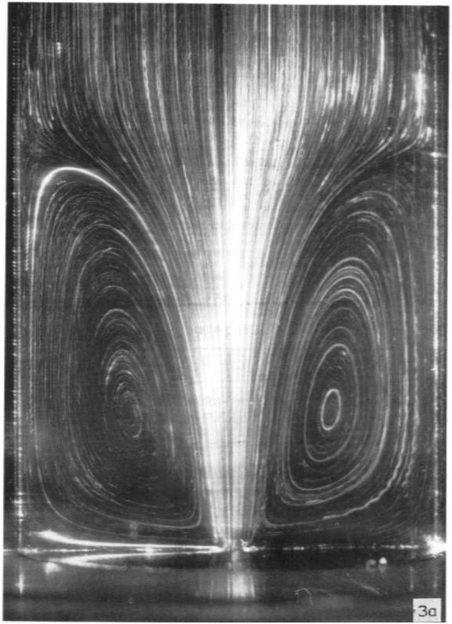


$$We = \frac{\lambda U}{L}$$

planar

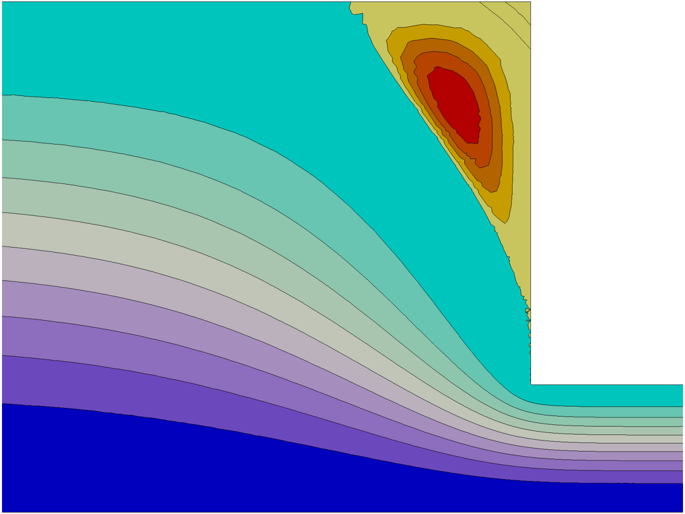
axisymmetric

Polymer solution flow



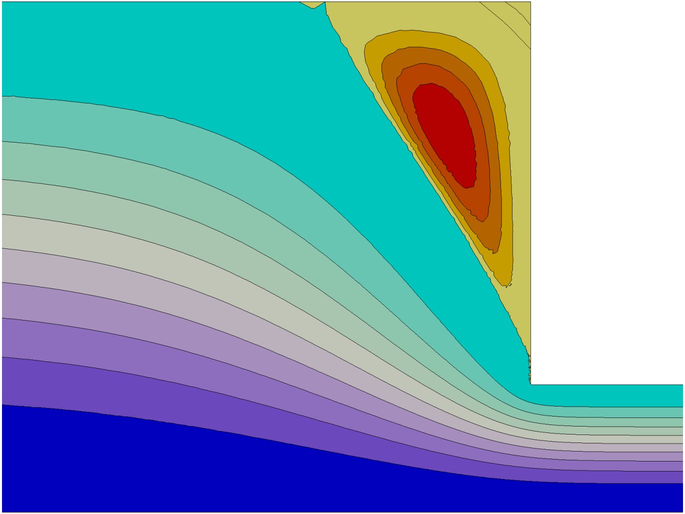
Oldroyd model: $We = \lambda U/L = 0.1$

Computations with Rheolef



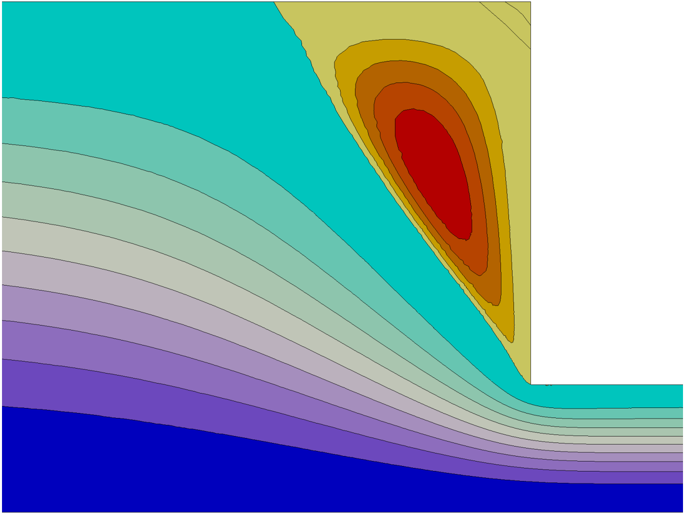
Oldroyd model: $We = \lambda U/L = 0.3$

Computations with Rheolef

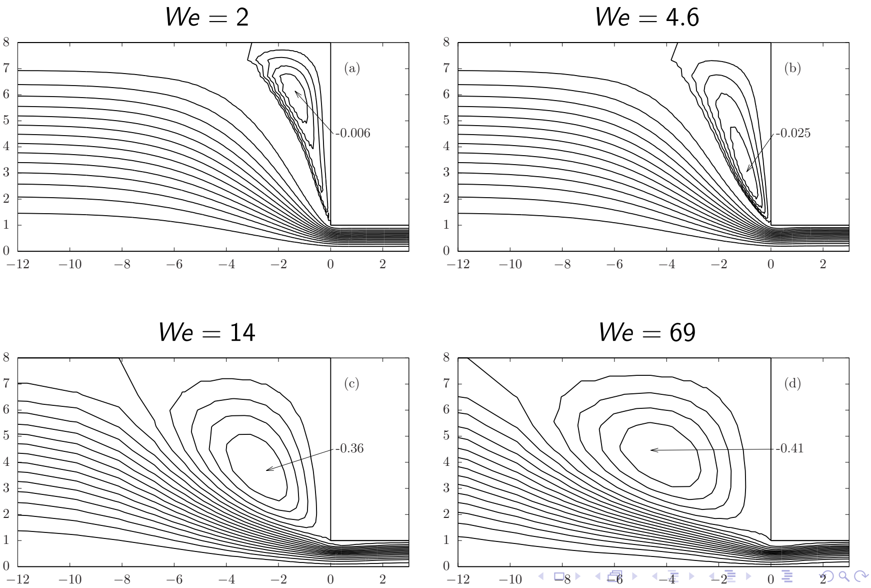


Oldroyd model: $We = \lambda U/L = 0.7$

Computations with Rheolef



Phan-Thien and Tanner model



Dowload all the PDFs: → zip

1.	NS(1) algorithm	Quelques résultats nouveaux sur les méthodes de projection → Luc Vandame	6 pages, 2001 → pdf
2.	NS(2) asympt. anal.	Derivation of a new dimensional viscous shallow water model → Michele Benedetti	15 pages, 2007 → pdf
3.	QN(1) appl. biology	Shape optimization in steady blood flow → Hadi Mazloum	17 pages, 2005 → pdf
4.	VP(1) appl. envir.	Submerged jet shearing of viscoplastic sludge → Eloïse Touron	9 pages, 2018 → pdf
5.	VP(2) asympt. anal.	Numerical modeling of shallow viscoplastic fluids → Karlotta Kilias	18 pages, 2014 → pdf
6.	VP(3) model	viscoplastic flows with stick-slip at the wall → Arianna Pellizzaro	15 pages, 2008 → pdf

7.	VP(4) algorithm	A damped Newton algorithm for computing viscoplastic fluid flows → Fabio Marini	10 pages, 2016 → pdf
