

Crypto Engineering '21



Hash functions

Pierre Karpman

`pierre.karpman@univ-grenoble-alpes.fr`

`https://membres-ljk.imag.fr/Pierre.Karpman/tea.html`

2021-09-30

Hash functions as a figure

↷ on the board

First definition

Hash function

A hash function is a mapping $\mathcal{H} : \mathcal{M} \rightarrow \mathcal{D}$

So it really is just a function...

Usually:

- ▶ $\mathcal{M} = \bigcup_{\ell < N} \{0, 1\}^{\ell}$, $\mathcal{D} = \{0, 1\}^n$, $N \gg n$
- ▶ N is typically $\geq 2^{64}$, $n \in \{128, 160, 192, 224, 256, 384, 512\}$

Also popular now: extendable-output functions (XOFs): $\mathcal{D} = \bigcup_{\ell < N'} \{0, 1\}^{\ell}$

- ▶ Hash functions are *keyless*
- ▶ So, how do you tell if one's **good**?

Random oracle

A function $\mathcal{H} : \mathcal{M} \rightarrow \mathcal{D}$ s.t. $\forall x \in \mathcal{M}, \mathcal{H}(x) \leftarrow \mathcal{D}$

- ▶ “The best we can ever get”
- ▶ Sometimes useful in proofs (“Random oracle model”, or ROM)
- ▶ Not possible to have one (except for small (co-)domains assuming a TRNG)
- ▶ But we can get *approximations* (e.g. SHA-3)
- ▶ Equivalent to the Ideal Cipher Model (Coron et al., 2008; + later patches)

Main security properties

What is hard for a RO should be hard for any HF

⇒

- 1 **First preimage**: given t , find m s.t. $\mathcal{H}(m) = t$
- 2 **Second preimage**: given m , find $m' \neq m$ s.t. $\mathcal{H}(m) = \mathcal{H}(m')$
- 3 **Collision**: find $(m, m' \neq m)$ s.t. $\mathcal{H}(m) = \mathcal{H}(m')$

Generic complexity:

1), 2): $\Theta(2^n)$;

3): $\Theta(2^{n/2}) \sim$ “Birthday paradox”

Why do we care? Applications!

Hash functions are useful for:

- ▶ Hash-and-sign (RSA signatures, (EC)DSA, ...)
- ▶ Message-authentication codes (HMAC, ...)
- ▶ Password hashing (with a grain of salt)
- ▶ Hash-based signatures (inefficient but PQ)
- ▶ As “RO instantiations” (OAEP, ...)
- ▶ As one-way functions (OWF)

So, how do you build hash functions?

- ▶ Objective #1: be secure
- ▶ Objective #2: be efficient
 - ▶ At most a few dozen cycles/byte!
 - ▶ \Rightarrow work with limited amount of memory

So...

- ▶ (#2) Build \mathcal{H} from a *small component*
- ▶ (#1) Prove that this is okay

What kind of small component?

Compression function

A compression function is a mapping $f : \{0, 1\}^n \times \{0, 1\}^b \rightarrow \{0, 1\}^n$

- ▶ A family of functions from n to n bits
- ▶ Not unlike a block cipher, only not invertible

Permutation

A permutation is an invertible mapping $p : \{0, 1\}^n \rightarrow \{0, 1\}^n$

Yes, very simple

- ▶ Like a block cipher with a fixed key, e.g. $p = \mathcal{E}(0, \cdot)$

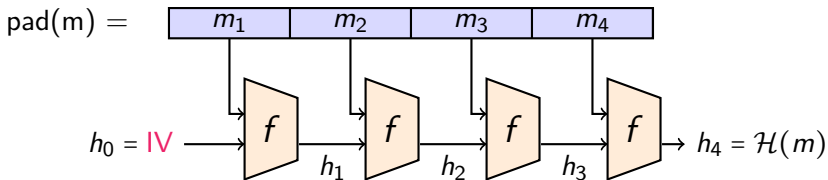
From small to big (compression function case)

Assume a good f

- ▶ Main problem: fixed-size domain $\{0, 1\}^n \times \{0, 1\}^b$
- ▶ Objective: *domain extension* to $\bigcup_{\ell < N} \{0, 1\}^\ell$
- ▶ (Not unlike using a mode of operation with a BC)

The classical answer: the Merkle-Damgård construction (1989)

MD: with a picture



That is: $\mathcal{H}(m_1||m_2||m_3||\dots) = f(\dots f(f(f(IV, m_1), m_2), m_3), \dots)$

$\text{pad}(m) \approx m||1000\dots 00(\text{length of } m) \leftarrow \text{strengthening}$

MD: does it work?

