Geophysical imaging

L. Métivier

Tuesday 6th October, 2020

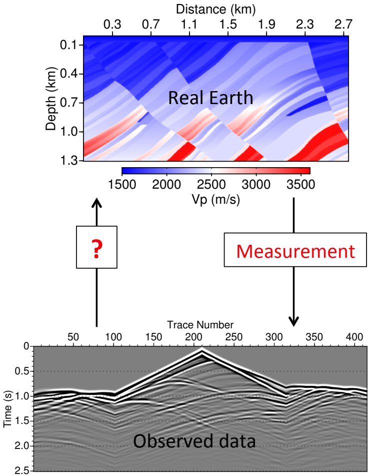
[1]LJK, ISTerre, CNRS, Univ. Grenoble Alpes, France

H202, ENSIMAG, Grenoble

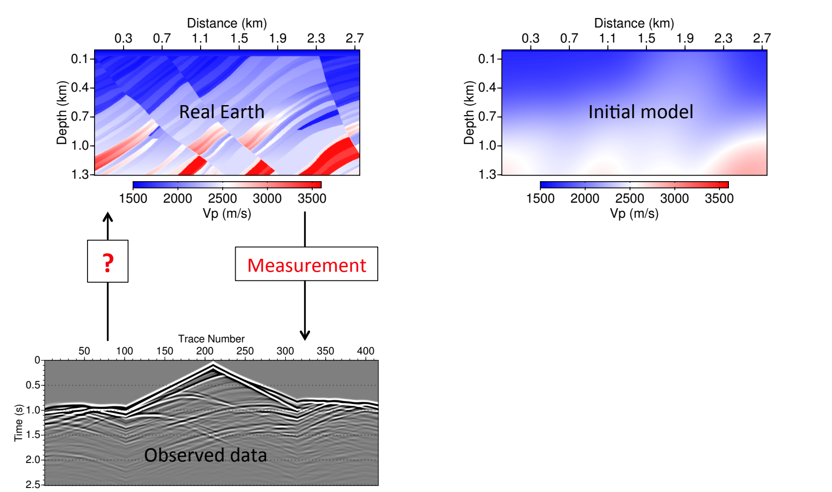
Inversion

Outline

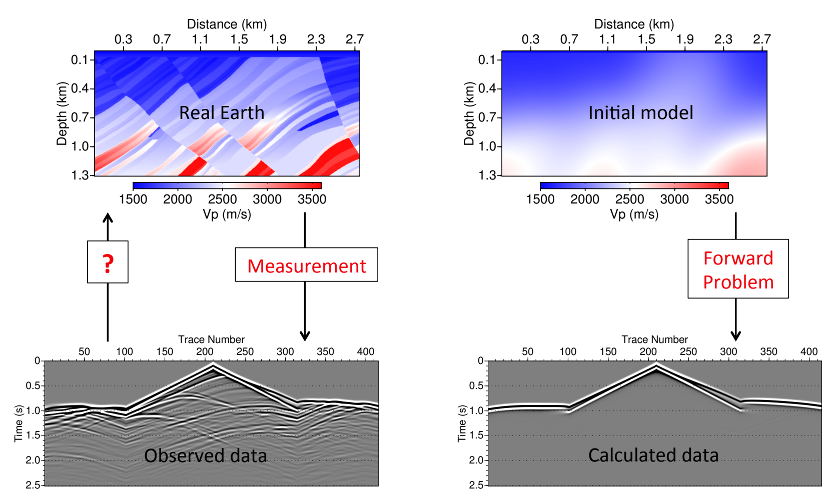
FWI loop


Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Real Earth
1500 2000 2500 3000 3500
Vp (m/s)
?
Measurement
Trace Number
50 100 150 200 250 300 350 400
Time (s)
0 0.5 1.0 1.5 2.0 2.5
Observed data

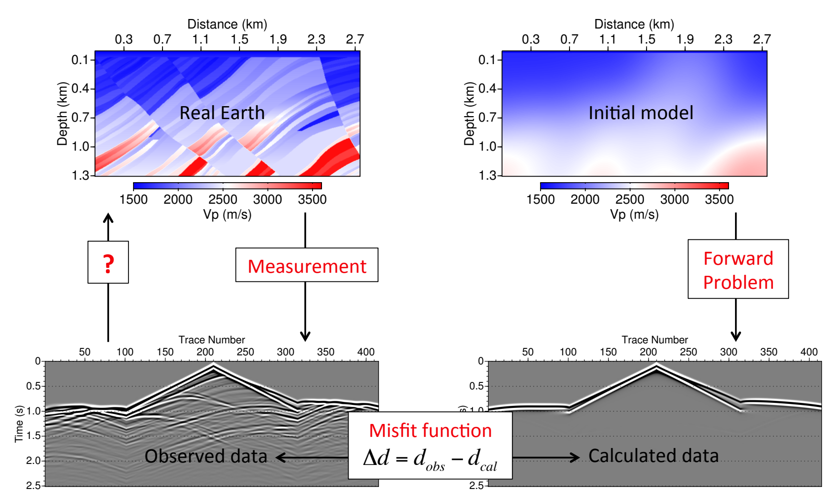
FWI loop


Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Real Earth
1500 2000 2500 3000 3500
Vp (m/s)
?
Measurement
Trace Number
50 100 150 200 250 300 350 400
Time (s)
0 0.5 1.0 1.5 2.0 2.5
Observed data
Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Initial model
1500 2000 2500 3000 3500
Vp (m/s)

FWI loop

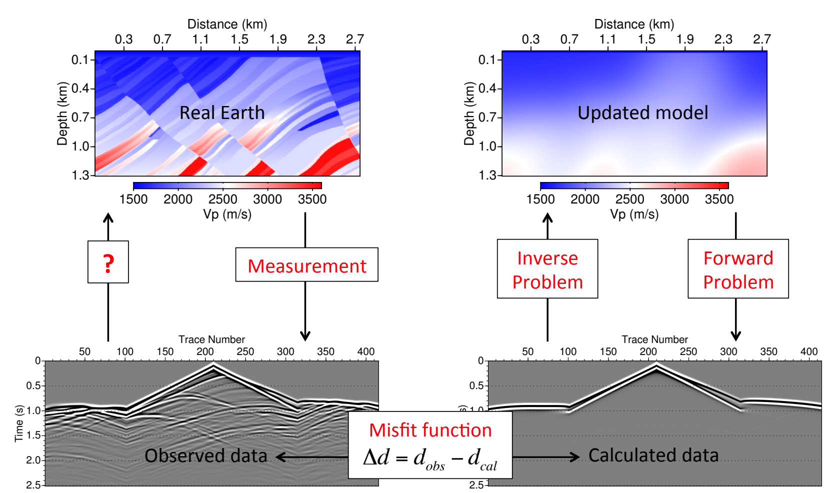

Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Real Earth
1500 2000 2500 3000 3500
Vp (m/s)
Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Initial model
1500 2000 2500 3000 3500
Vp (m/s)
?
Measurement
Forward Problem
Trace Number
50 100 150 200 250 300 350 400
Time (s)
0 0.5 1.0 1.5 2.0 2.5
Observed data
Trace Number
50 100 150 200 250 300 350 400
Time (s)
0 0.5 1.0 1.5 2.0 2.5
Calculated data

FWI loop



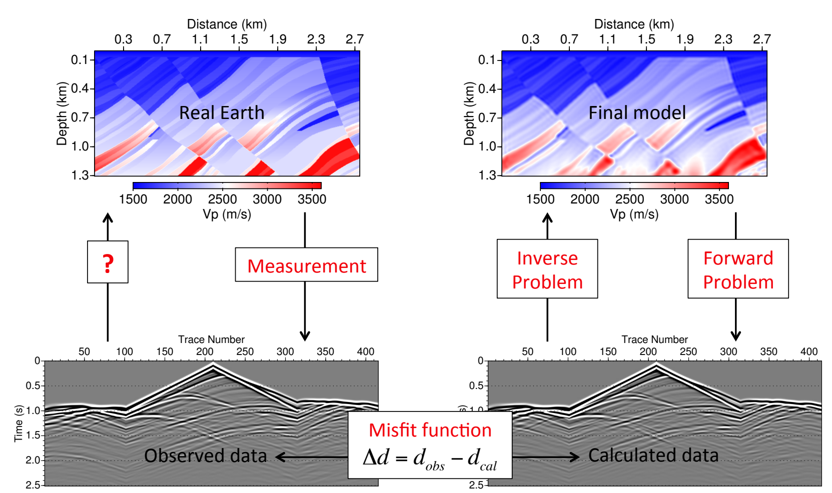
Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Real Earth
1500 2000 2500 3000 3500
Vp (m/s)
Initial model
?
Measurement
Forward Problem
Trace Number
50 100 150 200 250 300 350 400
Time (s)
0 0.5 1.0 1.5 2.0 2.5
Observed data
Misfit function
$\Delta d = d_{obs} - d_{cal}$
Calculated data

FWI loop



Distance (km)
0.3
0.7
1.1
1.5
1.9
2.3
2.7
Depth (km)
0.1
0.4
0.7
1.0
1.3
Real Earth
1500
2000
2500
3000
3500
Vp (m/s)
Distance (km)
Updated model
?
Measurement
Inverse Problem
Forward Problem
Trace Number
50
100
150
200
250
300
350
400
Time (s)
0
0.5
1.0
1.5
2.0
2.5
Observed data
Misfit function
$\Delta d = d_{obs} - d_{cal}$
Calculated data

FWI loop

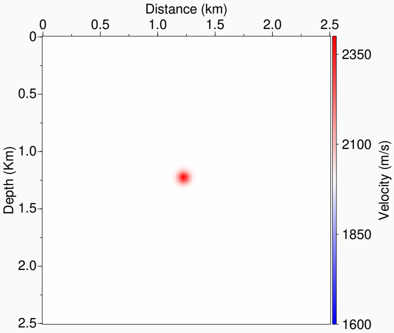


Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Real Earth
1500 2000 2500 3000 3500
Vp (m/s)
Distance (km)
0.3 0.7 1.1 1.5 1.9 2.3 2.7
Depth (km)
0.1 0.4 0.7 1.0 1.3
Final model
1500 2000 2500 3000 3500
Vp (m/s)
?
Measurement
Inverse Problem
Forward Problem
Trace Number
50 100 150 200 250 300 350 400
Time (s)
0 0.5 1.0 1.5 2.0 2.5
Observed data
Misfit function
$\Delta d = d_{obs} - d_{cal}$
Trace Number
50 100 150 200 250 300 350 400
0 0.5 1.0 2.5
Calculated data

