

# Analyse de primitives symétriques

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Thèse préparée à l'Inria Saclay & Rennes, l'École polytechnique, et la Nanyang  
Technological University

Sous la direction de Daniel Augot, Pierre-Alain Fouque et Thomas Peyrin

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## Introduction

New linear mappings for block ciphers

Practical freestart collisions for the full SHA-1

Conclusion

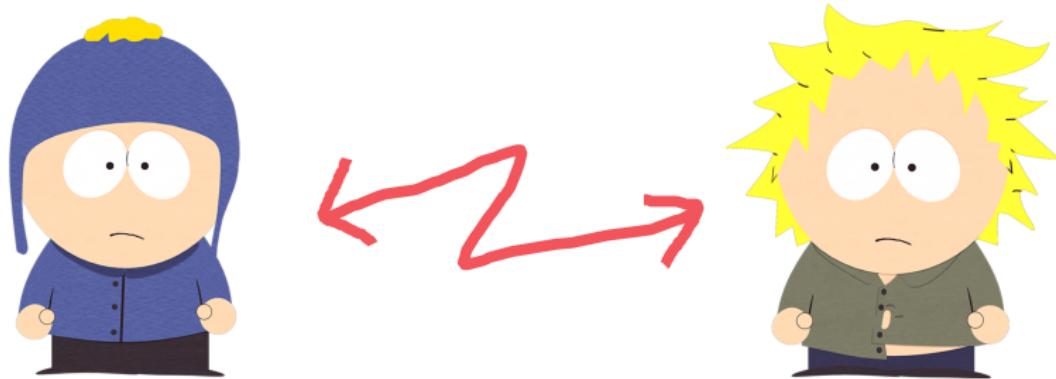
# Motivating cryptography

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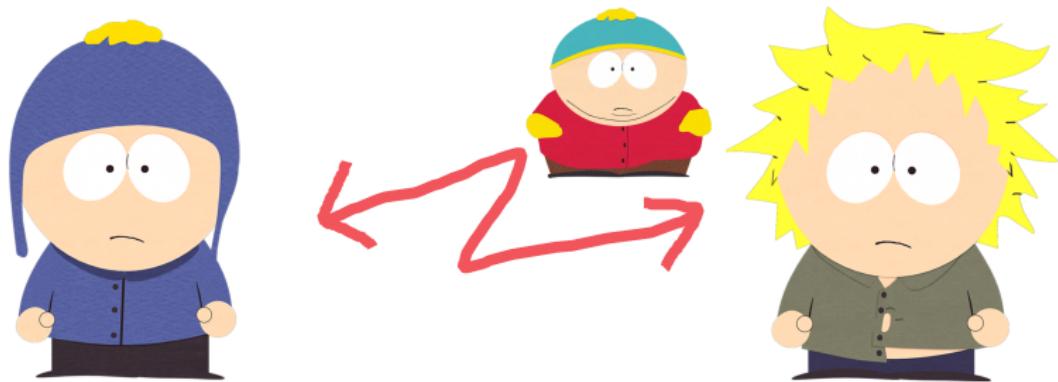
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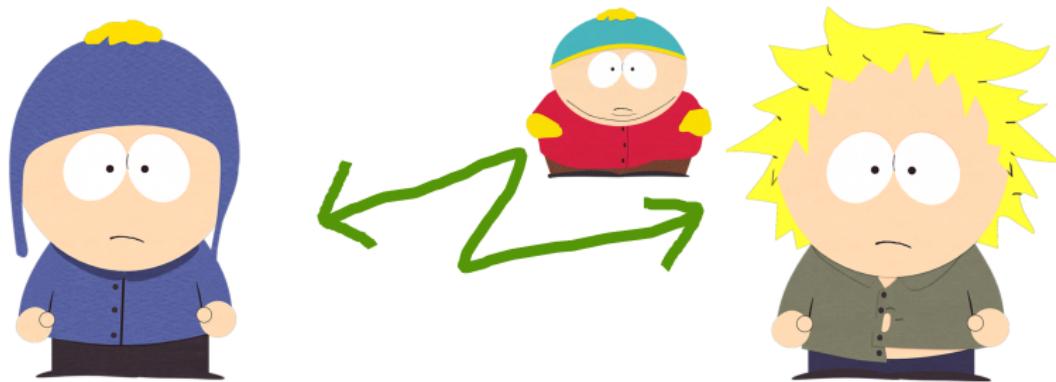
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# A hierarchy of cryptographic components