Efficiency?

- ▶ Only sequential calls to f
- ▶ \Rightarrow fine

Security?

- ▶ Still to be shown
- ▶ Objective: *reduce* security of \mathcal{H} to that of f
 - ▶ “If f is good, then \mathcal{H} is good”
- ▶ True for collision and first preimage, **false** for second preimage

MD (partial) security proof

Method: simple contrapositive argument

- ▶ Attack $\{1^{\text{st}}\text{preim.}, \text{coll.}\}$ on $\mathcal{H} \Rightarrow$ attack $\{1^{\text{st}}\text{preim.}, \text{coll.}\}$ on f

First preimage case

If $\mathcal{H}(m_1 \| m_2 \| \dots \| m_\ell) = t$, then $f(\mathcal{H}(m_1 \| m_2 \| \dots \| m_{\ell-1}), m_\ell) = t$

Collision case (sketch)

If $\mathcal{H}(m_1 \| m_2 \| \dots \| m_\ell) = \mathcal{H}(m'_1 \| m'_2 \| \dots \| m'_\ell)$, show that $\exists i$ s.t.
 $(h_i := \mathcal{H}(m_1 \| m_2 \| \dots \| m_{i-1}), m_i) \neq (h'_i := \mathcal{H}(m'_1 \| m'_2 \| \dots \| m'_{i-1}), m'_i)$
and $f(h_i, m_i) = f(h'_i, m'_i)$

- ▶ Proper message padding (such as stenghtening) necessary to make it work!

What about 2nd preimages??

No proof (with optimal resistance), can't have one:

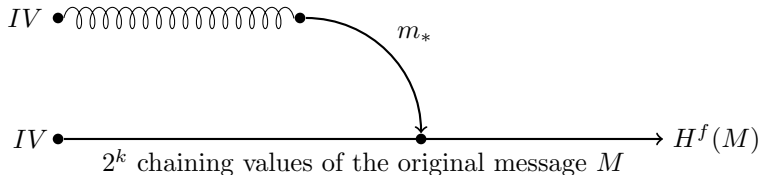
- ▶ Generic attack on messages of 2^k blocks for a cost $\approx k2^{n/2+1} + 2^{n-k+1}$ (Kelsey and Schneier, 2005)
- ▶ Idea: exploit internal collisions in the h_i s

This is not nice, but:

- ▶ Requires (very) long messages to gain something
- ▶ At least as expensive as collision search
 - ▶ Always going to be the case, as preimage \Rightarrow collision
- ▶ If n is chosen s.t. generic collisions are out of reach, we're somewhat fine

\Rightarrow Didn't make people give up MD hash functions (MD5, SHA-1, SHA-2 family)

Attack with an expandable message



<https://www.iacr.org/authors/tikz/>

Is that unavoidable?

No! Simple patch: Chop-MD/Wide-pipe MD (Coron et al., 2005) and (Lucks, 2005)

- ▶ Build \mathcal{H} from $f : \{0, 1\}^{2n} \times \{0, 1\}^b \rightarrow \{0, 1\}^{2n}$, truncate output to n bits (say)
- ▶ Collision in the output $\not\Rightarrow$ collision in the internal state
- ▶ Very strong provable guarantees (Coron et al.)
 - ▶ Secure domain extender for fixed-size RO
- ▶ Concrete instantiations: SHA-512/224, SHA-512/256 (2015)

But careful with models!