Outline

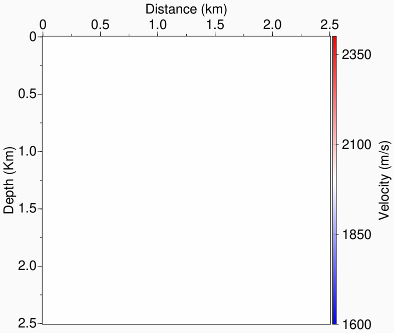
A simple test

Exact velocity model to recover

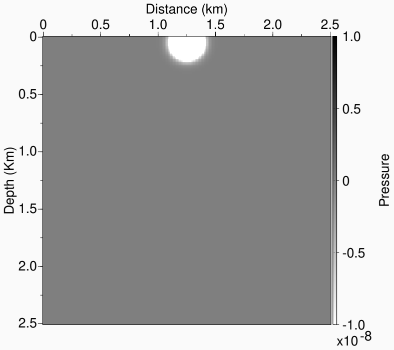


A simple test

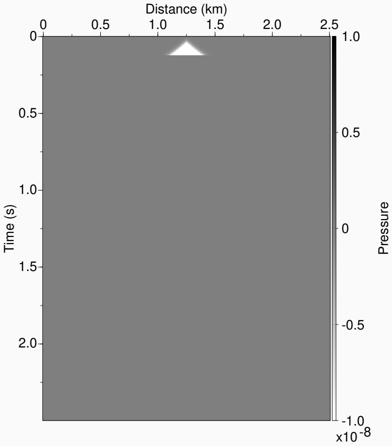
Initial velocity model



Collecting data with surface acquisition and a single source

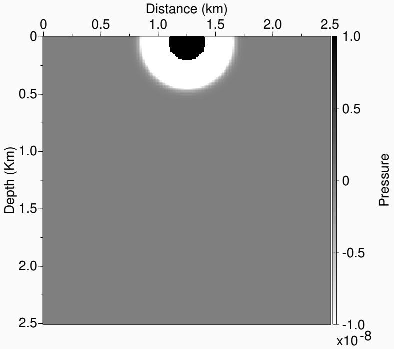


Wavefield propagating in the exact medium

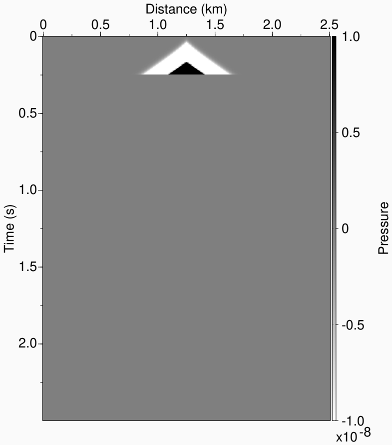


Recorded seismogram

Collecting data with surface acquisition and a single source

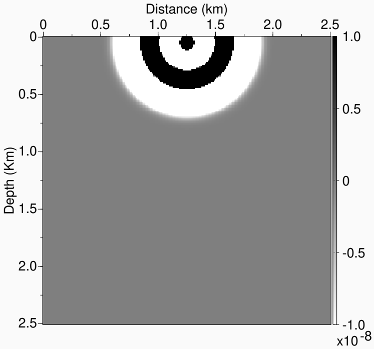


Wavefield propagating in the exact medium

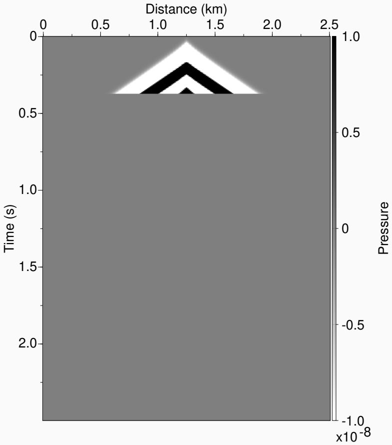


Recorded seismogram

Collecting data with surface acquisition and a single source

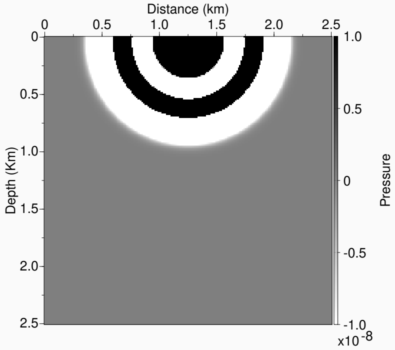


Wavefield propagating in the exact medium

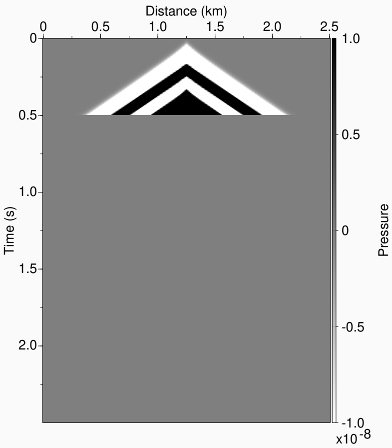


Recorded seismogram

Collecting data with surface acquisition and a single source

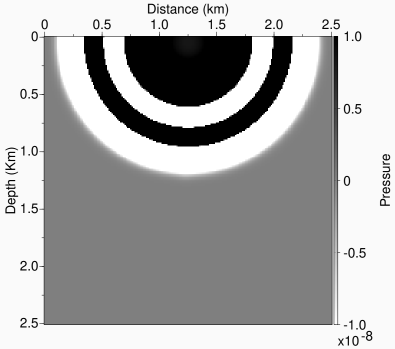


Wavefield propagating in the exact medium

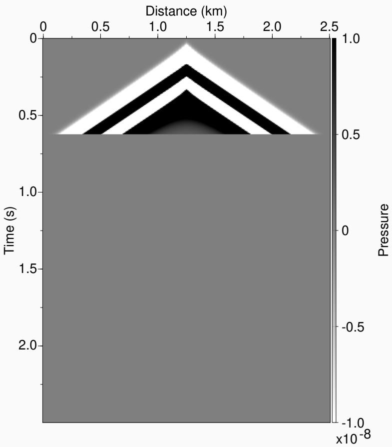


Recorded seismogram

Collecting data with surface acquisition and a single source

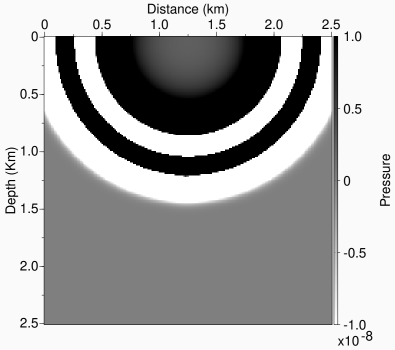


Wavefield propagating in the exact medium

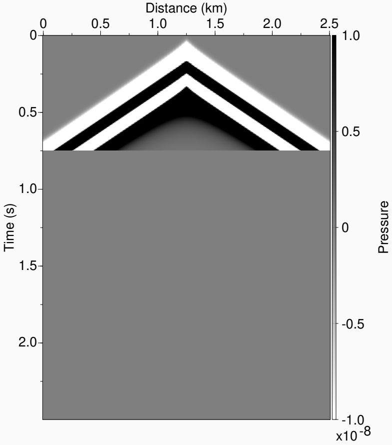


Recorded seismogram

Collecting data with surface acquisition and a single source

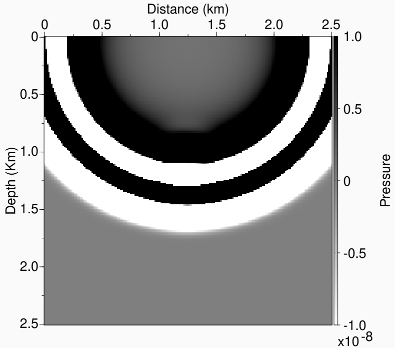


Wavefield propagating in the exact medium

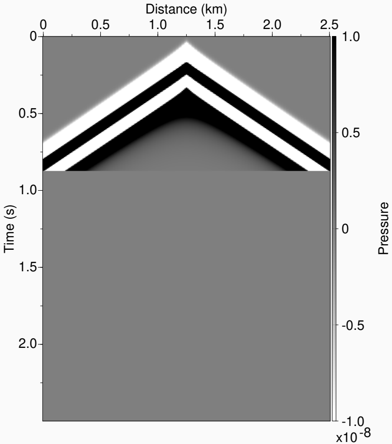


Recorded seismogram

Collecting data with surface acquisition and a single source



Wavefield propagating in the exact medium



Recorded seismogram

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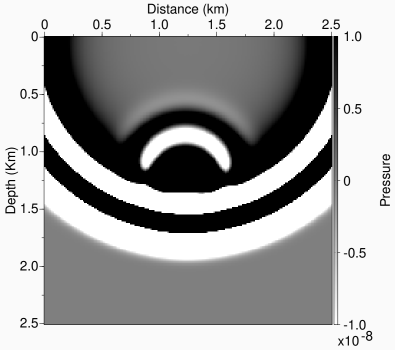
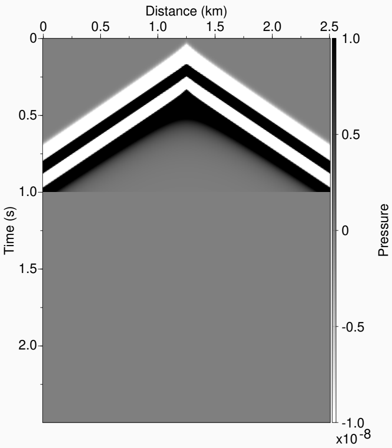


Wavefield propagating in the exact medium



Recorded seismogram

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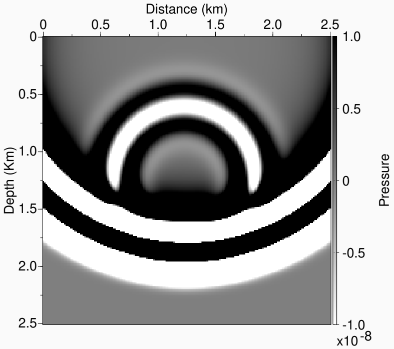
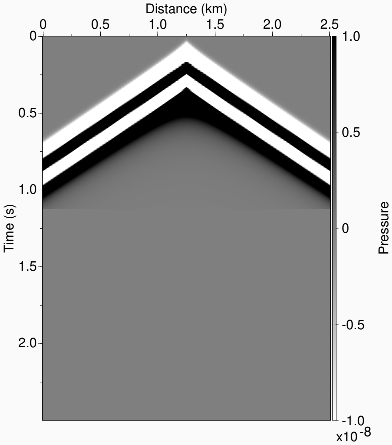


Wavefield propagating in the exact medium



Recorded seismogram

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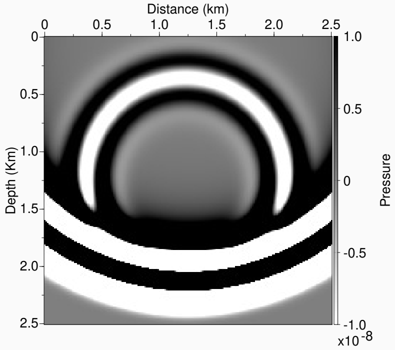
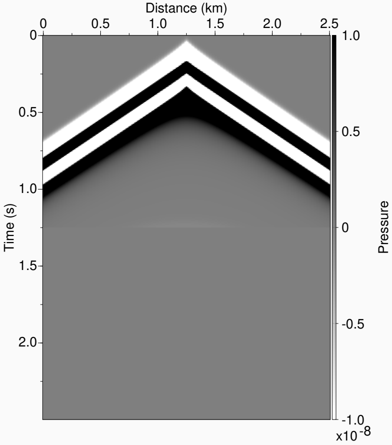


Wavefield propagating in the exact medium



Recorded seismogram

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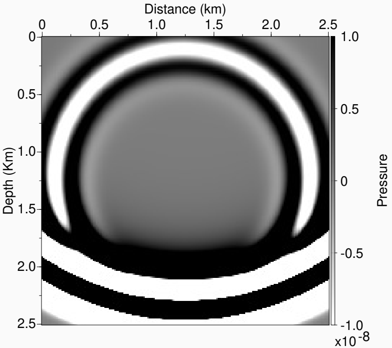
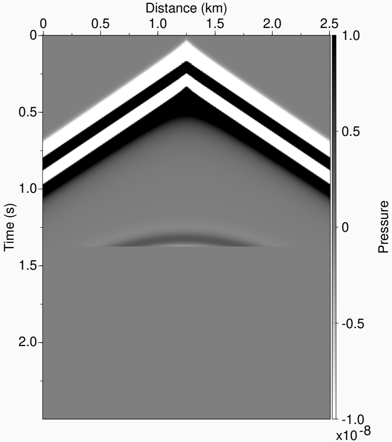
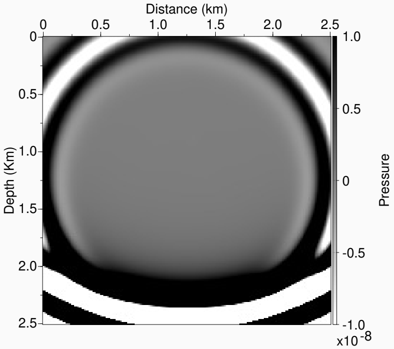


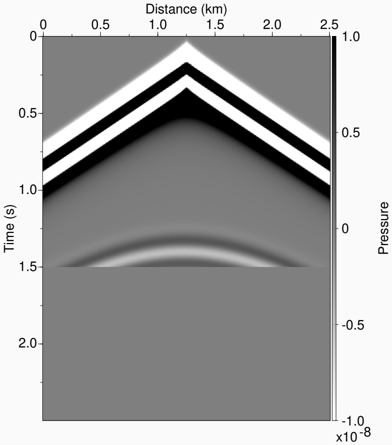
Wavefield propagating in the exact medium



Recorded seismogram



Wavefield propagating in the exact medium



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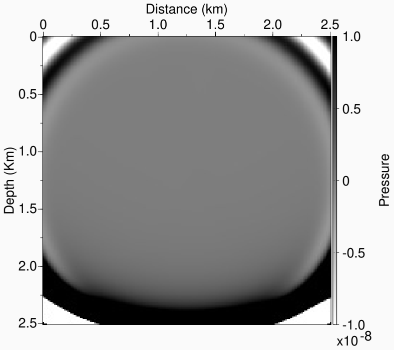
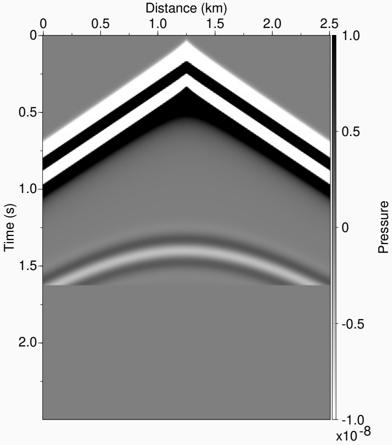


Wavefield propagating in the exact medium



Recorded seismogram

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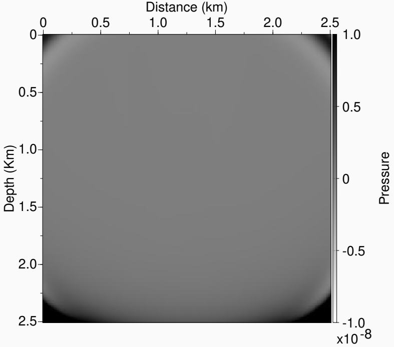
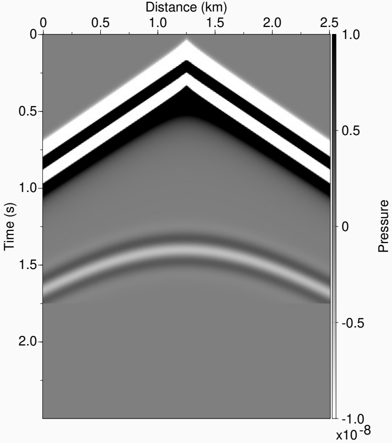


Wavefield propagating in the exact medium



Recorded seismogram

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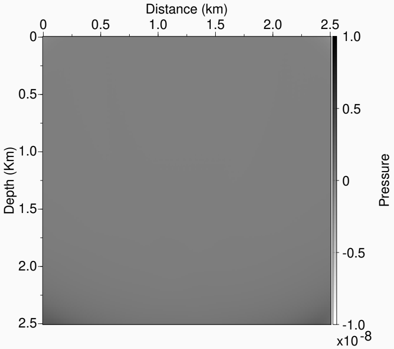
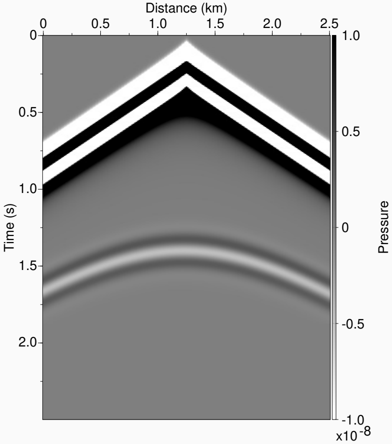
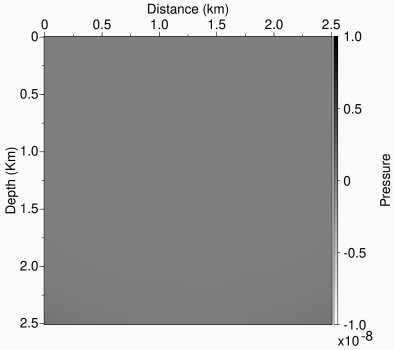


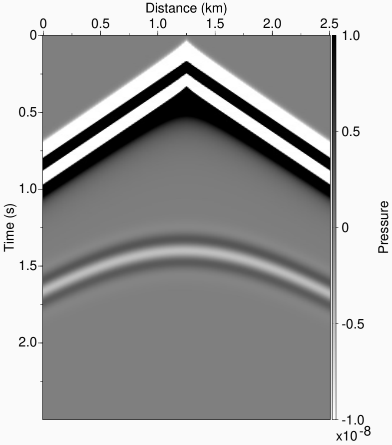
Wavefield propagating in the exact medium



Recorded seismogram

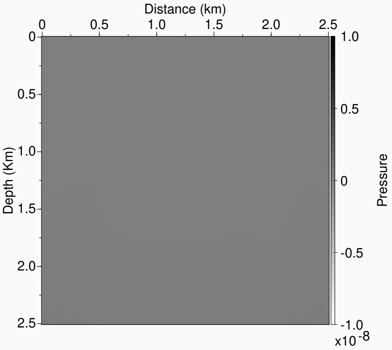
Collecting data with surface acquisition and a single source