8.	VE(1) theor. phys.	Oldroyd B, and not A ? → Maxime Renard	7 pages, 2021 → pdf
9.	VE(2) numer. meth.	Constitutive laws for the matrix-logarithm → Stefano Muzzolon	5 pages, 2004 → pdf
10.	VE(3) numer. math.	Stationary Oldroyd model with diffusive stress	13 pages, 2015 → pdf
11.	EVP(1) appli.	Steady Couette flows of elastoviscoplastic fluids	27 pages, 2012 → pdf
12.	EVP(2) appl. foam	A new operator splitting algorithm for elastoviscoplastic flows → Yiqun Liu	9 pages, 2013 → pdf
13.	EVP(3) appl. earth	Robust characteristics method for elastoviscoplastic problems	23 pages, 2007 → pdf
14.	EVP(4) suspension	Particle migration in channel flow of an elastoviscoplastic fluid → Camille Touron	8 pages, 2020 → pdf

Exams

14h	Octave Crespel	Quelques résultats nouveaux sur les méthodes de projection
14h20	Basile Dubois Bonnaire	Shape optimization in steady blood flow
14h40	Jules Treton	Submerged jet shearing of viscoplastic sludge
15h	Hanne Christiaensen	Numerical modeling of shallow viscoplastic fluids
15h20	Sara Avesani	Viscoplastic flows with stick-slip at the wall
15h40	Yannick Godammer	Constitutive laws for the matrix-logarithm of the conformation tensor
16h	Guilia Quarta	Stationary Oldroyd model with diffusive stress
16h20	Yann Vincent	A new operator splitting algorithm for elastoviscoplastic flows
16h40	Antoine Sese	Particle migration in channel flow of an elastoviscoplastic fluid