- ▶ Coron et al. show very strong provable guarantees for Chop-MD
 - ▶ Secure domain extender for fixed-size RO
- ▶ But this in fact doesn't guarantee weaker ones, such as preservation of collision-resistance (Bellare & Ristenpart, 2006)!
 - ▶ One can do “stupid things” with a non-ideal compression function
 - ▶ \rightsquigarrow Chop-MD with a (real) CR c.f. is not (necessarily) CR!
 - ▶ (In essence, one needs strengthening in the padding)

Practical impact of the MD proof

- ▶ If one can't attack f underlying \mathcal{H} , all is well
- ▶ Else, ...???
- ▶ \Rightarrow Attacking f is a meaningful goal for cryptographers (\approx (semi-)freestart attacks)
- ▶ Ideally, *never* use a \mathcal{H} with broken f

The MD5 failure

- ▶ MD5: designed by Rivest (1992)
- ▶ 1993: very efficient collision attack on the compression function (den Boer and Bosselaers); mean time of 4 minutes on a 33 MHz 80386
- ▶ MD5 still massively used...
- ▶ 2005: very efficient collision attack on the *hash function* (Wang and Yu)
- ▶ Still used...
- ▶ 2007: practically threatening collisions (Stevens et al.)
- ▶ Still used...
- ▶ 2009: even worse practical collision attacks (Stevens et al.)
- ▶ Hmm, maybe we should move on?

Was this avoidable?

Yes!

- ▶ Early signs of weaknesses \Rightarrow move to alternatives ASAP!
- ▶ What were they (among others)?
 - ▶ 1992: **RIPEMD** (RIPE); practically broken (collisions) 2005 (Wang et al.)
 - ▶ 1993: **SHA-0** (NSA); broken (collisions) 1998 (Chabaud and Joux); practically broken 2005 (Biham et al.)
 - ▶ 1995: **SHA-1** (NSA); broken (collisions) 2005 (Wang et al.); practically broken 2017 (Stevens et al. (and me!))
 - ▶ 1996: **RIPEMD-128** (Dobbertin et al.); broken (collisions) 2013 (Landelle and Peyrin)
 - ▶ 1996: **RIPEMD-160** (Dobbertin et al.); unbroken so far
 - ▶ 2001: **SHA-2** (NSA); unbroken so far

Lesson to learn?

- ▶ Don't use broken algorithms
- ▶ Care about “theoretical” attacks
- ▶ An attack that's “too expensive” may become practical in the future

Perfect bad example: Git

- ▶ Don't use SHA-1 in 2005!
- ▶ Don't hide needed security properties!

Also:

- ▶ Don't use MD5, SHA-1..., even if you just care about preimage attacks

Back to design: how to do f ?

- 1 Start like a block cipher
- 2 Add feedforward to prevent invertibility

Examples:

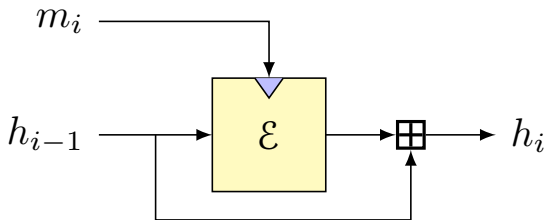
“Davies-Meyer”: $f(h, m) = \mathcal{E}_m(h) \boxplus h$

“Matyas-Meyer-Oseas”: $f(h, m) = \mathcal{E}_h(m) \boxplus m$

- ▶ Systematic analysis by Preneel, Govaerts and Vandewalle (1993). “PGV” constructions
- ▶ Then rigorous proofs **in the ideal cipher model** (Black et al., 2002), (Black et al., 2010)

Re: Davies-Meyer

Picture:



Used in MD4/5 SHA-0/1/2, etc.

Why is the “message” the “key”?

- ▶ We're not encrypting with \mathcal{E} so we can do what we want
- ▶ Disconnect chaining value and message length!
- ▶ \mathcal{E} 's block length: fixed by security level
- ▶ \mathcal{E} 's key length: fixed by “message” length
- ▶ Large “key” \Rightarrow more efficient
- ▶ Example: MD5's “block cipher” (also bad): 128-bit blocks, 512-bit keys

DM incentive: use very simple *message expansion* (“key schedules”)

- ▶ To be efficient!
- ▶ Warning: can be a source of weakness (MD5, SHA-0, SHA-1. Should that be surprising?)

