Practical Free-Start Collision Attacks on full SHA-1

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Title deconstruction

$\underbrace{\text{Practical}}_{\text{We can compute it}}$ $\underbrace{\text{Free-Start}}_{\text{Not unlike a false start}}$ $\underbrace{\text{Collision}}_{\text{As in } \mathfrak{f}(x)=\mathfrak{f}(x')}$

$\underbrace{\text{Attacks}}_{\text{We're the baddies}}$ $\underbrace{\text{on full}}_{\text{The real thing this time!}}$ $\underbrace{\text{SHA-1}}_{\text{Not a cat}}$

Hash functions

Hash function

A (binary) hash function is a mapping $\mathcal{H} : \{0,1\}^* \rightarrow \{0,1\}^n$

- Many uses in crypto: hash n' sign, MAC constructions...
- It is a keyless primitive
- Sooo, what's a good hash function?

Three security notions (informal)

First preimage resistance

Given t, find m such that $\mathcal{H}(m) = t$
Best generic attack is in $\mathcal{O}(2^n)$

Second preimage resistance

Given m, find $m' \neq m$ such that $\mathcal{H}(m) = \mathcal{H}(m')$
Best generic attack is in $\mathcal{O}(2^n)$

Collision resistance

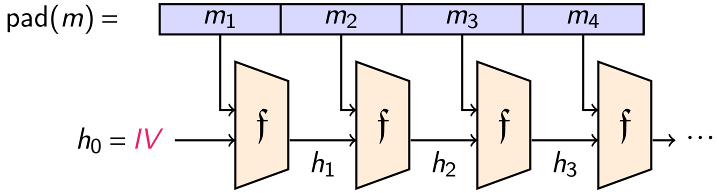
Find $m, m' \neq m$ such that $\mathcal{H}(m) = \mathcal{H}(m')$
Best generic attack is in $\mathcal{O}(2^{\frac{n}{2}})$

Merkle-Damgård construction

A domain of $\{0,1\}^*$ is annoying, so...

1. Start from a compression function $\mathfrak{f} : \{0,1\}^n \times \{0,1\}^b \rightarrow \{0,1\}^n$
2. Use a domain extender $\approx$
   $\mathcal{H}(m_1||m_2||\ldots||m_\ell) = \mathfrak{f}(\mathfrak{f}(\ldots\mathfrak{f}(IV, m_1)\ldots), m_\ell)$
3. Reduce the security of $\mathcal{H}$ to the one of $\mathfrak{f}$
   - $\mathsf{A}(\mathcal{H}) \Rightarrow \mathsf{A}(\mathfrak{f})$
   - $\neg\mathsf{A}(\mathfrak{f}) \Rightarrow \neg\mathsf{A}(\mathcal{H})$
   - $(\mathsf{A}(\mathfrak{f}) \Rightarrow ???)$
     - Invalidates the security reduction, tho

MD in a picture



Additional security notions for MD

Semi-free-start collisions

The attacker may choose IV, but it must be the same for m and m'

Free-start preimages & collisions

No restrictions on IV whatsoever

Free-start preimages & collisions (variant)

Attack $\mathfrak{f}$ instead of $\mathcal{H}$

What did we do?

- First try: collisions on 76/80 steps of the compression function of SHA-1 (95% of SHA-1)
- And it's practical
- Cost $\approx 2^{50.3}$ SHA-1, one inexpensive GPU is enough for fast results

- Second try: collisions on the full compression function of SHA-1 (100% of SHA-1)
- Still practical
- Cost $\approx 2^{57.5}$ SHA-1, 64 GPUs for a result in less than two weeks
- ‽Not "the same attack as 1) with more computation power"