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A **protocol** (e.g. TLS) uses among others

- ▶ A key exchange algorithm (e.g. Diffie-Hellman)
  - “**public-key**” **cryptography**
    - ▶ instantiated with a secure group (e.g. ANSSI FRP256V1)
- ▶ An authenticated-encryption mode of operation (e.g. GCM)
  - “**symmetric-key**” **cryptography**
    - ▶ instantiated with a secure **block cipher** (e.g. the AES)
- ▶ A digital signature algorithm (e.g. ECDSA)
  - “**public-key**” + “**symmetric-key**” **cryptography**
    - ▶ instantiated with a secure group and a secure **hash function** (e.g. SHA-3)

# Primitive-centered crypto in a nutshell

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- ▶ **Design** new primitives
  - ▶ Fast, lightweight, quantum-resistant (isogeny-based, etc.), ...
- ▶ **Analyse** new proposals
- ▶ Analyse standards
  - ▶ AES, SHA-{1,2,3}, ...

# Focus of this thesis

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We studied various aspects of

- design
- analysis
- implementation

of block ciphers and hash functions

# List of publications

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Efficient and Provable White-Box Primitives

with Pierre-Alain Fouque, Paul Kirchner and Brice Minaud  
ASIACRYPT 2016

Freestart collision for full SHA-1

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## Block cipher

A block cipher is a mapping  $\mathcal{E} : \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{C}$  s.t. for all  $k \in \mathcal{K}$ ,  $\mathcal{E}(k, \cdot)$  is invertible and  $\mathcal{E}(k, \cdot)$ ,  $\mathcal{E}^{-1}(k, \cdot)$  are efficiently computable.

In most cases:

- $\mathcal{M} = \mathcal{C} = \{0, 1\}^n$ ,  $n \in \{64, 128\}$
- $\mathcal{K} = \{0, 1\}^\kappa$ ,  $\kappa \in \{64, 80, 128, 256\}$

# Security of block ciphers

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Ideal block cipher

Key-recovery security

Differential cryptanalysis (Biham and Shamir, 1990)

Exploit statistical properties of  $x \mapsto \mathcal{E}(k, x) \oplus \mathcal{E}(k, x \oplus \Delta)$

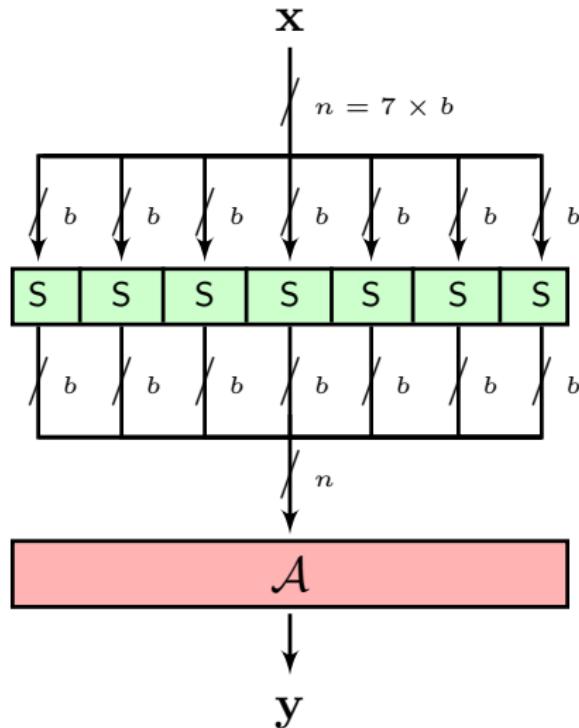
# Substitution-Permutation-Network block ciphers

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SPN round function:  $\mathcal{R} = \mathcal{A} \circ \mathcal{S}$

- ▶  $\mathcal{S}$  is the parallel application of  $s$   $b$ -bit *S-boxes*
- ▶  $\mathcal{A}$  is an *affine transformation* over  $\mathbf{F}_2^n$ ,  $n = s \times b$