Major PGV Warning

PGV constructions are proved secure in the *ideal cipher model*,
BUT

- ▶ Real ciphers are not ideal!
- ▶ Real ciphers *don't have to be ideal* to be okay ciphers (e.g. a “good” PRP is usually enough)
 - ▶ IDEA (Lai and Massey, 1991): weak key classes (Daemen et al., 1993)
 - ▶ TEA (Wheeler and Needham, 1994): equivalent keys (Kelsey et al., 1996)

What can go wrong?

Bad case of crypto design

Microsoft needed a hash function for ROM integrity check of the XBOX

- ▶ Used TEA in DM mode (Steil, 2005)
- ▶ Because of an earlier break of their RC4-CBC-MAC scheme (ibid.)
- ▶ Terrible idea, because of existence of equivalent keys!
- ▶ $\text{TEA}(k, m) = \text{TEA}(\hat{k}, m) \Rightarrow \text{DM-TEA}(h, k) = \text{DM-TEA}(h, \hat{k}) \Rightarrow$ easy collisions!
- ▶ Got hacked...
- ▶ IDEA for a hash function: also bad (Wei et al., 2012)

Never design your own crypto!

It's not all that bad, tho

- ▶ AES in a PGV construction so far unbroken (see e.g. Sasaki (2011))
 - ▶ But small parameters ?
- ▶ Ditto, SHA-256 as a block cipher: “SHACAL-2” (Handschuh and Naccache, 2001)
 - ▶ Enormous keys, 512 bit!



Now just a few examples

The MD4/MD5/SHA-0/SHA-1 family

- ▶ Merkle-Damgård construction
- ▶ Davies-Meyer compression function
 - ▶ 512-bit messages, 128-bit (resp. 160-bit) chaining values for MD4/5 (resp. SHA-0/1)
 - ▶ Very simple message expansion
- ▶ All broken (MD4 could be broken *by hand*)

RIPEMD family

- ▶ Somewhat similar, but uses two parallel lines merged at the end

SHA-2

- ▶ Somewhat similar, but heavier message expansion

And now for something different

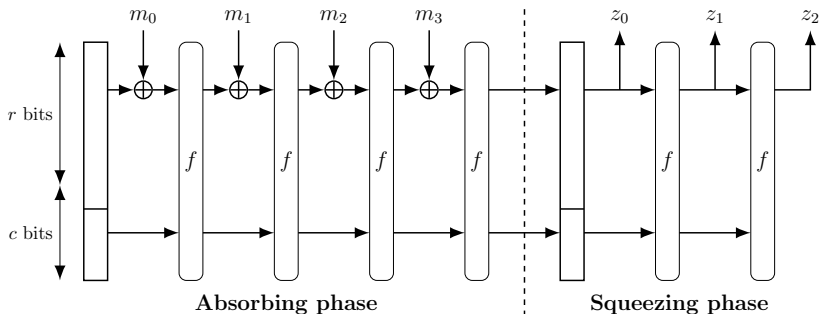
If you need a hash function today \Rightarrow SHA-3 (initially Keccak, (Bertoni et al., 2008))

- ▶ Winner of an academic competition run by NIST (2008–2012)
- ▶ Sponge construction (not Merkle-Damgård)
- ▶ Based on a permutation (not a compression function)

Sponge:

- 1 Compute $i := p(p(\dots p(m_1 || 0^c) \oplus m_2 || 0^c) \dots)$
- 2 Output $\mathcal{H}(m) := [i]_r || [p(i)]_r || \dots || [p^n(i)]_r$

Picture of a sponge



<https://www.iacr.org/authors/tikz/>

Sponge nice features

- ▶ Indifferentiable from a RO (same, as Wide-pipe MD) (Bertoni et al., 2008)
- ▶ Quite flexible
 - ▶ For fixed permutation size: speed/security tradeoff
- ▶ Natively a XOF
- ▶ Can be extended to do (authenticated) encryption
- ▶ Simpler to design a permutation; less of a waste?

- ▶ Close structure: JH construction, another SHA-3 competitor (Wu, 2008)

Conclusion

- ▶ Don't design crypto yourself!
 - ▶ There is no generic way to design a hash function
 - ▶ Every small detail counts (recall e.g. SHA-0, TEA)
- ▶ Use SHA-3 (SHA-2 still okay)
- ▶ NEVER USE MD5/SHA-1
 - ▶ Even if you only care about preimage attacks