The collision

	Message 1
IV_1	50 6b 01 78 ff 6d 18 90 20 22 91 fd 3a de 38 71 b2 c6 65 ea
M_1	9d 44 38 28 a5 ea 3d f0 86 ea a0 fa 77 83 a7 36
	33 24 48 4d af 70 2a aa a3 da b6 79 d8 a6 9e 2d
	54 38 20 ed a7 ff fb 52 d3 ff 49 3f c3 ff 55 1e
	fb ff d9 7f 55 fe ee f2 08 5a f3 12 08 86 88 a9
Compr(IV_1,M_1)	f0 20 48 6f 07 1b f1 10 53 54 7a 86 f4 a7 15 3b 3c 95 0f 4b

	Message 2
IV_2	50 6b 01 78 ff 6d 18 91 a0 22 91 fd 3a de 38 71 b2 c6 65 ea
M_2	3f 44 38 38 81 ea 3d ec a0 ea a0 ee 51 83 a7 2c
	33 24 48 5d ab 70 2a b6 6f da b6 6d d4 a6 9e 2f
	94 38 20 fd 13 ff fb 4e ef ff 49 3b 7f ff 55 04
	db ff d9 6f 71 fe ee ee e4 5a f3 06 04 86 88 ab
Compr(IV_2,M_2)	f0 20 48 6f 07 1b f1 10 53 54 7a 86 f4 a7 15 3b 3c 95 0f 4b

The SHA-1 hash function

- Designed by the NSA in 1995 as a quick fix to SHA-0
- Part of the MD4 family
- Hash size is 160 bits $\Rightarrow$ collision security should be 80 bits
- Message blocks are 512-bit long
- Compression function in MD mode

SHA-1 round function

Block cipher in Davies-Meyer mode

5-branch ARX Feistel

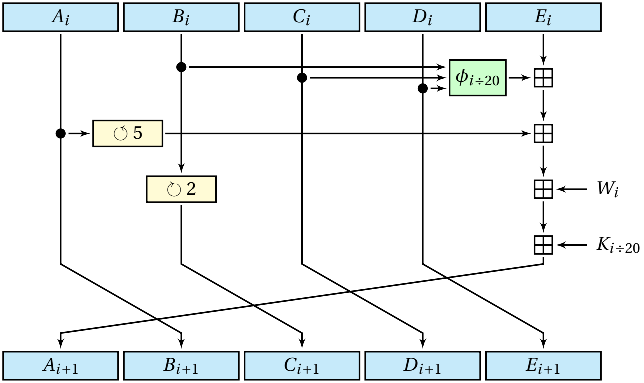
$$A_{i+1} = A_i^{\circlearrowleft 5} + \phi_{i \div 20}(A_{i-1}, A_{i-2}^{\circlearrowright 2}, A_{i-3}^{\circlearrowright 2}) + A_{i-4}^{\circlearrowright 2} + W_i + K_{i \div 20}$$

with a linear message expansion:

$W_{0 \dots 15} = M_{0 \dots 15}$, $W_{i \geq 16} = (W_{i-3} \oplus W_{i-8} \oplus W_{i-14} \oplus W_{i-16})^{\circlearrowleft 1}$ $\leftsquigarrow$ The only difference between SHA-0 and SHA-1

80 steps in total

Round function in a picture



Wang collisions

SHA-1 is not collision-resistant (Wang, Yin, Yu, 2005)

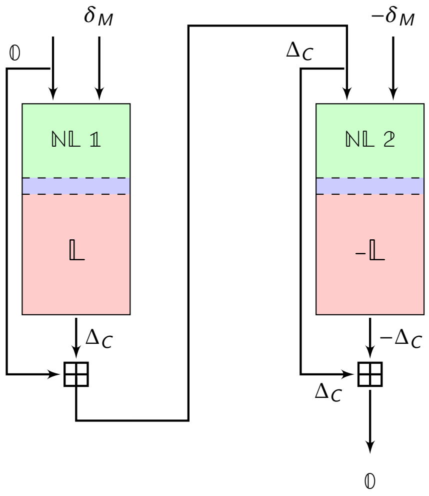
Differential collision attack

- Find a message difference that entails a good *linear* diff. path
- Construct a *non-linear* diff. path to bridge the *IV* with the linear path
- Use *message modification* to speed-up the attack
- Requires a pair of two-block messages