# An SPN round function in a picture



## The wide-trail strategy (Daemen, 1995)

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- ▶ Use structure in  $\mathcal{A}$  and  $\mathcal{S}$  to lower-bound the number of *active S-boxes* in a differential characteristic or linear approximation
- ▶ Introduce a notion of *diffusion* for a round function
- ▶ Canonical example: the [AES](#) (Daemen & Rijmen, 2002)

# New linear mappings for block ciphers

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*Diffusion Matrices from Algebraic-Geometry Codes with Efficient SIMD Implementation,*  
with Daniel Augot and Pierre-Alain Fouque,  
SAC 2014

# The objective

- ▶ Find  $\mathcal{A}$  over  $\mathbb{F}_{2^b}^s$  with very good *diffusion*, with  $b$  small (i.e. 4)

A diffusion measure: the branch number (Daemen 1995)

Let  $M \in \mathcal{M}_s(\mathbb{F}_{2^b})$ ,  $\mathbf{x} \in \mathbb{F}_{2^b}^s$  and  $\text{wt}(\mathbf{x})$  be the number of non-zero coordinates of  $\mathbf{x}$ .

The *differential branch number* of  $M$  is

$$\min_{\mathbf{x} \neq 0} (\text{wt}(\mathbf{x}) + \text{wt}(M\mathbf{x}))$$

The *linear branch number* of  $M$  is

$$\min_{\mathbf{x} \neq 0} (\text{wt}(\mathbf{x}) + \text{wt}(M^t\mathbf{x}))$$

## Branch number & minimal distance

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- ▶  $M \in \mathcal{M}_s(\mathbf{F}_{2^b})$  has differential branch number  $d$   
 $\Leftrightarrow [I_s \ M]$  generates a  $[2s, s, d]_{\mathbf{F}_{2^b}}$  code
- ▶ Singleton bound:  $d$  is at most  $s + 1$ ; equality  $\Rightarrow$  MDS code,  
**MDS matrix**
- ▶ **MDS conjecture**: there is no  $[2s, s, s + 1]_{\mathbf{F}_{2^b}}$  MDS code with  
 $2s > 2^b$

# The idea

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- ▶ Take for  $\mathcal{A}$  a single  $M$  with high branch number (SHARK structure, Rijmen et al., 1996)
- ▶ For  $b = 4$ ,  $s \geq 16$  (block size  $\geq 64$ ),  $M$  cannot be MDS
- ▶ ⇒ Use *Algebraic geometry codes*: trade length for minimal distance (Goppa, 1981), (Tsfasman, Vlăduț, Zink, 1982)

AG codes as **evaluation codes**:

- ▶ Say we want an  $[n, k]_{\mathbf{F}_q}$  code

- 1 Let  $\mathcal{X}$  be a smooth plane **curve of genus  $g$**  with  $\#\mathcal{X}(\mathbf{F}_q) > n$
- 2 Inject  $\mathbf{F}_q^k$  to  $\mathcal{L}(rP)$  of dim.  $k$  with  $P \in \mathcal{X}(\mathbf{F}_q)$
- 3  $\mathcal{C}(m)$  is the evaluation of  $m$  on  $n$  distinct points of  $\mathcal{X} \setminus \{P\}$
- 4 For well-chosen  $n$  and  $k$ ,  $r = k - 1 + g$  (Riemann, Roch)  
 $\Rightarrow \text{wt}(\mathcal{C}(m \neq 0)) \geq n - (k - 1 + g)$

- ▶  $\Rightarrow$  the **min. distance is  $g$  less than MDS**, but  $\mathcal{X}$  can be chosen s.t.  $\#\mathcal{X}(\mathbf{F}_q) > q + 1$

# A concrete AG code

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- Let  $\mathcal{X}$  be of equation  $x^5 = y^2z^3 + yz^4$  in  $\mathbf{P}^2(\mathbf{F}_{2^4})$
- It is a maximal curve of genus 2 and has 33 points
- $\Rightarrow$  We can define a  $[32, 16, 15]_{\mathbf{F}_{2^4}}$  code  $\mathcal{C}$
- This gives many (up to  $32!$ )  $M \in \mathcal{M}_{16}(\mathbf{F}_{2^4})$  of diff. branch number 15
- The dual also has min. distance 15  $\Rightarrow M$  has lin. branch number 15