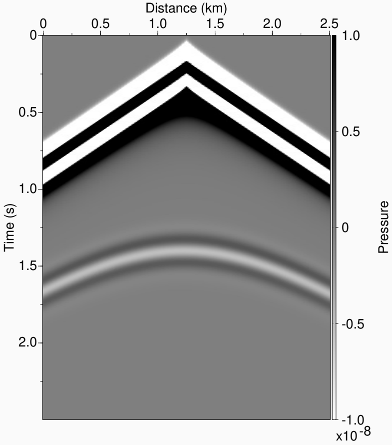


Wavefield propagating in the exact medium   Recorded seismogram

Collecting data with surface acquisition and a single source

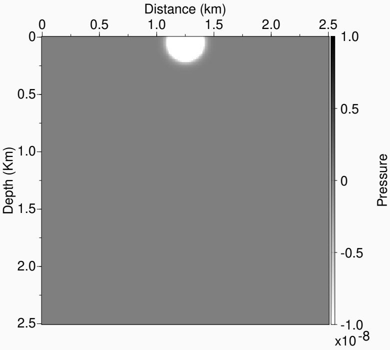


Wavefield propagating in the exact medium

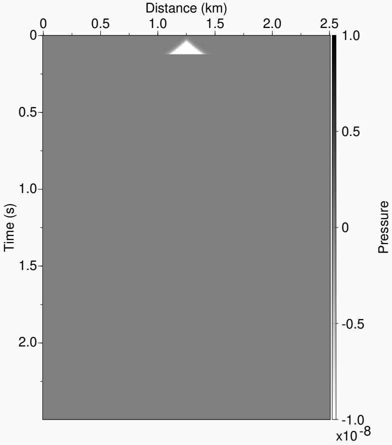


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

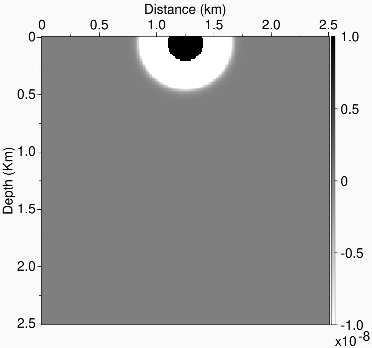


Wavefield propagating in the initial medium

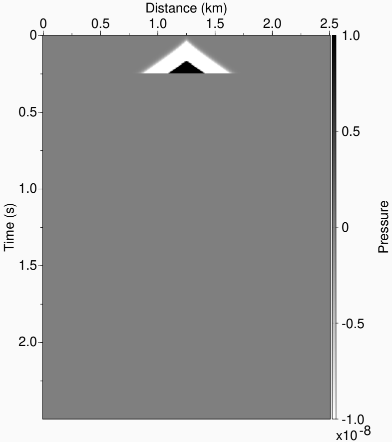


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

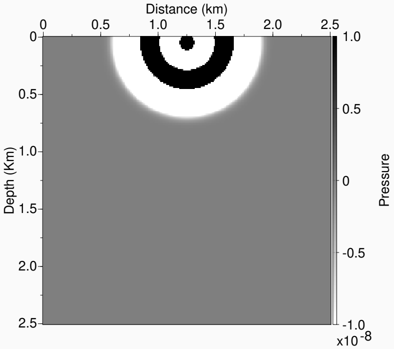


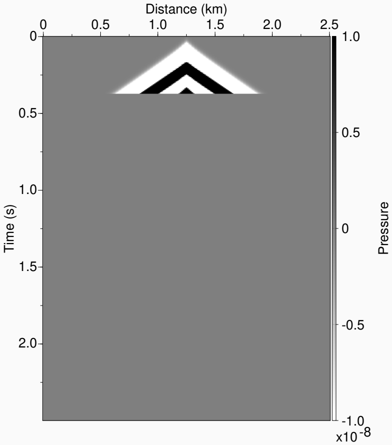
Wavefield propagating in the initial medium



Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

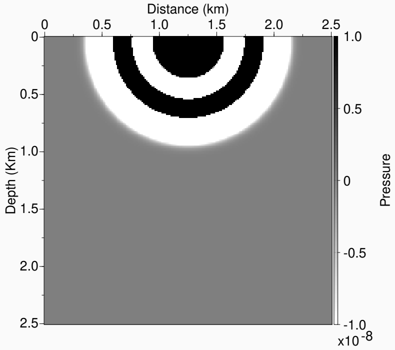




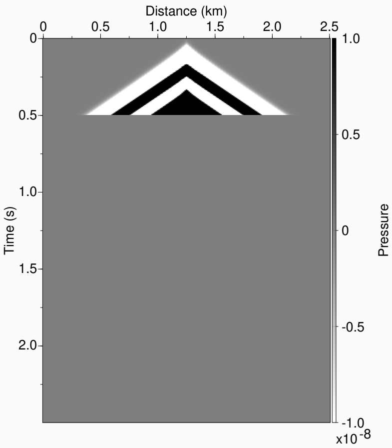
Wavefield propagating in the initial medium

Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

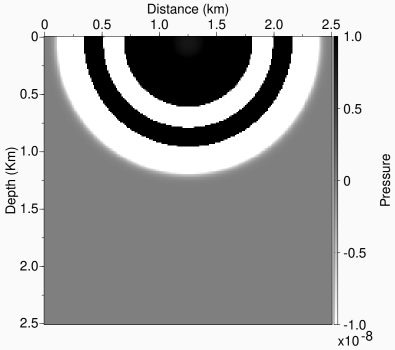


Wavefield propagating in the initial medium

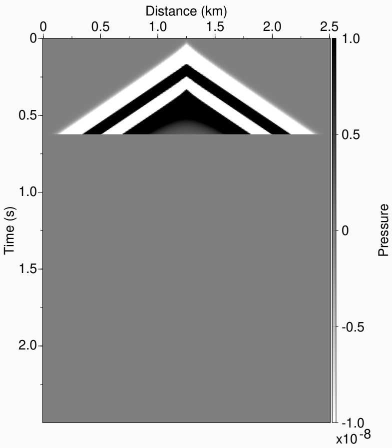


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

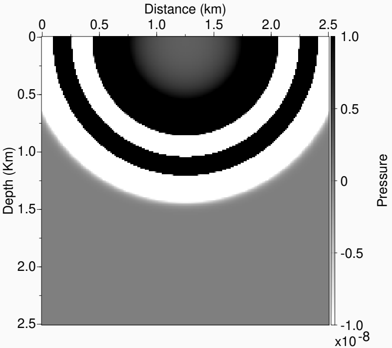


Wavefield propagating in the initial medium

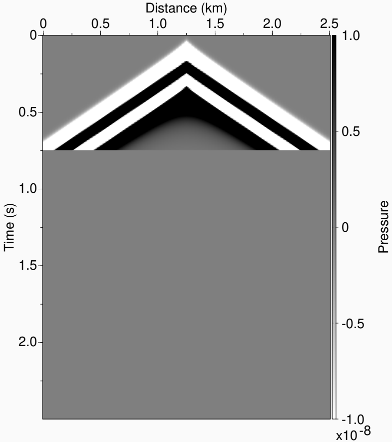


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

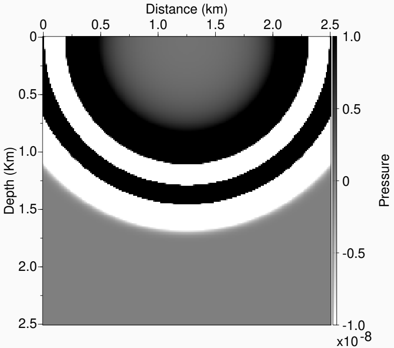


Wavefield propagating in the initial medium

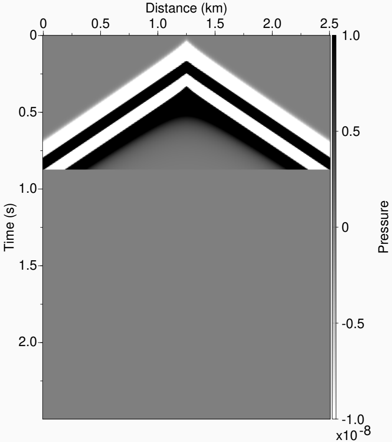


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

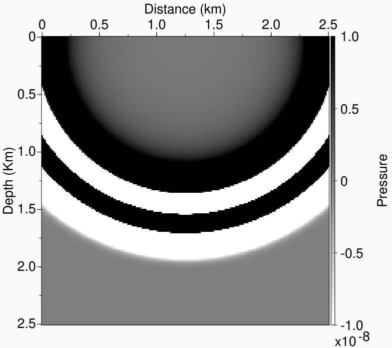


Wavefield propagating in the initial medium

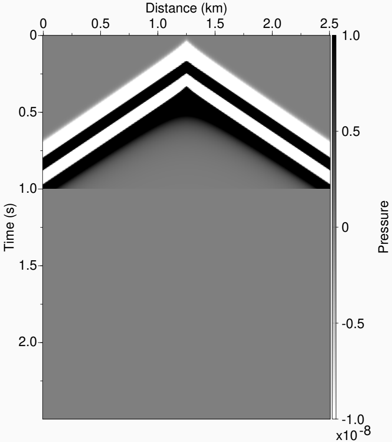


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

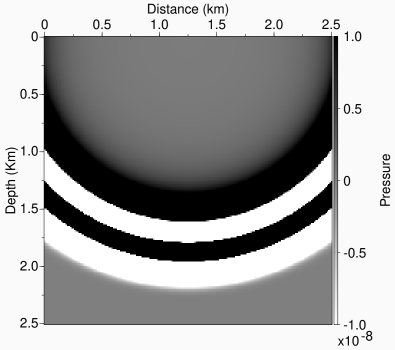


Wavefield propagating in the initial medium

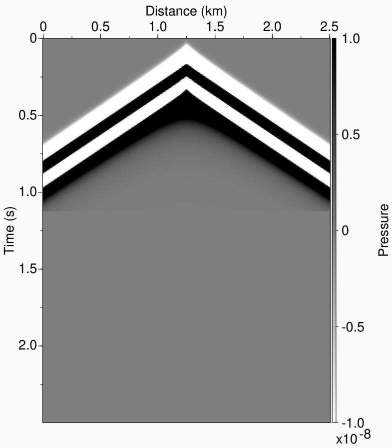


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

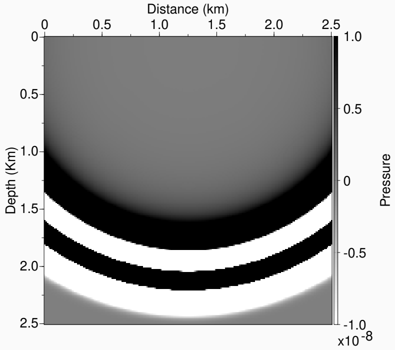


Wavefield propagating in the initial medium

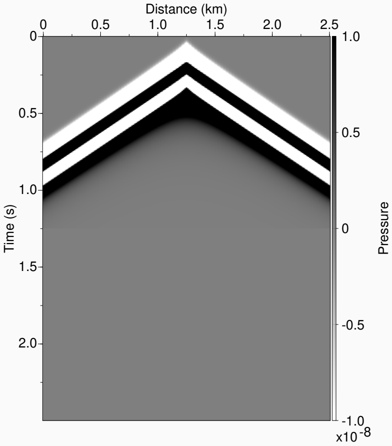


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

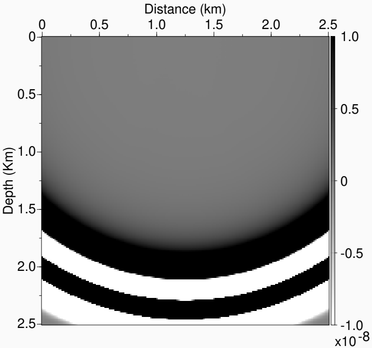


Wavefield propagating in the initial medium

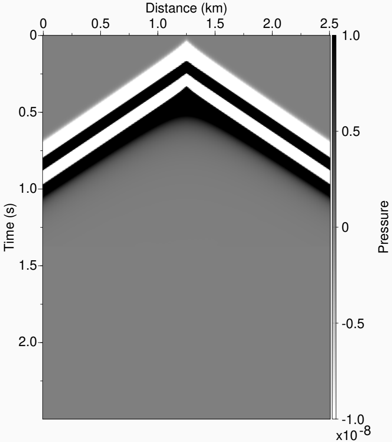


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

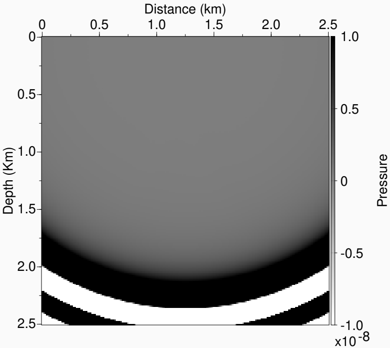


Wavefield propagating in the initial medium

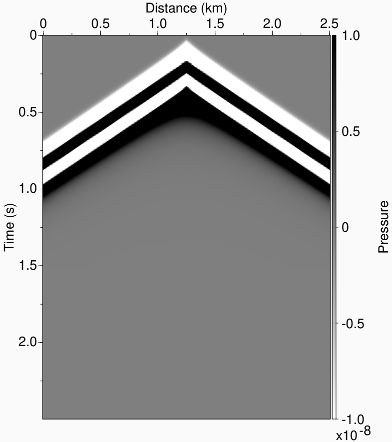


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

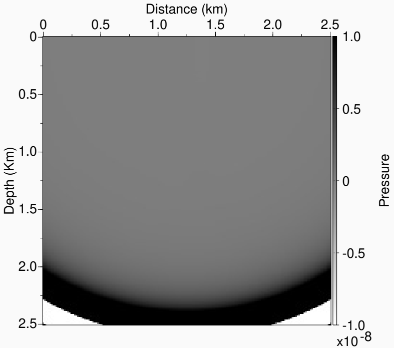


Wavefield propagating in the initial medium

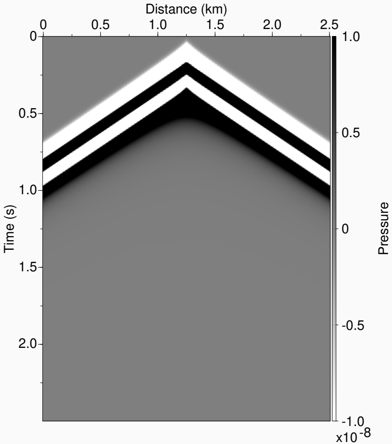


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

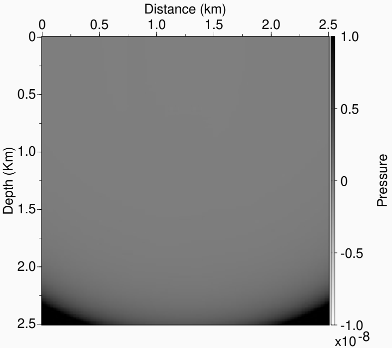


Wavefield propagating in the initial medium

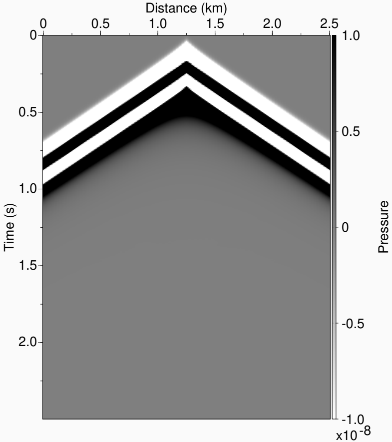


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

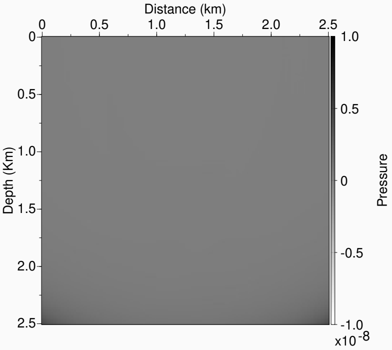


Wavefield propagating in the initial medium

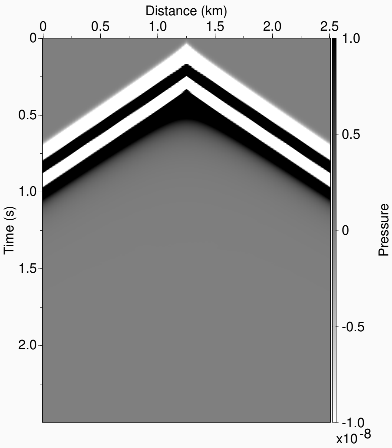


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

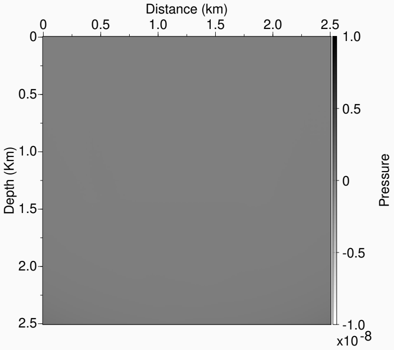


Wavefield propagating in the initial medium

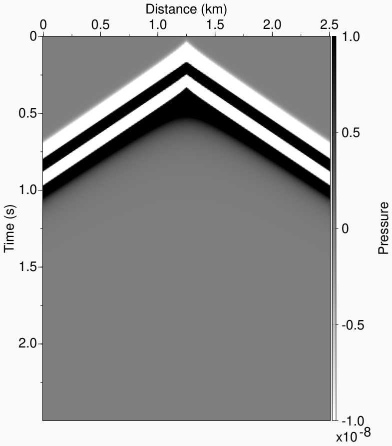


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

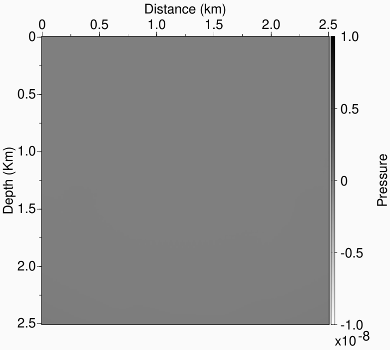


Wavefield propagating in the initial medium

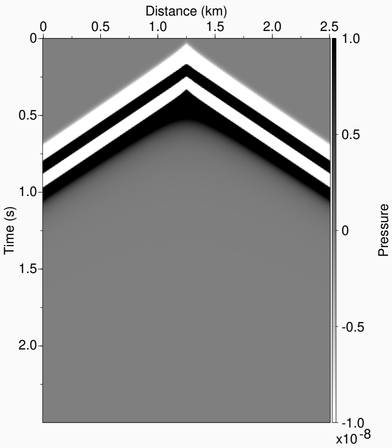


Recorded seismogram

Same experiment in the initial medium without Gaussian anomaly

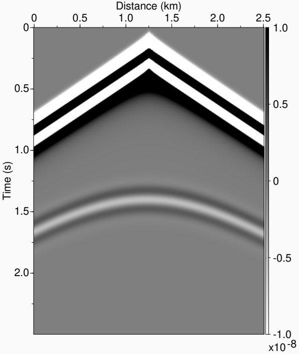


Wavefield propagating in the initial medium

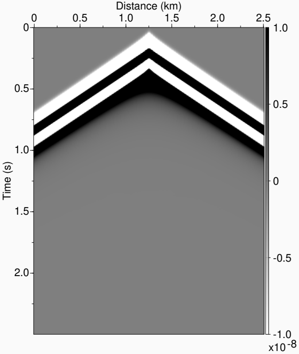


Recorded seismogram

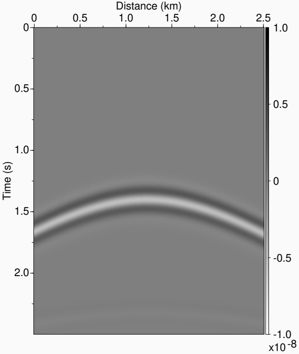
Comparison: observed and initial data, residuals



Observed data



Calculated data



Residuals

Gradient building through the Jacobian matrix

Remember the direct formula for the gradient

$$\nabla f(m)_i = \sum_{j=1}^{N} J_{ij}^T \Delta d_j = \sum_{j=1}^{N} J_{ji} \Delta d_j = \sum_{r=1}^{N_r} \int_0^T \frac{\partial d_{cal}}{\partial m_i}(x_r, t) \Delta d(x_r, t) \tag{1}$$

In addition we have

$$A(m)u = \varphi, \tag{2}$$

which if we derive with respect to a single parameter m_i gives

$$\frac{\partial A}{\partial m_i} u + A(m) \frac{\partial u}{\partial m_i} = 0 \tag{3}$$

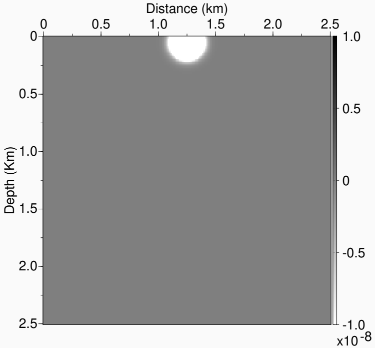
Therefore we have

$$\frac{\partial u}{\partial m_i} = -A(m)^{-1} \frac{\partial A}{\partial m_i} u. \tag{4}$$

and

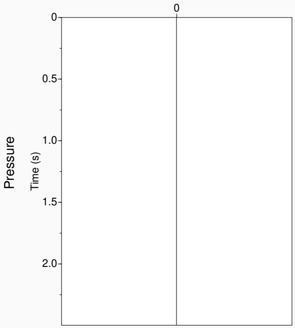
$$\frac{\partial d_{cal}}{\partial m_i}(x_r, t) = -RA(m)^{-1} \frac{\partial A}{\partial m_i} u \simeq -RA(m)^{-1} u_i(t) \tag{5}$$

Gradient building through the Jacobian matrix: step 1



Incident wavefield

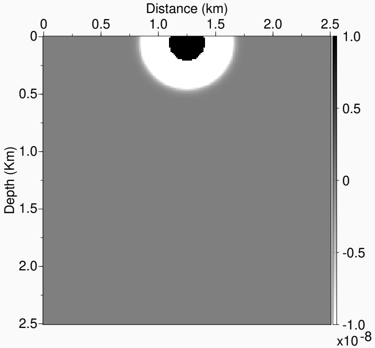
$u(x,t)$

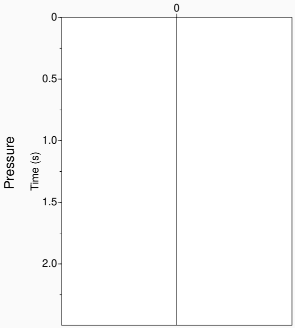


Recorded field in the center

$u_i(t)$

Gradient building through the Jacobian matrix: step 1

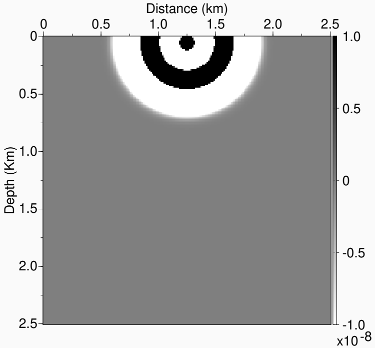




Incident wavefield
$u(x,t)$

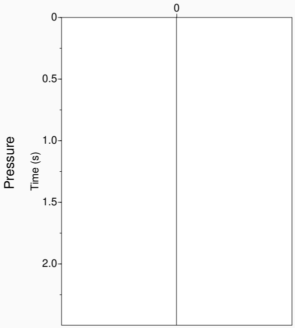
Recorded field in the center
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



Incident wavefield

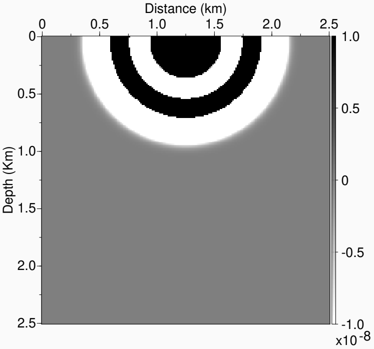
$u(x,t)$



Recorded field in the center

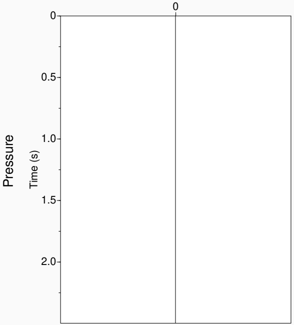
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



Incident wavefield

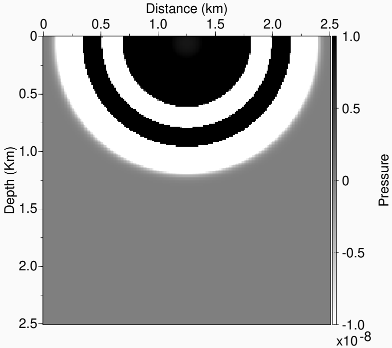
$u(x, t)$



Recorded field in the center

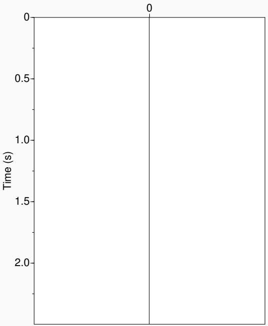
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



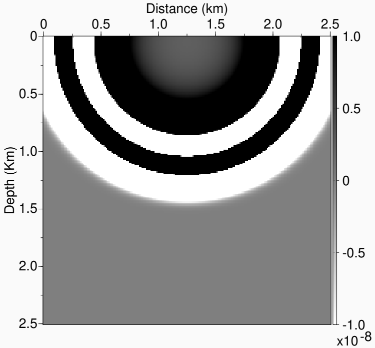
Incident wavefield

$u(x, t)$



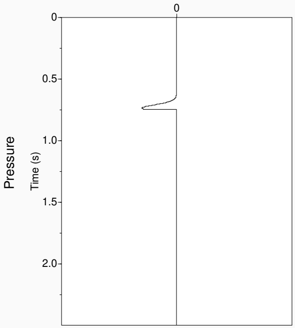
Recorded field in the center

$u_i(t)$



Incident wavefield

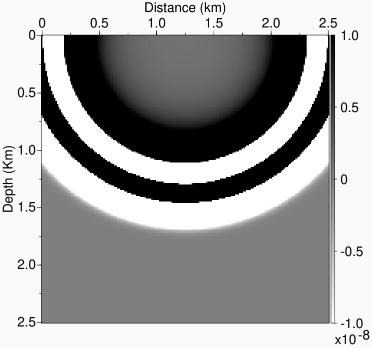
$u(x, t)$



Recorded field in the center

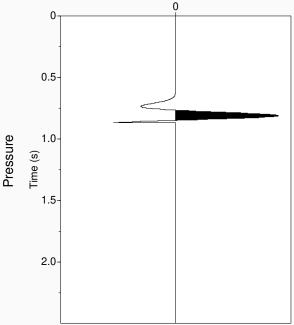
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



Incident wavefield

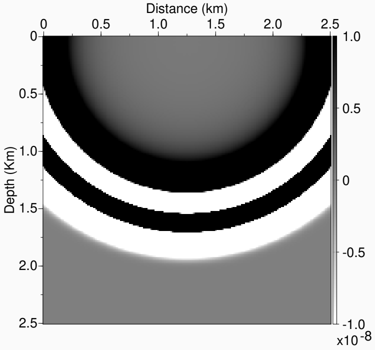
$u(x, t)$



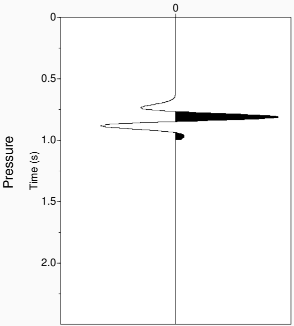
Recorded field in the center

$u_i(t)$

Gradient building through the Jacobian matrix: step 1

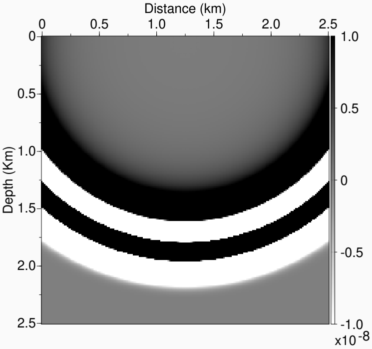


Incident wavefield
$u(x, t)$



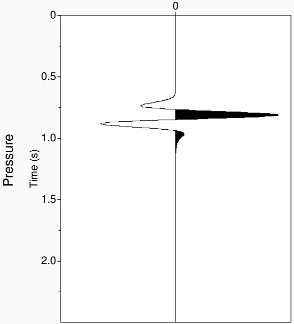
Recorded field in the center
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



Incident wavefield

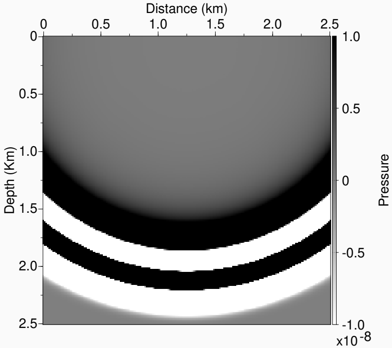
$u(x,t)$



Recorded field in the center

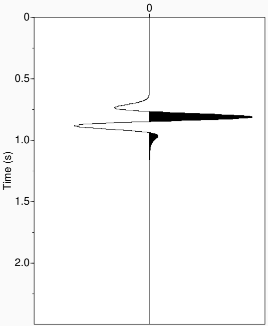
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



Incident wavefield

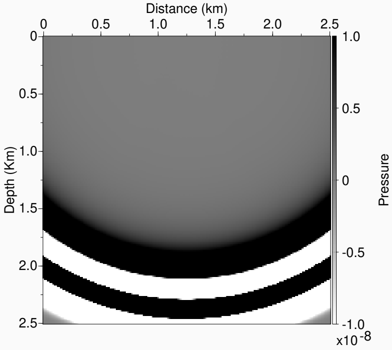
$u(x, t)$

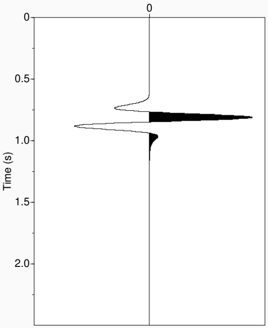


Recorded field in the center

$u_i(t)$

Gradient building through the Jacobian matrix: step 1





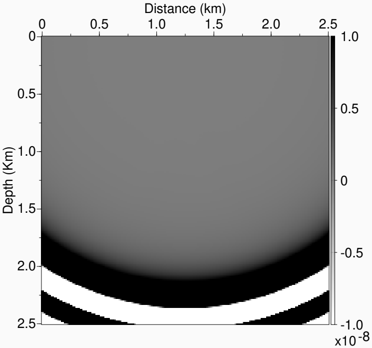
Incident wavefield

$u(x,t)$

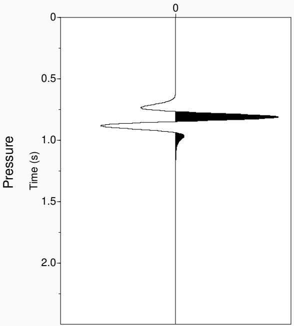
Recorded field in the center

$u_i(t)$

Gradient building through the Jacobian matrix: step 1

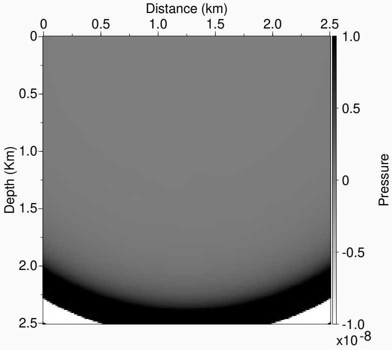


Incident wavefield
$u(x,t)$

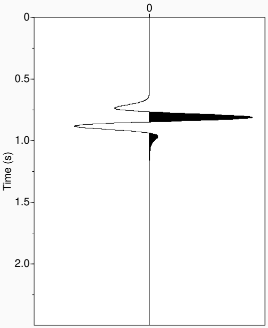


Recorded field in the center
$u_i(t)$

Gradient building through the Jacobian matrix: step 1

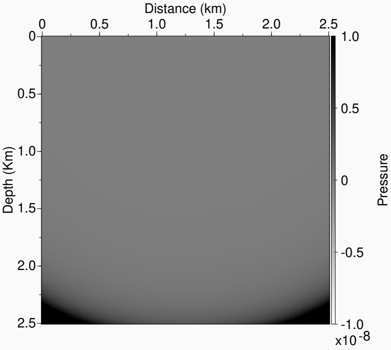


Incident wavefield
$u(x, t)$



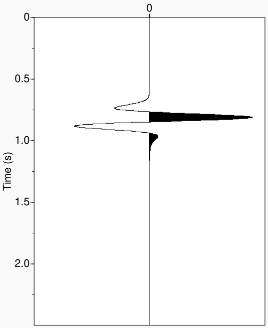
Recorded field in the center
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