Attack complexity $\equiv 2^{69}$

Eventually improved to $\equiv 2^{61}$ (Stevens, 2013)

Two-block attack in a picture



Preimage detour

SHA-1 is much more resistant to preimage attacks

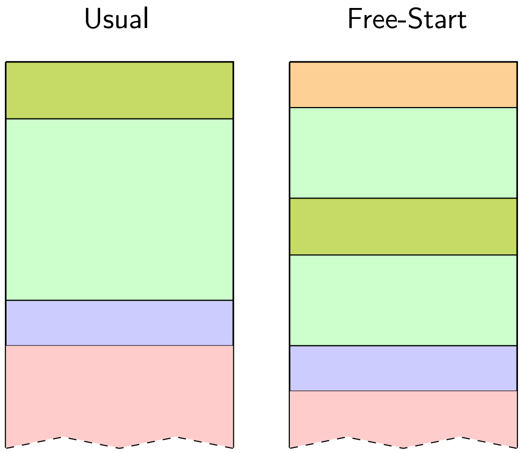
- No attack on the full function
- Practical attacks up to $\lessapprox$ 30 steps ($\lessapprox$ 37.5% of SHA-1) (De Cannière & Rechberger, 2008)
- Theoretical attacks up to 62 steps (77.5% of SHA-1) (Espitau, Fouque, Karpman, 2015)

Let's break stuff!

Why doing free-start again?

- Main reason is starting from a “middle” state + shift the message
- ⇒ Can use freedom in the message up to a later step
- ⇒ But no control on the *IV* value
- ⇒ Must ensure proper backward propagation

The point of free-start (in a picture)



But then we need to...

1. Find a good linear part
2. Construct a good shifted non-linear part
3. Find accelerating techniques

Let's do this for 80 steps!

Linear part selection

Criteria:

- High overall probability
- No (or few) differences in last five steps (= differences in IV)
- Few differences in early message words

⇒ Not many candidates

We picked $\mathrm{II}(59,0)$ (Manuel notation, 2011)
(This is just a shifted version of $\mathrm{II}(55,0)$ used for 76 steps)

Linear path in a picture (part 1/2)

```
                       A                                      W

41: x-------------------------------   ---------------------------x----
42: --------------------------------   --------------------------------
43: x-------------------------------   --x------------------------x----
44: --------------------------------   --x-----------------------------
45: x-------------------------------   ---------------------------x----
46: --------------------------------   -xx-----------------------------
47: -x------------------------------   ---------------------------xx---
48: --------------------------------   xxx-----------------------------
49: --------------------------------   x--x----------------------------
50: x-------------------------------   --xx-----------------------x----
51: --------------------------------   x-xx----------------------------
52: --------------------------------   --x-----------------------------
53: --------------------------------   --x-----------------------------
54: --------------------------------   x-x-----------------------------
55: x-------------------------------   ---------------------------x----
56: --------------------------------   x-------------------------------
57: --------------------------------   --x-----------------------------
58: --------------------------------   --x-----------------------------
59: --------------------------------   --x-----------------------------
60: --------------------------------   x-------------------------------
```

Linear path in a picture (part 2/2)

```
                   A                                  W

61: x-------------------------------   ---------------------------x----
62: --------------------------------   --------------------------------
63: x-------------------------------   --x------------------------x----
64: --------------------------------   x-x-----------------------------
65: --------------------------------   --------------------------------
66: --------------------------------   --x-----------------------------
67: --------------------------------   --x-----------------------------
68: --------------------------------   --------------------------------
69: --------------------------------   --------------------------------
70: --------------------------------   --------------------------------
71: --------------------------------   --------------------------------
72: --------------------------------   --------------------------------
73: --------------------------------   --------------------------------
74: --------------------------------   -------------------------------x
75: -------------------------------x   --------------------------x-----
76: --------------------------------   -------------------------------x
77: --------------------------------   -x----------------------------x-
78: ------------------------------x-   -x-----------------------x-----x
79: -------------------------------x   -x------------------------x---x-
80: --------------------------------
```