# Implementation matters

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For  $M$  to be used as  $\mathcal{A}$  in a block cipher, we need:

- Efficient implementations of multiplication by  $M$ ...
- ... That are “constant-time”, to protect against side-channel attacks

Thus we:

- Defined good vectorized algorithms for  $\times M$  using pshufb
- Optimized the structure of  $M$  for faster multiplication

A cost function for  $\times M$ : #pshufb

- Combinatorial in nature
- Low cost can be obtained if  $M$  is generated by a single row

# Tuning $M$ for fast implementations

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- ▶ Idea: use the **automorphisms of  $\mathcal{C}$**  to find a circulant  $M$
- ▶ Result: no luck, but still got  $M$  generated by 8 rows (cost 52)
- ▶ Second idea: **randomly sample** many generating matrices  
 $(\approx 2^{38})$
- ▶ Result: found many matrices of cost 43

# Structured block $M$ (cost 52)

5	2	1	3	8	5	1	5	12	10	14	6	7	11	4	11
2	2	4	1	5	12	2	1	9	15	8	11	7	6	9	3
1	4	4	3	1	2	15	4	5	13	10	12	9	6	7	13
3	1	3	3	5	1	4	10	14	2	14	8	15	13	7	6
8	5	1	5	5	2	1	3	7	11	4	11	12	10	14	6
5	12	2	1	2	2	4	1	7	6	9	3	9	15	8	11
1	2	15	4	1	4	4	3	9	6	7	13	5	13	10	12
5	1	4	10	3	1	3	3	15	13	7	6	14	2	14	8
12	9	5	14	7	7	9	15	7	6	11	3	15	5	13	7
10	15	13	2	11	6	6	13	6	6	7	9	5	10	2	14
14	8	10	14	4	9	7	7	11	7	7	6	13	2	8	4
6	11	12	8	11	3	13	6	3	9	6	6	7	14	4	12
7	7	9	15	12	9	5	14	15	5	13	7	7	6	11	3
11	6	6	13	10	15	13	2	5	10	2	14	6	6	7	9
4	9	7	7	14	8	10	14	13	2	8	4	11	7	7	6
11	3	13	6	6	11	12	8	7	14	4	12	3	9	6	6

# Unstructured fast $M$ (cost 43)

11	6	1	6	10	14	10	9	13	3	3	12	9	15	2	9
6	12	0	4	2	8	9	2	5	11	9	5	4	1	15	6
9	11	2	2	1	11	13	15	13	3	2	1	14	1	3	10
0	0	9	8	11	6	2	1	11	10	15	10	10	15	1	14
13	13	3	15	3	1	11	2	9	2	10	14	1	11	1	2
1	9	8	4	14	10	2	5	15	2	12	12	9	10	1	9
5	9	11	2	15	1	12	4	6	0	6	4	5	8	2	9
1	4	14	9	13	2	10	12	0	6	6	9	2	0	11	10
13	10	3	9	2	15	6	6	11	1	9	9	12	14	10	3
0	10	6	12	11	0	4	9	1	14	10	2	9	2	13	6
2	0	5	6	9	0	1	5	15	12	13	15	1	11	13	11
11	2	10	1	1	15	0	8	0	9	14	10	10	6	11	15
12	14	10	11	3	10	6	0	5	11	1	8	2	9	2	3
15	2	2	5	1	10	9	4	1	8	9	9	12	10	14	12
15	1	12	5	13	11	0	6	2	5	11	1	15	0	9	13
5	6	11	0	2	9	14	11	12	10	3	2	8	10	3	1

## Performance and applications

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- ▶ Eight ( $A \circ S$ ) rounds should resist attacks
- ▶ ⇒ On Sandy Bridge,  $\approx 30$  cycles per byte for good  $M$  (AVX assembly)
- ▶ Not so fast for general-purpose...
- ▶ ... But only needs 128 S-box applications ( $\approx 512$  N.L. gates)
  
- ▶ Potential application: cipher suitable for masking at very high order, e.g. beat Mysterion (Journault, Standaert, Varici, 2016)