Incident wavefield

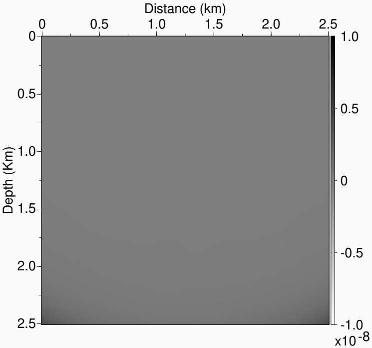
$u(x, t)$



Recorded field in the center

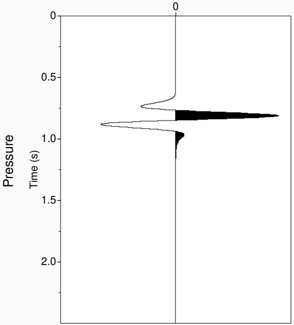
$u_i(t)$

Gradient building through the Jacobian matrix: step 1



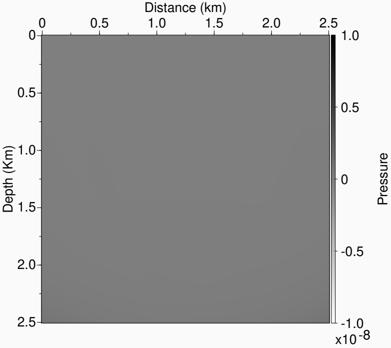
Incident wavefield

$u(x,t)$

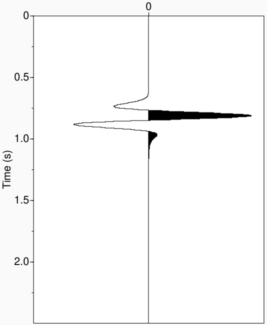


Recorded field in the center

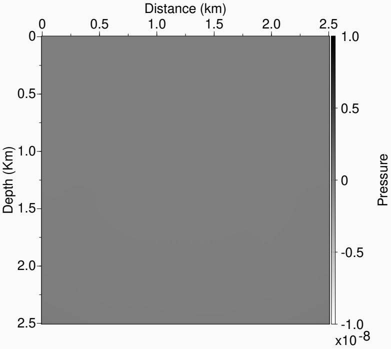
$u_i(t)$



Incident wavefield
$u(x, t)$

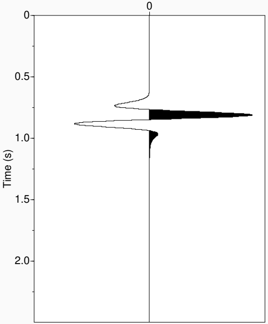


Recorded field in the center
$u_i(t)$



Incident wavefield

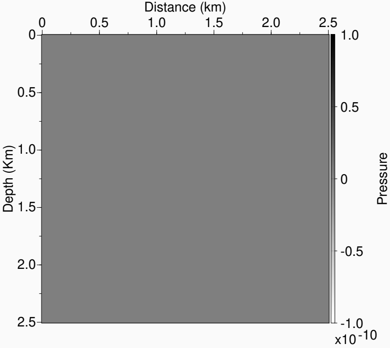
$u(x, t)$



Recorded field in the center

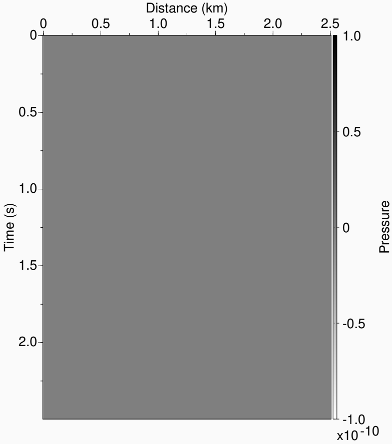
$u_i(t)$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

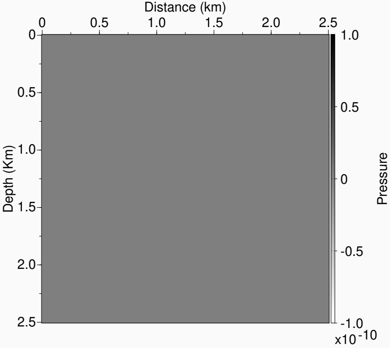
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

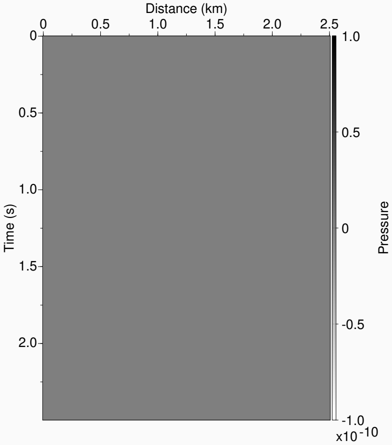
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

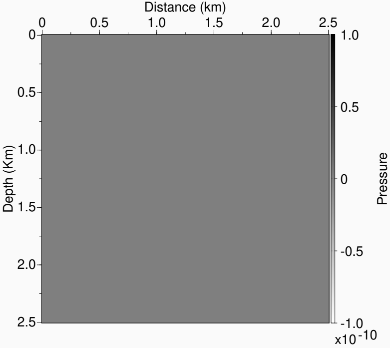
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

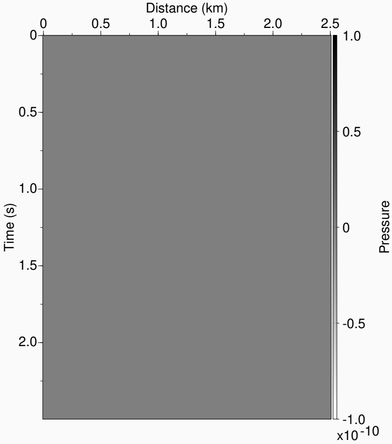
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

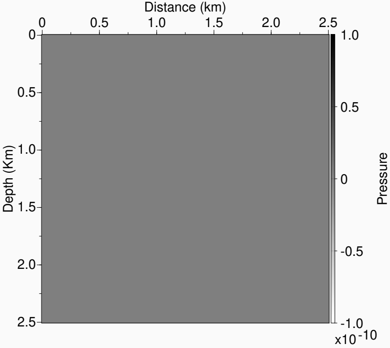
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

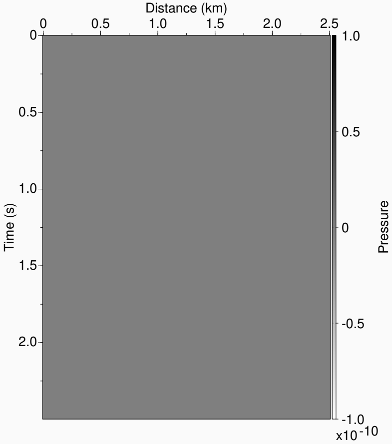
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

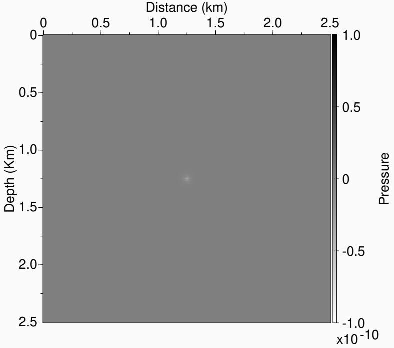
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

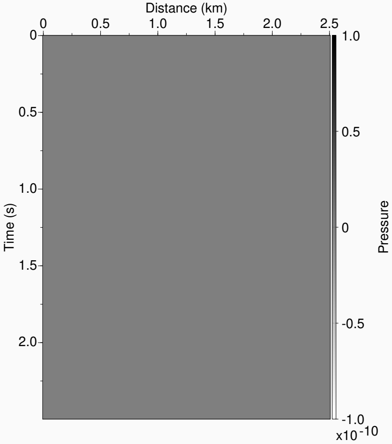
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

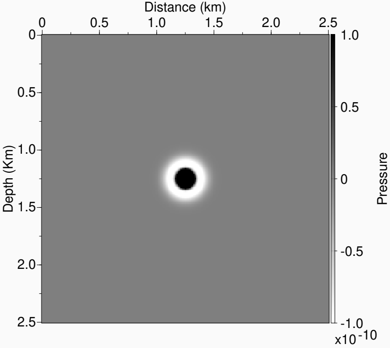
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

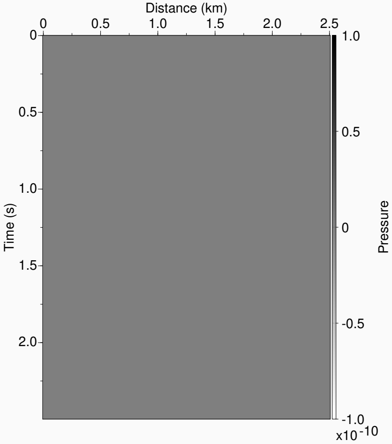
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

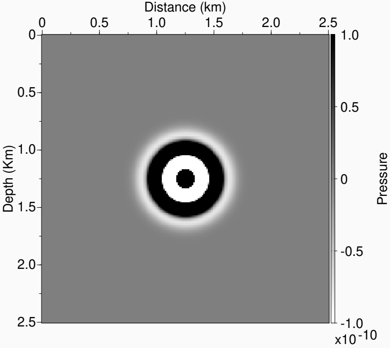
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

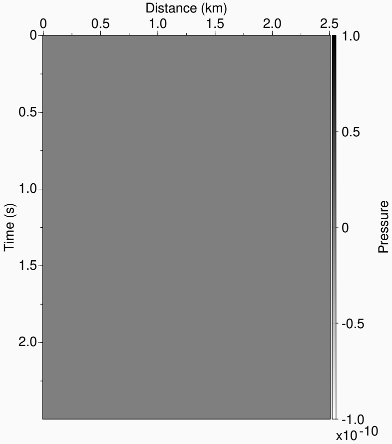
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

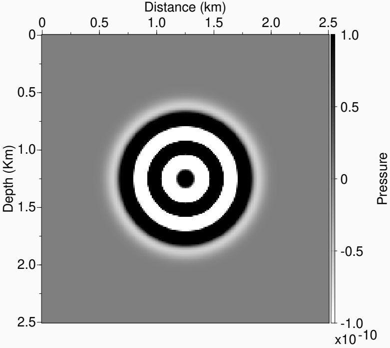
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

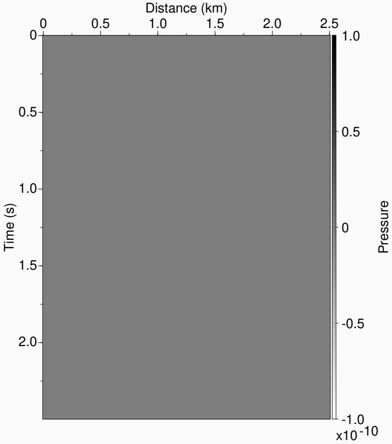
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

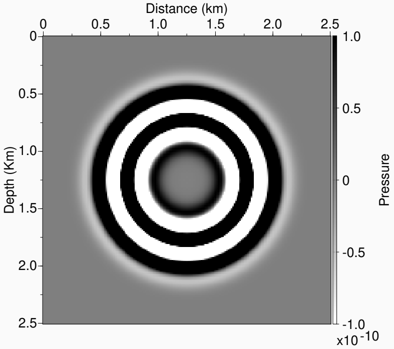
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

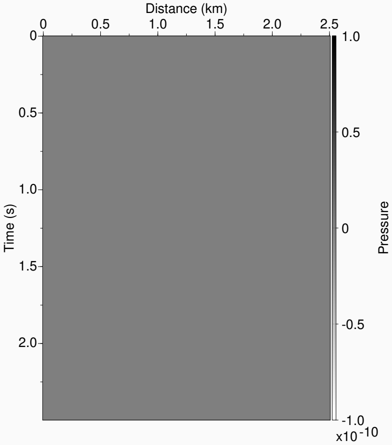
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

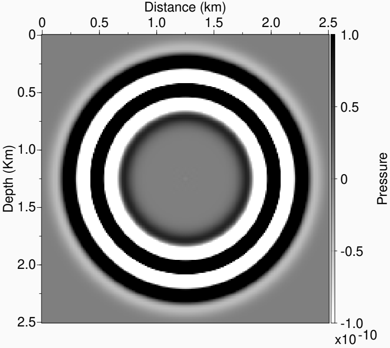
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

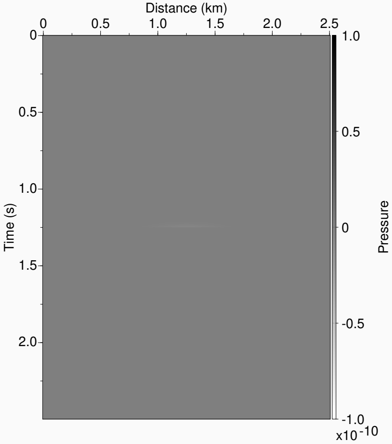
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

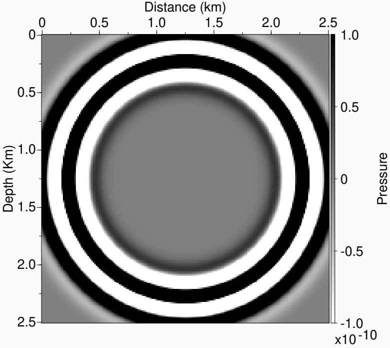
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

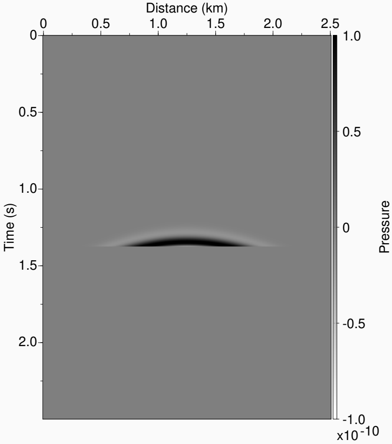
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

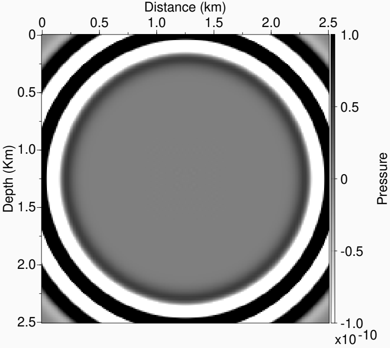
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

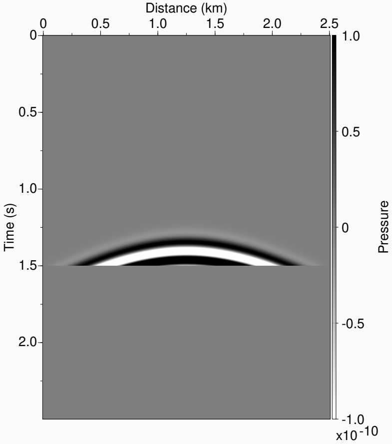
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix: step 2



