# Introduction to cryptology (GBIN8U16) Password Hashing

### Pierre Karpman pierre.karpman@univ-grenoble-alpes.fr https://www-ljk.imag.fr/membres/Pierre.Karpman/tea.html

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**Password Hashing** 

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A simple login/password interaction:

- **1** User U wants to log on system S; sends password p
- System S checks password associated with U in database  $D = \{(U_i, p_i)\}$ ; grants access if equal to p

A simple total break:

- 1 Adversary A steals database D (Quite realistic; happens a lot)
- $\Rightarrow$  Passwords must never be stored *in clear*!

A first attempt (aborted):

- Store p encrypted with, say, CBC-ENC
- U, S Need to store/know the secret key: nothing is solved

A first attempt:

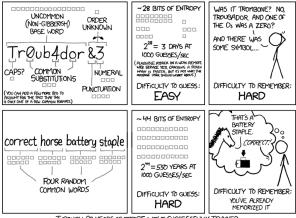
- Store p encrypted with, say, RSA-OAEP
- U, needs to know S's public key
- S has a single secret to store (but always used to decrypt; still bad)

A second atttempt:

- Store hashed passwords  $\mathcal{H}(p) \rightsquigarrow D = \{(U_i, \mathcal{H}(p_i))\}$
- > S checks that the received password hashes to the right value
- If  $\mathcal{H}$  is preimage-resistant,  $\mathcal{H}(p) \not\rightarrow p$ ?
- Basically sound, but still with some problems!

- ▶ Let  $\mathcal{H}: \{0,1\}^* \to \{0,1\}^n$ . For any set  $\mathcal{S}, \#\mathcal{S} \leq 2^{n/2}, x \in \mathcal{S}$  can be found in time  $\langle \#\mathcal{S} \text{ given } \mathcal{H}(x) \text{ (Question: how?)}$
- If  $\mathcal{H}(x)$  is used to identify x, any preimage works
- "Inverting"  $\mathcal{H}$  takes time  $\approx \min(2^n, \#S)$  (Assuming  $x \stackrel{\$}{\leftarrow} S$ )
- Not a problem of hash functions specifically, just the absence of (other) secret

## Password entropy: a global issue



THROUGH 20 YEARS OF EFFORT, WE'VE SUCCESSFULLY TRAINED EVERYONE TO USE PASSWORDS THAT ARE HARD FOR HUMANS TO REMEMBER, BUT EASY FOR COMPUTERS TO GUESS.

https://xkcd.com/936/

### Microsoft's LM hash? (1980's)

- 1 Truncate p to 14 ASCII characters
- 2 Convert it to uppercase
- **3** Split it in two halves  $p_0$ ,  $p_1$
- 4 LMHash $(p) = DES(p_0, c) || DES(p_1, c)$  for a fixed constant c
  - $\blacktriangleright$  DES :  $\{0,1\}^{56}\times\{0,1\}^{64}\rightarrow\{0,1\}^{64}$  is a block cipher

## What's wrong with that?

- Final The two halves of the hash are processed separately
- Only  $69^7 \lessapprox 2^{43}$  possible inputs per half
  - Only 2<sup>20</sup> seconds on one core of this laptop needed to exhaust them
- Impossible to securely store a strong password

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- $\blacktriangleright$  A "modern" answer: just take  ${\cal H}$  to be, say, SHA3-256
- Problem: multi-target attacks are (still) easy
  - An adversary may want to find one password among N
  - For every candidate p', check if  $\mathcal{H}(p') \in D$
  - The work is decreased by a factor  $\approx N$
  - N might be large (say, > 1000)

• One counter-measure: use different functions for every user

- Simple to implement: every user  $U_i$  selects a large random number  $r_i$ ;  $D = \{(U_i, r_i, \mathcal{H}(r_i || p_i))\} \leftarrow \text{If } \mathcal{H} \text{ is not SHA3, something different from prefixing might be necessary}$
- One has to check for every candidate p', for every user if p' is the right password

- If a password is "random enough", salted hash is fine
- But most/some might not be that
- Assume that one:
  - ▶ Has 2<sup>50</sup> password candidates for a user
  - ▶ Can compute 2<sup>23</sup> hashes/core/second
  - Has 128 available cores
  - ▶ ⇒ Only  $2^{20}$  seconds (< two weeks) to find *p* (that's not enough)
- One counter-measure: make hash functions *slower* 
  - Not slow enough to hinder the user
  - Slow enough to make exhaustive search too costly

- Instead of computing  $\mathcal{H}(r \| p)$  once, iterate many times!
- Example: PBKDF2
  - $h \approx \bigoplus_{i=0}^{c} h_i$ ;  $h_i = \mathcal{H}(h_{i-1} || p)$ ;  $h_0 = r$
  - Choose the iteration count c to be "large enough"
  - Typically  $c \approx 1000$
- Say it takes 10ms to hash one password  $\Rightarrow$  35 years on 10 000 cores to try 2<sup>50</sup> candidates for one user
- One problem:
  - The user *needs* to hash on a regular core
  - An adversary may try hashes on fast dedicated circuits

#### A reasonable assumption:

- A PBKDF2 hash function can be computed 2<sup>20</sup> times faster than on a CPU core, using dedicated hardware with low amortized cost
- ▶ 10ms to hash one password on CPU  $\Rightarrow < 2^{-26}$  on efficient hardware  $\Rightarrow < 2^{20}$  seconds on 10 machines to try  $2^{50}$  passwords

How to solve this?