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## Hash function

A hash function is a mapping  $\mathcal{H} : \mathcal{M} \rightarrow \mathcal{D}$

In practice:

- $\mathcal{M} = \bigcup_{\ell < N} \{0, 1\}^\ell$ ,  $\mathcal{D} = \{0, 1\}^n$ ,  $N \gg n$
- Typically  $N = 2^{64}$ ,  $n \in \{128, 160, 224, 256, 384, 512\}$
- It is a **keyless** primitive
- What's a **good hash function?**

# Three security notions

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First preimage resistance

Second preimage resistance

Collision resistance

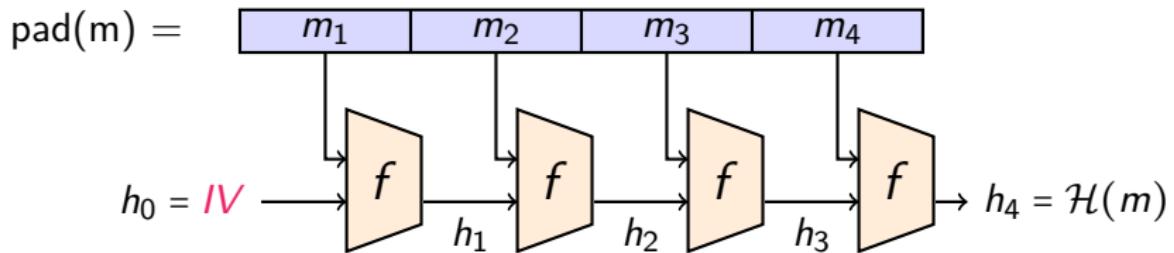
Find  $m, m' \neq m$  s.t.  $\mathcal{H}(m) = \mathcal{H}(m')$

Best generic attack is in  $\mathcal{O}(2^{\frac{n}{2}})$

# Merkle-Damgård construction in a picture

(Merkle, 1989), (Damgård, 1989)

Define variable-input-length  $\mathcal{H}$  from fixed-input-length  $f$ :



Security reduction of  $\mathcal{H}$  to  $f$

$\mathcal{H}_i$ :  $\mathcal{H}$  with  $IV$  set to  $i$

## Freestart collisions

A freestart collision is a pair  $((i, m), (i', m'))$  s.t.  $\mathcal{H}_i(m) = \mathcal{H}_{i'}(m')$

## Freestart collisions (variant)

Attack  $f$  instead of  $\mathcal{H}$

# Practical freestart collisions for SHA-1

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*Freestart collision for full SHA-1,*  
with Thomas Peyrin and Marc Stevens,  
EUROCRYPT 2016

# The objective

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- ▶ Show that SHA-1 is *really* not secure
- ▶ Find a **practical** attack on the full SHA-1 for a well-defined security notion
- ▶ And implement it

# The idea

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- ▶ Collision attacks are **near-practical** (Wang, Yin, Yu, 2005)
- ▶ ⇒ Move to a freestart model to make the attack **faster**

# The SHA-1 hash function

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- ▶ Designed by the NSA in 1995
- ▶ Hash size is 160 bits ⇒ collision security should be 80 bits
- ▶ Compression function in Merkle-Damgård mode

# SHA-1 compression function

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Block cipher in Davies-Meyer mode

Block cipher: 5-branch ARX Feistel

$$A_{i+1} = A_i^{\odot 5} + \phi_{i \div 20}(A_{i-1}, A_{i-2}^{\odot 2}, A_{i-3}^{\odot 2}) + A_{i-4}^{\odot 2} + W_i + K_{i \div 20}$$