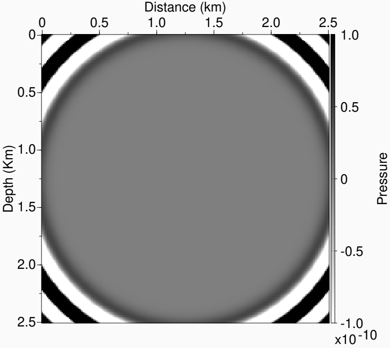
Diffracted wavefield

$\frac{\partial u}{\partial m_i}$



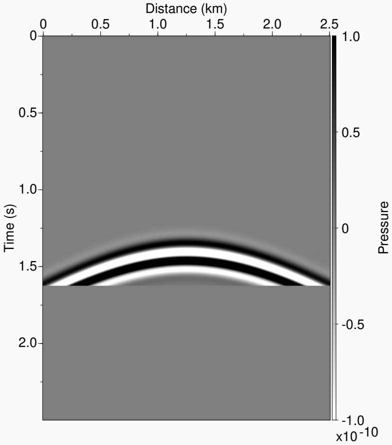
Recorded diffracted field

$\frac{\partial d_{cal}}{\partial m_i}$



Diffracted wavefield

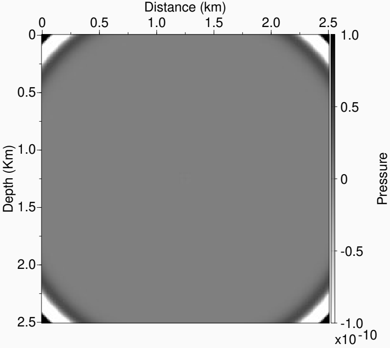
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

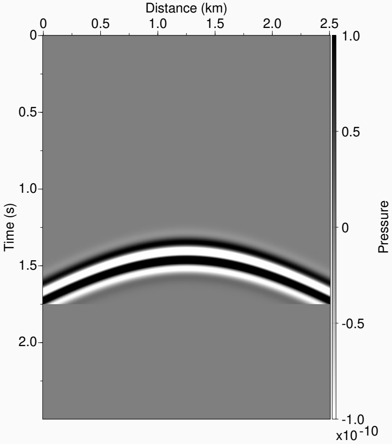
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

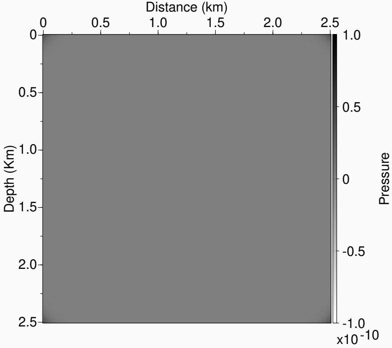
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

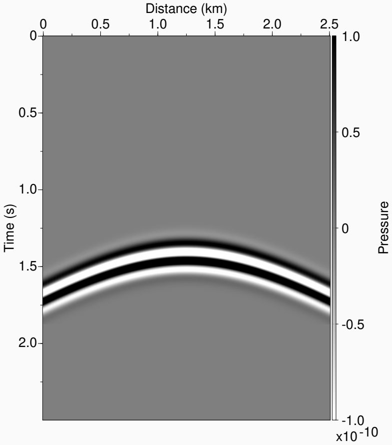
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

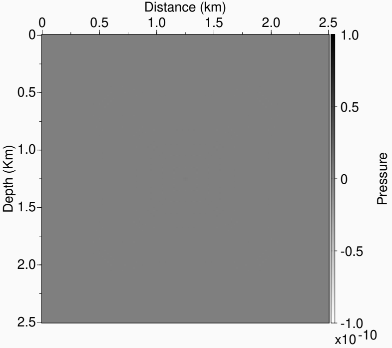
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

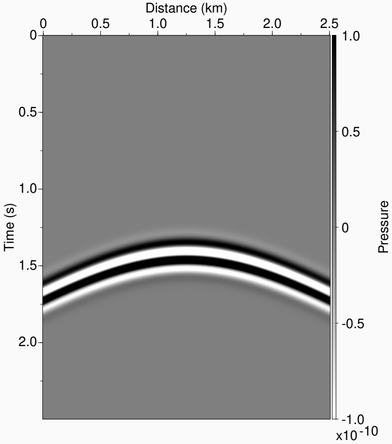
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

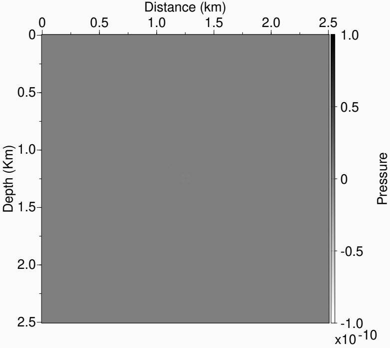
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

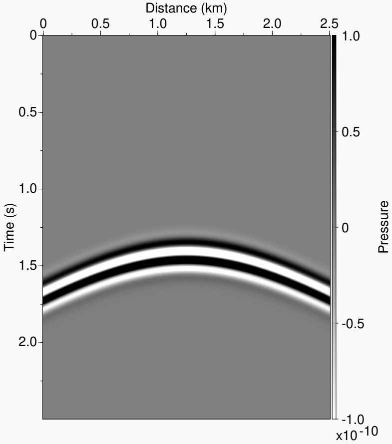
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix: step 2



Diffracted wavefield

$$\frac{\partial u}{\partial m_i}$$

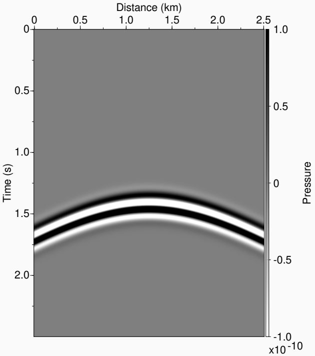


Recorded diffracted field

$$\frac{\partial d_{cal}}{\partial m_i}$$

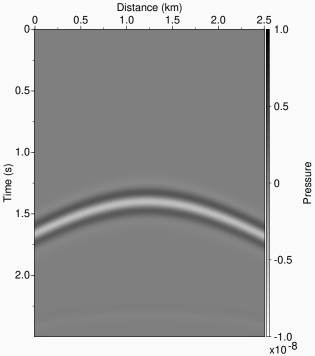
Gradient building through the Jacobian matrix: step 3

Correlation with the actual residuals : constructive correlation, the gradient will be non-zero for the center point



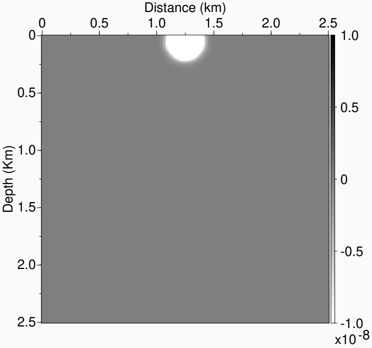
Recorded diffracted field

$\frac{\partial d_{cal}}{\partial m_i}$



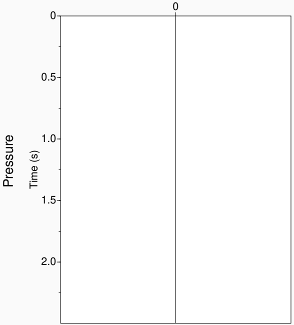
Data residuals

$d_{cal}(x_r, t) - d_{obs}(x_r, t)$



Incident wavefield

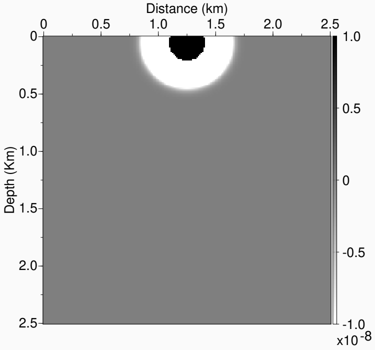
$u(x, t)$

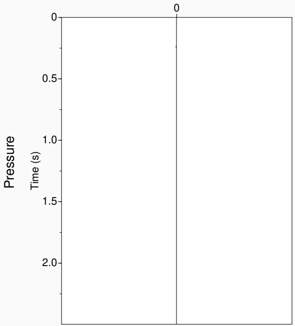


Recorded field at $\mathbf{x} = (0.875, 0.25)$

$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1





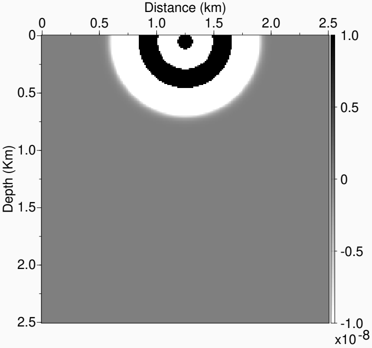
Incident wavefield

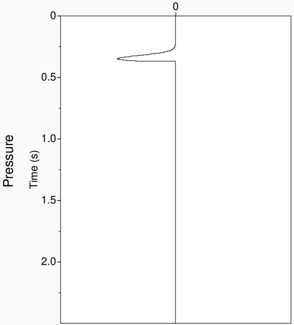
$u(x,t)$

Recorded field at $\mathbf{x} = (0.875, 0.25)$

$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

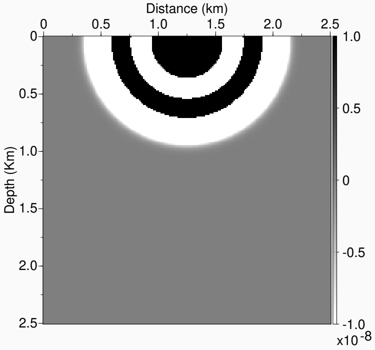




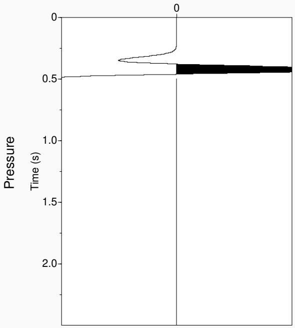
Incident wavefield $u(x,t)$

Recorded field at $\mathbf{x} = (0.875, 0.25)$ $u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

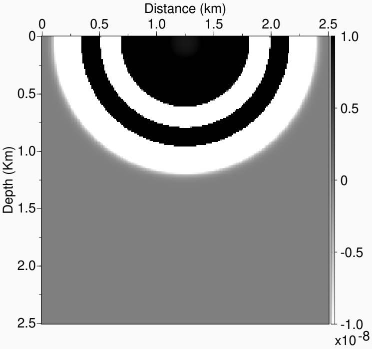


Incident wavefield
$u(x,t)$



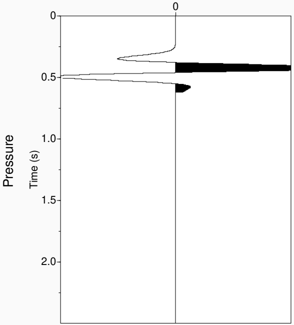
Recorded field at $\mathbf{x} = (0.875, 0.25)$
$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1



Incident wavefield

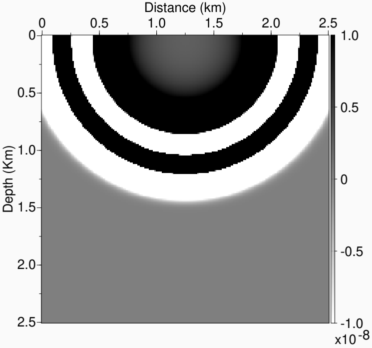
$u(x, t)$

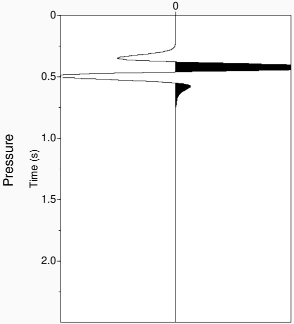


Recorded field at $\mathbf{x} = (0.875, 0.25)$

$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1





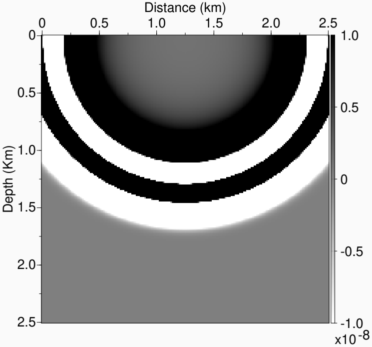
Incident wavefield

$u(x, t)$

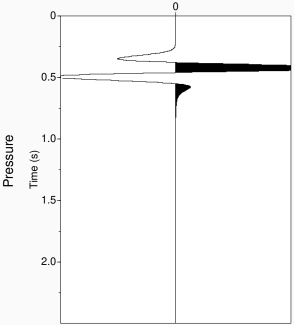
Recorded field at $\mathbf{x} = (0.875, 0.25)$

$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

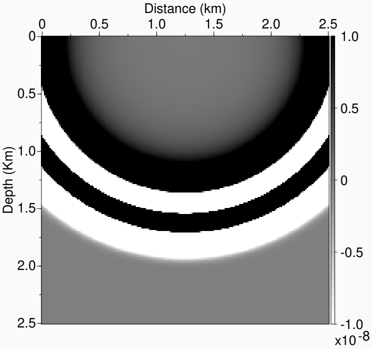


Incident wavefield
$u(x,t)$



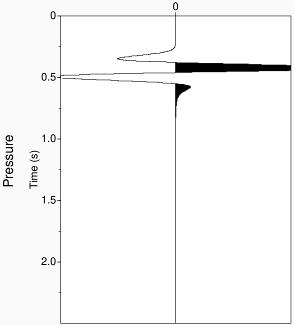
Recorded field at $\mathbf{x} = (0.875, 0.25)$
$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1



Incident wavefield

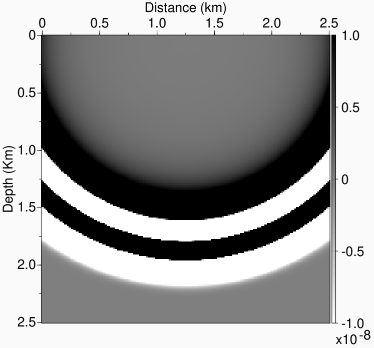
$u(x, t)$



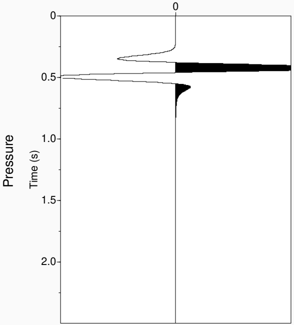
Recorded field at $\mathbf{x} = (0.875, 0.25)$

$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

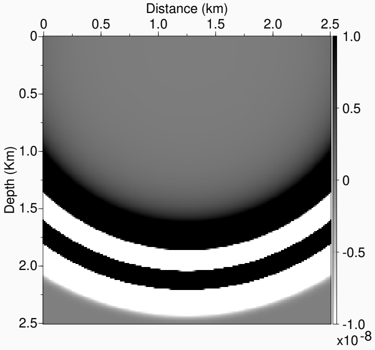


Incident wavefield
$u(x,t)$

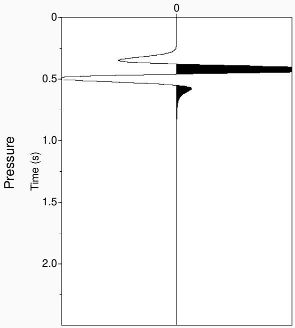


Recorded field at $\mathbf{x} = (0.875, 0.25)$
$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