Non-linear part construction

- Start with prefix of high backward probability for the first 4 steps
- Use improved JLCA for the rest
- $\Rightarrow$ Good overall path with “few” conditions (246 up to #30)

Non-linear path in a picture

```
                     A                                    W
-4: ................................
-3: ................................
-2: .............................^-.
-1: 1...1....................0.....+
00: 01..0....................1......    x.+...+....................+....
01: 11+^..+.............^......+....    ..-..-.....................-++..
02: ..-11-1.1......^.....1+110.1.0..    ..+..--....................-.+..
03: .0.0-0011.^.10...+01.01111^0.1.1    ..-..--....................-+.-.
04: .1.11+-1+^^^+1^^^011^^.-+++++-.+    ...........................+....
05: .+.+.-++++++++++++++++++.+0-1111    .....-.....................+++..
06: .0.0.1.011.111.11110-0100-1.10-+    x+..++.....................-.+..
07: 1-.+.1.010100010000000111+.-.0.+    ....-+........................+.
08: 0+.0.0................0..+.-.0.1    x-.........................+....
09: .+.0.0...................0.+...^    x.-+.-.....................-++..
10: .+.........................+.0..    ..-+++.......................-..
11: ...-............................    x.++++.....................-+.+.
12: ...0.1.......................1..    ..-........................-....
13: .1...0........................!^    ..+..+.....................-++..
14: +-..............................    x++.+-.....................-.+..
15: 1.1-..........................!.    ....+-........................+.
16: +.10.1..........................    x+.........................-....
```

Accelerating techniques

- Message modification: correct bad instances
- Neutral bits: generate more good instances when one's found
- We choose NBs because:
  - Easy to find
  - Easy to implement
  - Good parallelization potential (more of that later)
  - Includes both "single" NBs and boomerangs

Neutral bits (with an offset)

- We start with an offset (remember?)
- ⇒ Use neutral bits with an offset too
- In our attack, offset = 5
  - free message words = W5...20 instead of W0...15
- ⇒ Must also consider backward propagation

Our 60 "single" neutral bits

```
A18:
W14 ....................xxxx........
W15 ...............xxxx.............
A19:
W14 ........................x.x.....
W15 ...................xxxxx........
W16 ...............xxxxx............
A20:
W15 ........................x..x....
W16 ....................xxxx........
W17 ............xxxxxx..............
A21:
W17 ..................xxxx..........
W18 ................x...............
A22:
W18 .................xxxxxx.........
W19 .................x...x..........
A23:
W18 .......................xxx.x....
W19 ...................xx.x.........
W20 ................x...............
A24:
W19 .......................xxx......
W20 ..................xxx...........
A25:
W20 .........................x......
```

Our 4 boomerangs

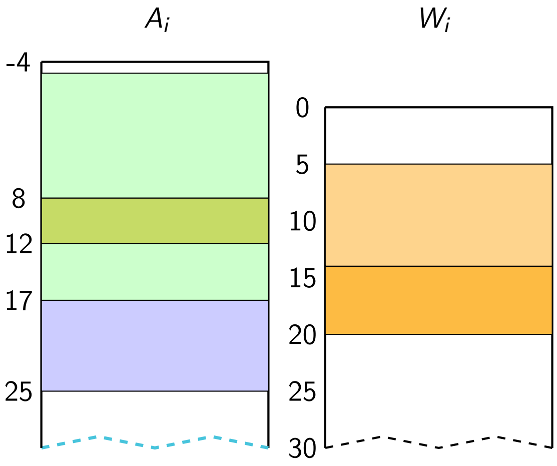
```
W10:  . . . . . . . . . . . . . . . . . . . . . . B A . . . . . . . .
W11:  . . . . . . . . . . . . . . . . . b a . . . . D C . . . . . . .
W12:  . . . . . . . . . . . . . . . . . . d c . . . . . . . . . . . .
W13:  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
W14:  . . . . . . . . . . . . . . . . . . . . . . . . . a . . . . . .
W15:  . . . . . . . . . . . . . . . . . . . . . . . . b a . . . . . .
W16:  . . . . . . . . . . . . . . . . . . . . . . . . . d c . . . . .
```