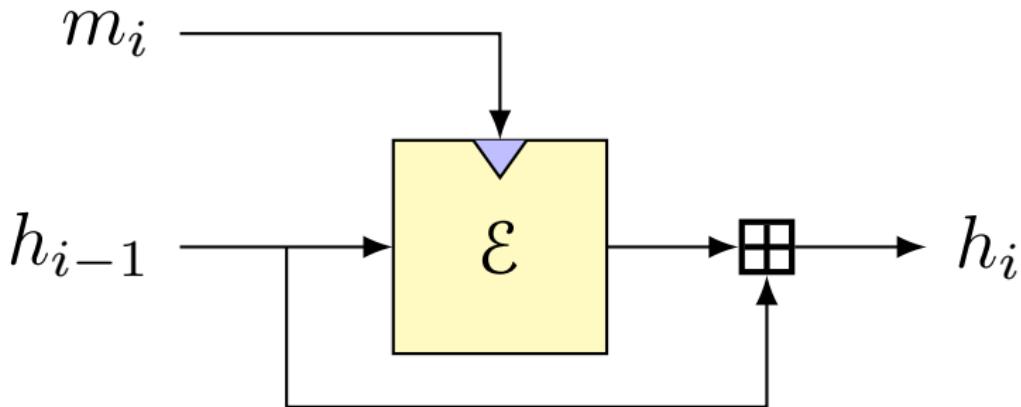
with a linear message (key) expansion:

$$W_{0 \dots 15} = M_{0 \dots 15}, \quad W_{i \geq 16} = (W_{i-3} \oplus W_{i-8} \oplus W_{i-14} \oplus W_{i-16})^{\odot 1}$$

80 steps in total

# Davies-Meyer construction in a picture

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# Collision attacks for the hash function

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SHA-1 is **not collision-resistant** (Wang, Yin, Yu, 2005)

Attack complexity  $\approx 2^{69}$

Eventually improved to  $\approx 2^{61}$  (Stevens, 2013)

## Differential collision attack (Wang et al., 2005)

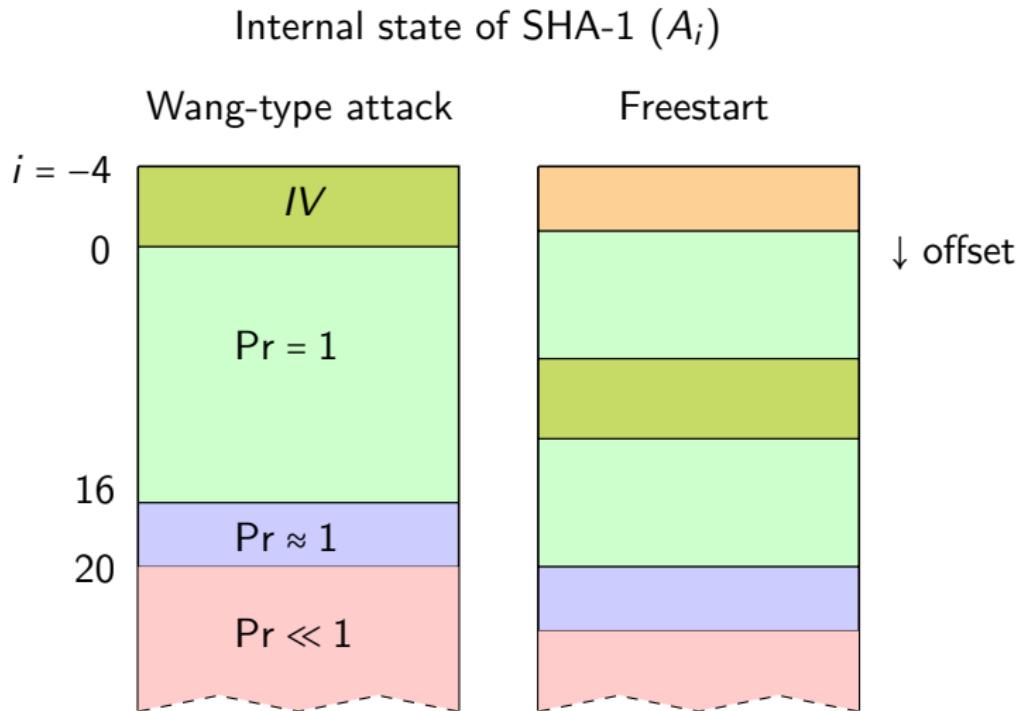
- 1 Find a *good linear* differential path for the message
- 2 Construct a *non-linear* diff. path to connect the *IV* to the linear path

# Time for an attack

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# The point of freestart (in a picture)



# The steps of the attack (Wang-type)

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- 1 Find a good linear part (for the differential path)
- 2 Construct a good shifted non-linear part (for the diff. path)
- 3 Instantiate accelerating techniques

Let's do this for 80 steps!