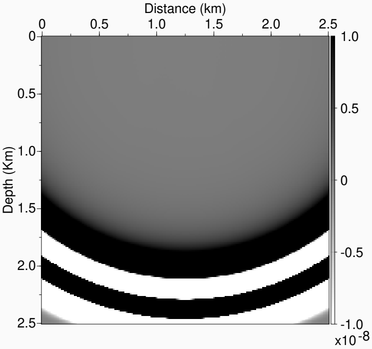


Incident wavefield
$u(x,t)$

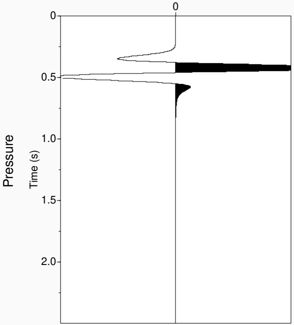


Recorded field at $\mathbf{x} = (0.875, 0.25)$
$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

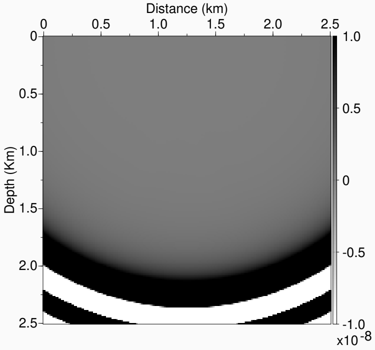


Incident wavefield
$u(x, t)$

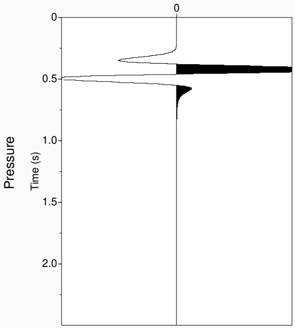


Recorded field at $\mathbf{x} = (0.875, 0.25)$
$u_i(t)$

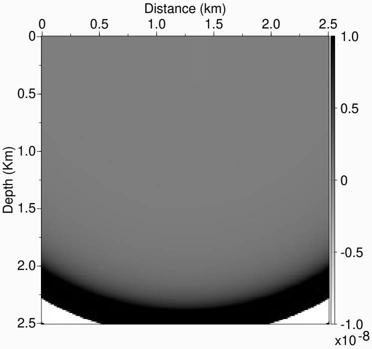
Gradient building through the Jacobian matrix for another point : step 1



Incident wavefield
$u(x, t)$

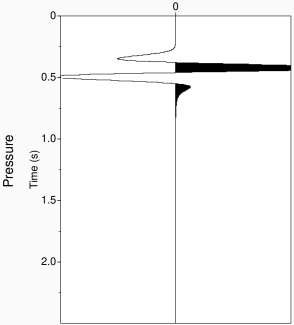


Recorded field at $\mathbf{x} = (0.875, 0.25)$
$u_i(t)$



Incident wavefield

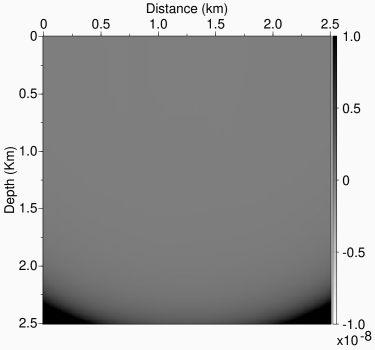
$u(x,t)$

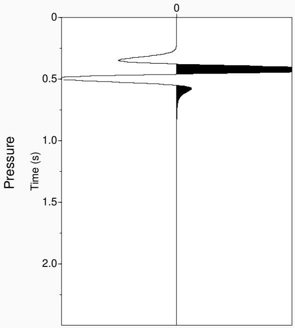


Recorded field at $\mathbf{x} = (0.875, 0.25)$

$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

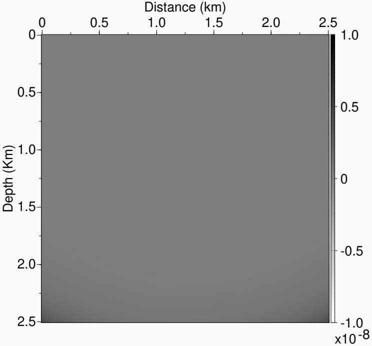


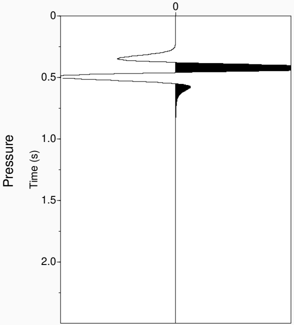


Incident wavefield $u(x, t)$

Recorded field at $\mathbf{x} = (0.875, 0.25)$ $u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1

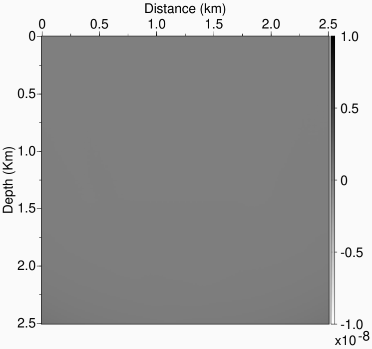




Incident wavefield $u(x,t)$

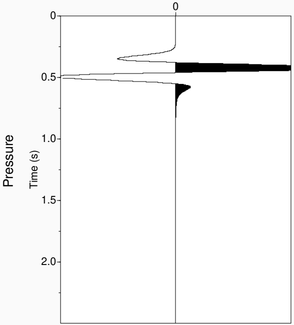
Recorded field at $\mathbf{x} = (0.875, 0.25)$ $u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1



Incident wavefield

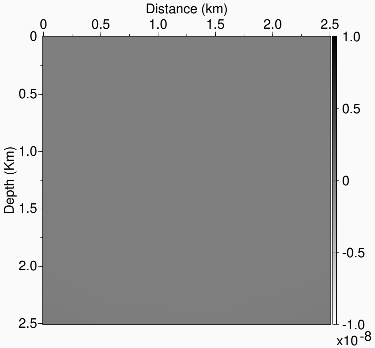
$u(x, t)$

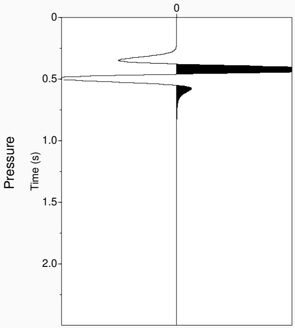


Recorded field at $\mathbf{x} = (0.875, 0.25)$

$u_i(t)$

Gradient building through the Jacobian matrix for another point : step 1





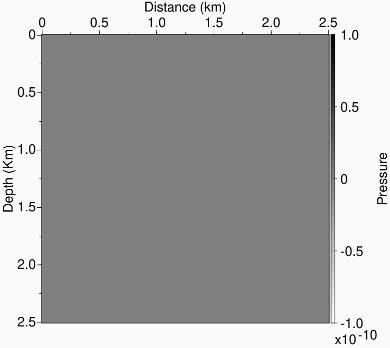
Incident wavefield

$u(x, t)$

Recorded field at $\mathbf{x} = (0.875, 0.25)$

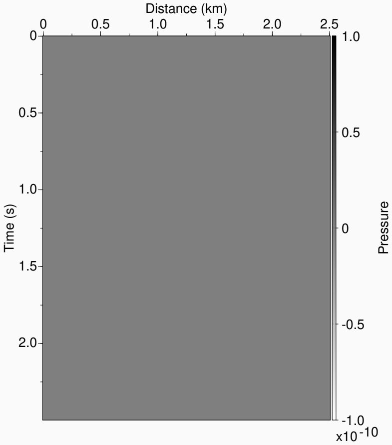
$u_i(t)$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

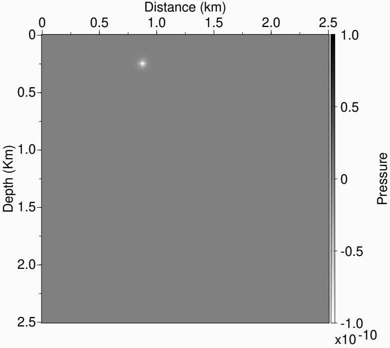
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

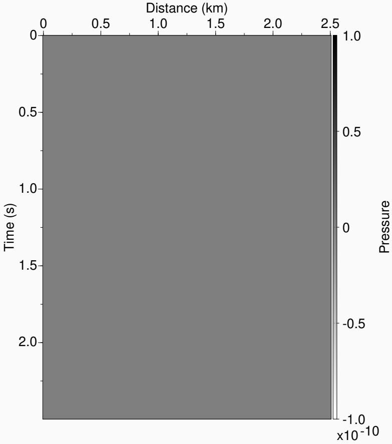
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

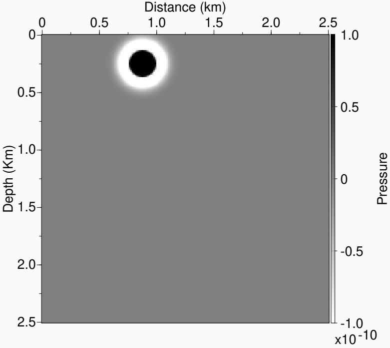
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

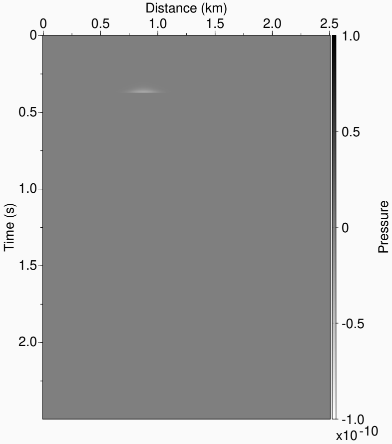
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

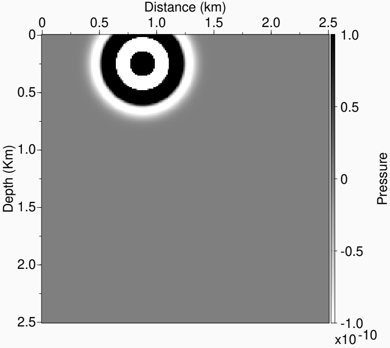
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

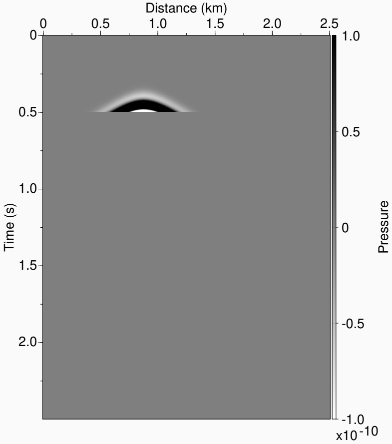
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

$$\frac{\partial u}{\partial m_i}$$



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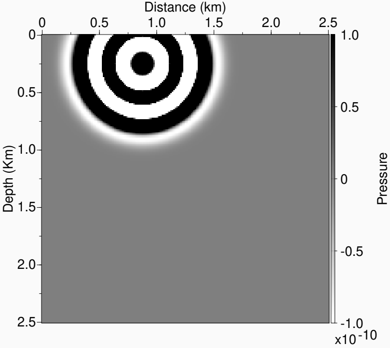
Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

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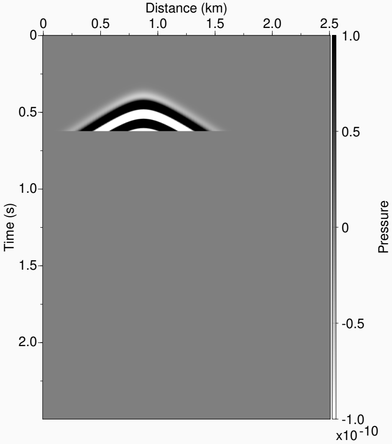

Recorded diffracted field

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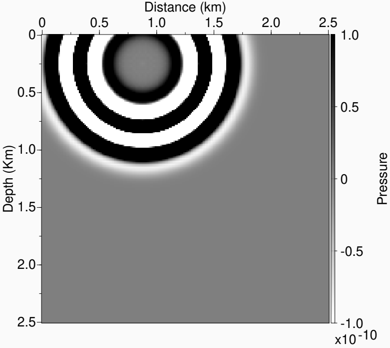
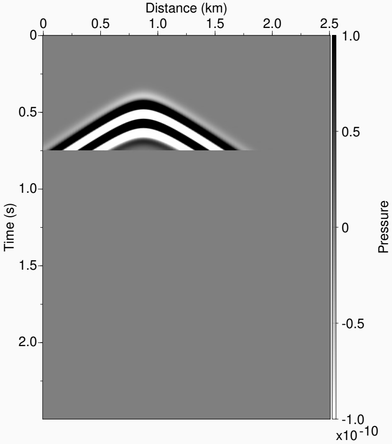
Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

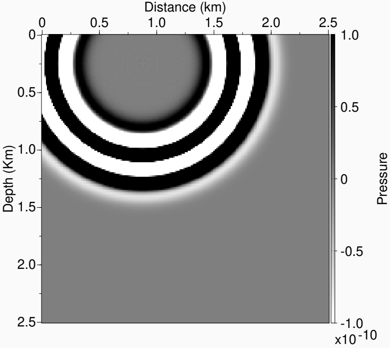
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

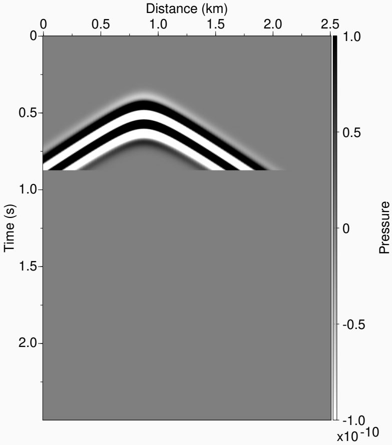
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

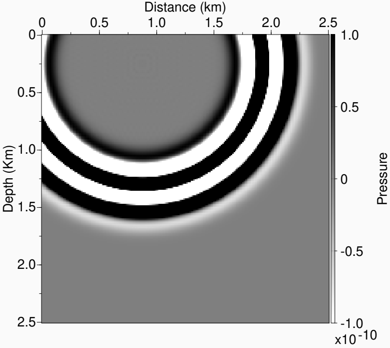
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

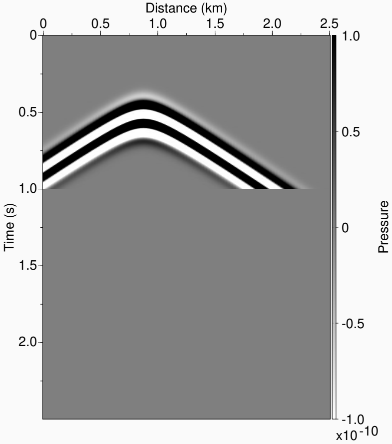
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

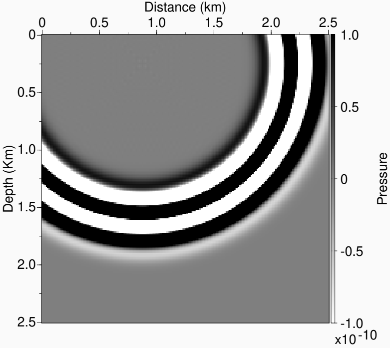
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