- Cannot make the user wait one day to check a password
- So use hashing that's *slow everywhere*

An assumption: memory is slow for everybody

- So use a "memory-hard" hash function that needs a lot of memory to be computed
- A framework: the output must depend on "many" intermediate values, accessed many times → a (quadratic) tradeoff
  - Either store all intermediate values (costs memory)
  - Or recompute them as needed (costs time)
- Only increases memory consumption (not time) of hashing a password for a generic user
- Makes dedicated hardware not more efficient than regular CPU (hopefully)

Scrypt (Percival, 2009), the (very rough) idea:

- Use the password and salt to generate a large buffer
- Access the buffer in a sequential and unpredictible way to generate the output

A bit more precisely:

**1** 
$$h_i = \mathcal{H}(h_{i-1}); h_0 = r || p$$
, for *i* up to  $n-1$ 

2  $s_i = \mathcal{H}(s_{i-1} \oplus h_{s_{i-1} \mod n}), s_0 = \mathcal{H}(h_{n-1}), \text{ for } i \text{ up to } n$ 

8 Return s<sub>n</sub>

The visible intuitive tradeoff from two slides ago:

- Either store all the  $h_i \rightsquigarrow$  time = memory  $\approx n$  calls to  $\mathcal{H}$
- Either recompute  $h_{s_{i-1} \mod n}$  once  $s_{i-1}$  is known  $\sim$  constant memory, time  $\approx n \times n/2$

 $\Rightarrow$  Only a few MB of generated values might be enough to defeat special-purpose hardware

 One can in fact prove that the above tradeoff is roughly optimal (Alwen & al., 2016) HKDF (Boyen, 2007) uses a memory-hard function with an (optionally) *unknown* iteration count

- **I** A user computes an iterated function on the password *p*
- Interrupts the process when wanted; obtains a hash h of p and a verification string v
- 3 The hash and the iteration count can be retrieved from p and v
- The user may tune the iteration count on its own to its requirements
- Without that knowledge, an adversary is less efficient

# HKDF: How?

Preparation phase: Input: <i>p</i> , <i>r</i> , <i>t</i>
Output: h, v, r
$\mathbf{I}  z = \mathcal{H}(r  p)$
<b>2</b> For $i = 1, \ldots, t \triangleleft t$ may be user-defined
$y_i = z$
4 For $* = 1, \dots, q \triangleleft q$ controls the time/space ratio
5 $j = 1 + (z \mod i)$
$\mathbf{G} \qquad \mathbf{z} = \mathcal{H}(\mathbf{z}    \mathbf{y}_j)$
7 Return r; $v = \mathcal{H}(y_1  z)$ ; $h = \mathcal{H}(z  r)$

Extraction phase: Input: <i>p</i> , <i>r</i> , <i>v</i> Output: <i>h</i>
•
$1  z = \mathcal{H}(r  p)$
<b>2</b> For $i = 1,, \infty$
$3   y_i = z$
4 For $* = 1,, q$
5 $j = 1 + (z \mod i)$
$6 \qquad \mathbf{z} = \mathcal{H}(\mathbf{z}  \mathbf{y}_j)$
If $(\mathcal{H}(y_1  z) = v)$ Then Break
8 Return $h = \mathcal{H}(z  r)$

- Both functions use password-dependent memory accesses
- May leak information about the password
- So (memory-hard) functions with password-independent accesses may sometimes be preferable
- For (some) more on password hashing: https://password-hashing.net/

## To finish: something a bit different



It may be useful to have a hash function that:

- Is slow to execute (i.e. it is slow to compute  $y \coloneqq \mathcal{H}(x)$  given x)
- ▶ Is fast to verify (i.e. it is fast to check that  $y = \mathcal{H}(x)$  given x and y)

An application:

Collaborative random-number generation

### Randomness beacon

A *Randomness beacon* is a system that publishes (pseudo-)random numbers at regular interval

#### Example:

https://beacon.nist.gov/home

Some applications:

- Remote random consensus ("Shall we go to a pizzeria or a crêperie?")
- (Faster) challenge generation in authentication protocols
- Lotteries
- Jury/assembly selection

# Collaborative beacons

One can distinguish:

- "Oracle" beacons (have to be trusted)
- "Collaborative" beacons (everyone can contribute)
- A design strategy (Lenstra & Wesolowski, 2015):
  - **I** Use a slow hash function with fast verification that takes wall time  $> \Delta$  to be computed (hopefully on the best platform)
  - 2 Gather public seeds from time  $t \Delta$  to t
  - 3 At time t, hash all collected seeds, then publish the hash
  - 4 Everyone can efficiently test the result and its dependence on the seeds
    - An adversary does not have time to precompute a hash and insert a seed that biases the result

Sloth: A slow hash function in a nutshell:

- If p ≡ 3 mod 4 is a (large) prime, if x ∈ 𝔽<sup>×</sup><sub>p</sub> is a square mod p, the fastest know way to compute a square root of x is as x<sup>(p+1)/4</sup>
- ► Exactly one of x or -x is a square ⇒ one can map any number to a well-defined square root
- Computing a square root takes ≈ log(p) more time than "verifying" one
- So (to make things more modular):
  - Compute an iterative chain of square roots
  - Interleaved with, say, block cipher applications to break the algebraic structure

- Sloth is not memory-hard, but CPUs are good at big-number arithmetic
  - Dedicated hardware may not be a threat
  - (Some password-hashing functions are based on the same assumption (Pornin, 2014))
- A Twitter-accessible beacon: https://twitter.com/random\_zoo