Let's sum up

- Initialize the state with an offset
- Initialize message words with an offset
- Use neutral bits with an offset
- ⇒ many neutral bits up to late steps (yay)
- ⇒ don't know the *IV* in advance (duh)

- Linear path ⇒ differences in the *IV*
- Everything done in one block
- ⇒ Attack on the compression function

Same thing in a picture



If it's practical you must run it

- Attack expected to be practical, but still expensive
- Why not using GPUs?
- One main challenge: how to deal with the branching?

Target platform

- Nvidia GTX-970
- Recent, high-end, good price/performance
- $13 \times 128 = 1664$ cores @ $\propto 1$ GHz
- High-level programming with CUDA
- Throughput for 32-bit arithmetic: all 1/cycle/core except $\circlearrowright$
- $\approx$ SGD 500

Architecture imperatives

- Execution is bundled in warps of 32 threads
- Single Instruction Multiple Threads:
  Control-flow divergence is serialized $\Rightarrow$ minimize branching
- Hide latency by grouping threads into larger blocks
- But careful about register / memory usage

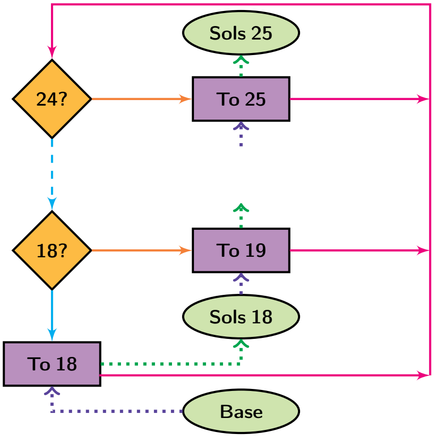
Our snippet-based approach

1. Store partial solutions up to some step in shared buffers
2. Every thread of a block loads one solution
3. … tries all neutral bits for this step
4. … stores successful candidates in next step buffer

Our snippet-based approach (cont.)

1. Base solutions up to #17 generated on CPU
2. Use single neutral bits up to #25 on GPU
3. Use boomerangs on #28 and #30 on GPU
4. Further checks up to #60 on GPU
5. Final collision check on CPU

Snippets in a picture (w/o boomerangs)



GPU results (76 steps)

- Hardware: one GTX-970
- One partial solution up to #56 per minute on average
- $\Rightarrow$ Expected time to find a collision $\lessapprox$ 5 days
- Complexity $\equiv 2^{50.3}$ SHA-1 compression function

GPU v. CPU

- On one CPU core @ 3.2 GHz, the attack takes ≈ 606 days
- ⇒ One GPU ≡ 140 cores
- (To compare with ≡ 40 (Grechnikov & Adinetz, 2011))
- For raw SHA-1 computations, ratio is 320
- ⇒ Lose only ×2.3 from the branching (not bad)

GPU results (80 steps)

- Hardware: 64 GTX-970
- $\Rightarrow$ Expected time to find a collision $\lessapprox$ 10 days
- Complexity $\equiv 2^{57.5}$ SHA-1 compression function
- On Amazon Elastic C2 cost $\approx$ USD 2K (with older GPUs)

What about a full *hash function* collision?

- Estimated complexity: $\lessapprox 2^{61}$
- GPU framework translates swimmingly to this case
- 64-GTX970 cluster $\Rightarrow \approx$ 110-220 days ($\approx$ 4-8 months)
- On Amazon Elastic C2 $\Rightarrow \approx$ USD 22-44K

For more details

Pierre Karpman, Thomas Peyrin, and Marc Stevens:
Practical Free-Start Collision Attacks on 76-step SHA-1,
CRYPTO 2015
Eprint 2015/530

Marc Stevens, Pierre Karpman, and Thomas Peyrin:
Freestart collision for full SHA-1,
EUROCRYPT 2016
Eprint 2015/967

C'est fini !