# Linear part selection

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## Criteria:

- ▶ High overall probability
  - ▶ No (or few) differences in last five steps (= differences in  $/V$ )
  - ▶ Few differences in early message words
- ⇒ Not many candidates

We picked II(59,0) (Manuel notation, 2011)

# Linear path in a picture (last 20 steps)

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$i$	$A_i$	$W_i$
61	●oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo●oooo
62	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
63	●oooooooooooooooooooooooooooo	oooo●oooooooooooooooooooo●oooo
64	oooooooooooooooooooooooooooo	●oooooooooooooooooooooooo
65	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
66	oooooooooooooooooooooooooooo	oooo●oooooooooooooooooooo
67	oooooooooooooooooooooooooooo	oooo●oooooooooooooooooooo
68	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
69	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
70	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
71	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
72	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
73	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo
74	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo●
75	oooooooooooooooooooooooooooo●	oooooooooooooooooooooooooooo●oooo
76	oooooooooooooooooooooooooooo	oooooooooooooooooooooooooooo●
77	oooooooooooooooooooooooooooo	oooo●oooooooooooooooooooo●oooo
78	oooooooooooooooooooooooooooo●	oooo●oooooooooooooooooooo●oooo●
79	oooooooooooooooooooooooooooo●	oooo●oooooooooooooooooooo●oooo●o
80	oooooooooooooooooooooooooooo	

# Non-linear part construction

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- ▶ Start with **prefix of high backward probability** for the first 4 steps
- ▶ Use a mix of **automated search** for the rest

## Non-linear path in a picture

# Accelerating techniques

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Attack process:

- ▶ Generate many “**partial solutions**”: message pairs following the diff. path up to some step
- ▶ Hope that one yields a **collision**

To make this efficient, use:

- ▶ **Message modification** (Wang et al., 2005)
- ▶ **Neutral bits** (Biham and Chen, 2004):  
Generate more good instances when one's found
- ▶ We choose neutral bits because:
  - ▶ **Easy** to find
  - ▶ **Easy** to implement

## Neutral bits (with an offset)

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- ▶ We start with an **offset**
- ▶ ⇒ Use neutral bits with an offset too
- ▶ In our attack, **offset = 5**
  - ▶ free message words = **W5...20** instead of W0...15

## Let's sum up

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- ▶ Initialize the state **with an offset**
- ▶ Initialize message words **with an offset**
- ▶ Use neutral bits **with an offset**
- ▶ ⇒ many neutral bits up to **late steps** (**good!**)
- ▶ ⇒ don't know the **IV** in advance (**well...**)

## If it's practical you must run it

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- ▶ Attack expected to be practical, but still **expensive**
- ▶ Why not using **GPUs**?
- ▶ One main challenge: how to deal with the **branching**?

# Architecture imperatives

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Target platform: Nvidia GTX-970

- Execution is bundled in **warps** of 32 threads
- **Single Instruction Multiple Threads:**  
Control-flow **divergence is serialized**  $\Rightarrow$  **minimize branching**
- Hide latency by grouping warps into **blocks**

# Our snippet-based approach

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- 1 Store **partial solutions** up to some step in **shared buffers**
- 2 Every thread of a block loads one solution
- 3 ... tries **all neutral bits** for this step
- 4 ... stores **successful candidates** in next step buffer

# GPU results

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- ▶ Hardware: 64 GTX-970
- ▶ ⇒ Expected time to find a collision  $\lesssim 10$  days
  - ▶ Energy cost  $\equiv$  boil 50 kL of 20°C water
- ▶ Complexity  $\equiv 2^{57.5}$  SHA-1 compression function
- ▶ Conjectured 250-500 days for hash function collision with the same cluster

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# Summary of this thesis

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The work done in this thesis included:

- ▶ **Design** of block ciphers
  - ▶ Fly (lightweight), PuppyCipher & CoureurDesBois (whitebox), (Samneric (large matrices))
- ▶ **Analysis** of encryption schemes
  - ▶ Prøst-OTR (related-key), ASASA family
- ▶ And of hash functions
  - ▶ SHA-1, BLAKE(2)

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