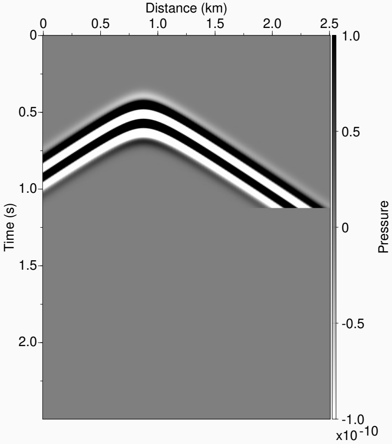
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

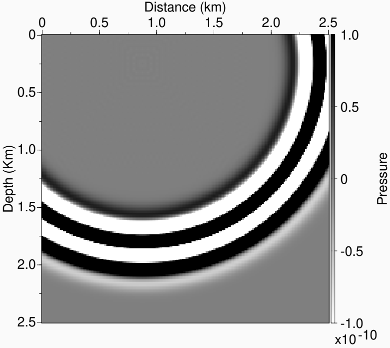
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

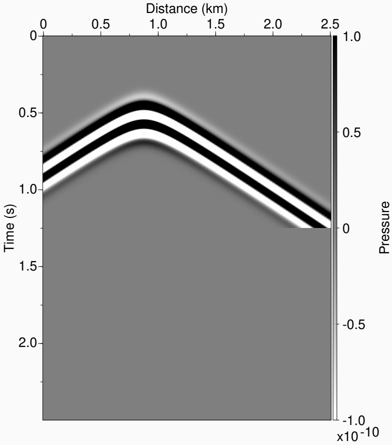
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

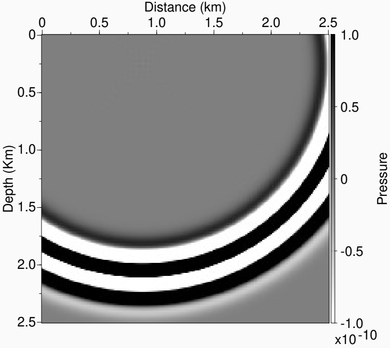
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

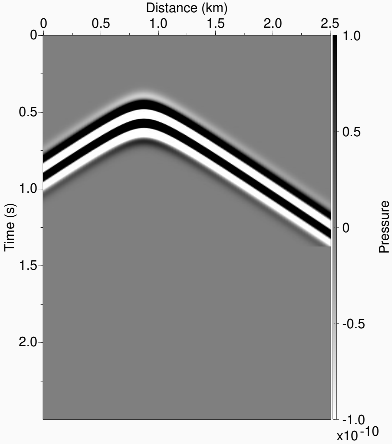
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

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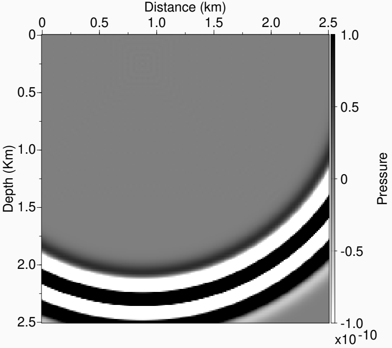
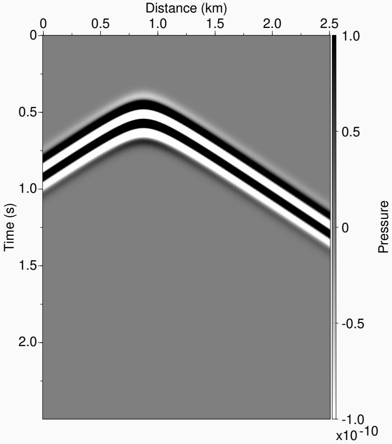
Gradient building through the Jacobian matrix for another point: step 2



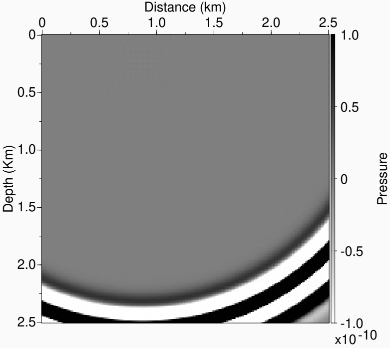
Diffracted wavefield

$$\frac{\partial u}{\partial m_i}$$



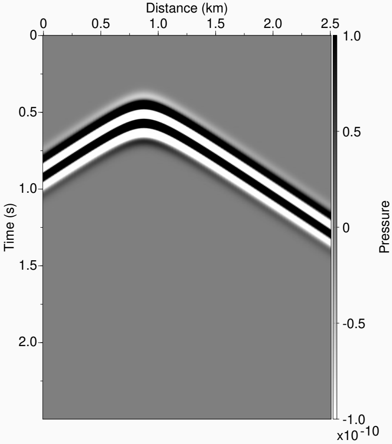
Recorded diffracted field

$$\frac{\partial d_{cal}}{\partial m_i}$$



Diffracted wavefield

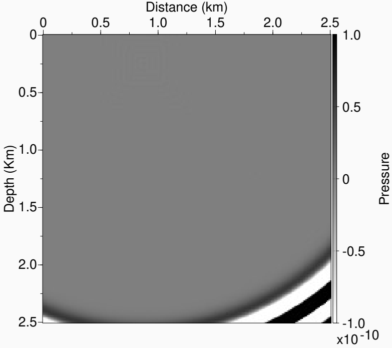
$$\frac{\partial u}{\partial m_i}$$



Recorded diffracted field

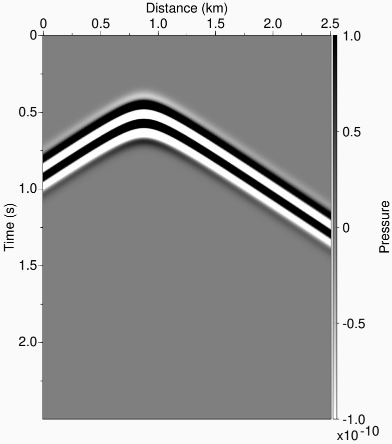
$$\frac{\partial d_{cal}}{\partial m_i}$$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

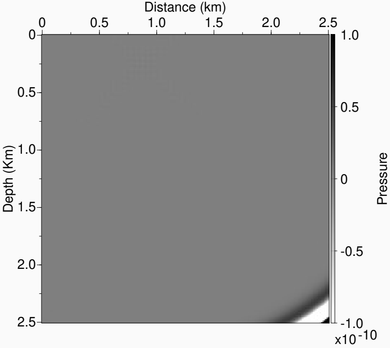
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

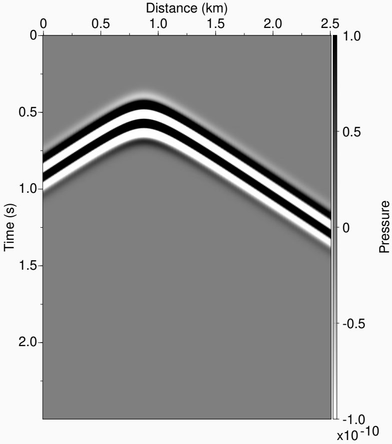
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix for another point: step 2



Diffracted wavefield

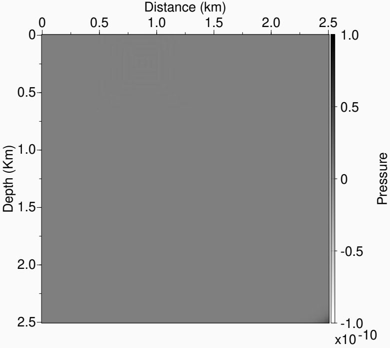
$\frac{\partial u}{\partial m_i}$



Recorded diffracted field

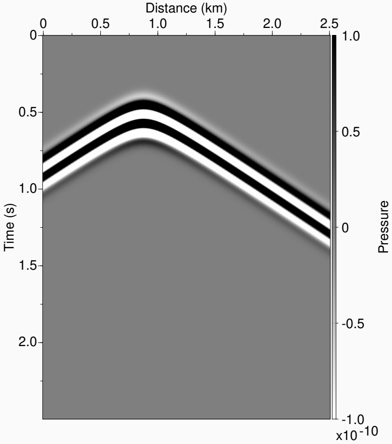
$\frac{\partial d_{cal}}{\partial m_i}$

Gradient building through the Jacobian matrix for another point: step 2



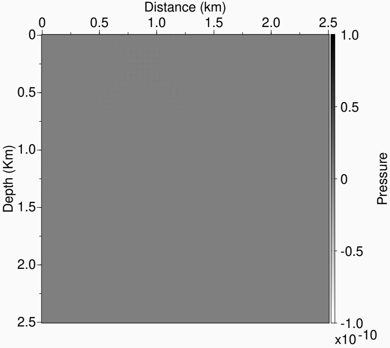
Diffracted wavefield

$$\frac{\partial u}{\partial m_i}$$



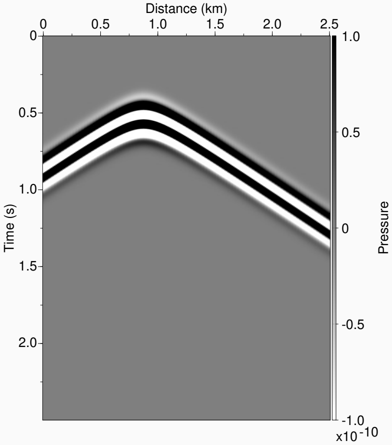
Recorded diffracted field

$$\frac{\partial d_{cal}}{\partial m_i}$$



Diffracted wavefield

$$\frac{\partial u}{\partial m_i}$$

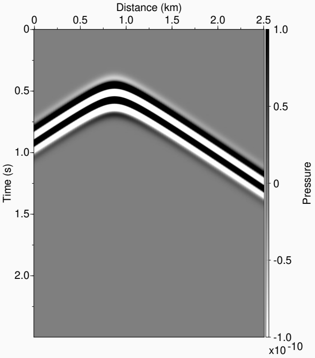


Recorded diffracted field

$$\frac{\partial d_{cal}}{\partial m_i}$$

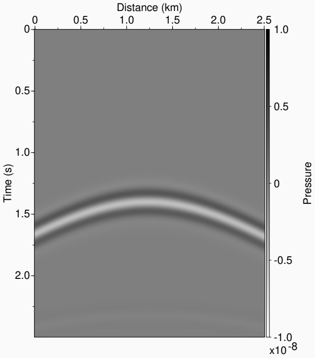
Gradient building through the Jacobian matrix for another point: step 3

Correlation with the actual residuals : non-constructive correlation, the gradient will be zero for this point



Recorded diffracted field

$\frac{\partial d_{cal}}{\partial m_i}$

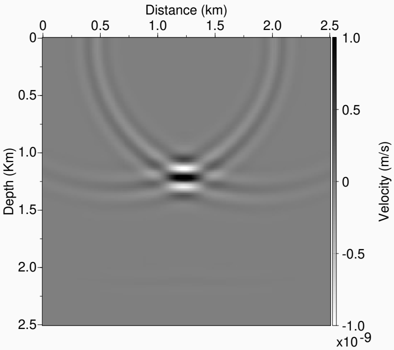


Data residuals

$d_{cal}(x_r, t) - d_{obs}(x_r, t)$

Gradient building through the Jacobian matrix: final gradient

After scanning all the point and computing the correlation between the scattering response and the residuals, we obtain the following gradient



We have a strong anomaly at the correct position, and artifacts coming from the usage of a single source in this experiment.

Gradient building through the Jacobian matrix: comments

The method is not practicable.

- It requires to solve one wave propagation problem per discrete point of the medium to compute the diffracted wavefield

Yet it provides a good understanding what the gradient means.

- Basically, all the points of the medium are scanned and the scattering response is recorded for each of them and correlated with the residuals. Strong gradient thus means positioning a diffracting point such that the diffraction patterns correlates well with the residuals (and thus decreases the misfit between calculated and observed data)

The construction pattern is

- from the source to the diffracting point
- from the diffracting point to the receivers

Next we analyze the gradient building through the adjoint state strategy

Gradient building through the adjoint state strategy

Remember the formula for the gradient is

$$-\frac{2}{V_P^3}\int_0^T \frac{\partial^2 u(x,t)}{\partial t^2}\lambda(x,t)dt \tag{6}$$

where $u(x,t)$ is the incident wavefield satisfying

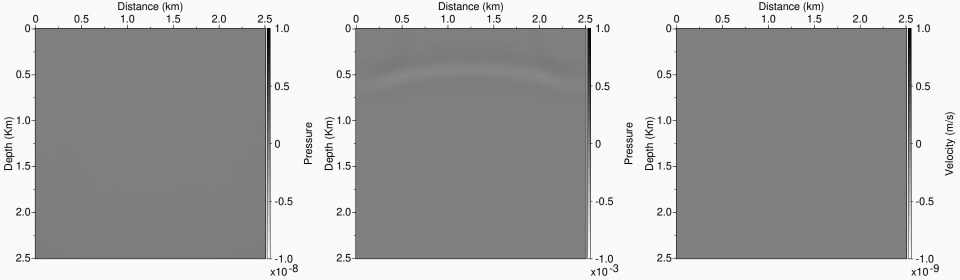
$$\frac{1}{V_P^2}\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial^2 u(x,t)}{\partial x^2} = \varphi(x,t) \tag{7}$$

with homogeneous initial conditions and $\lambda(x,t)$ is the adjoint wavefield satisfying

$$\frac{1}{V_P^2}\frac{\partial^2 \lambda(x,t)}{\partial t^2} - \frac{\partial^2 \lambda(x,t)}{\partial x^2} = -\sum_{r=1}^{N_r}\left(d_{cal}(x_r,t) - d_{obs}(x_r,t)\right)\delta(x - x_r) \tag{8}$$

with homogeneous final conditions.

Gradient building through the adjoint state strategy: illustration



Incident wavefield

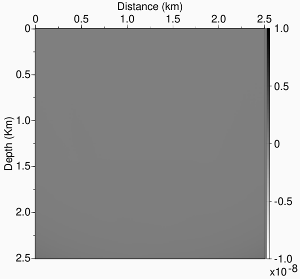
$u(x, T-t)$

Adjoint wavefield

$\lambda(x, T-t)$

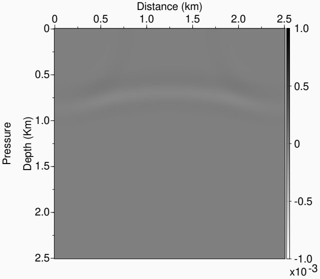
Gradient

$-\frac{2}{V_P^3}\int_0^T \frac{\partial^2 u(x,t)}{\partial t^2}\lambda(x,t)dt$



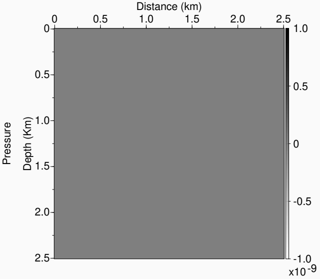
Incident wavefield

$u(x, T - t)$



Adjoint wavefield

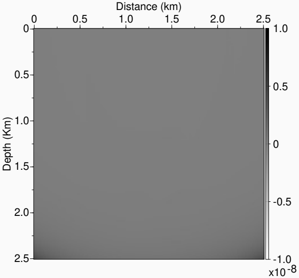
$\lambda(x, T - t)$



Gradient

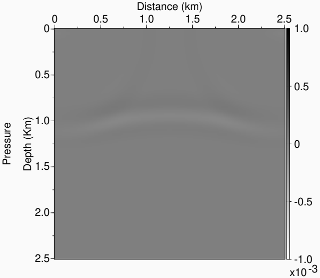
$-\frac{2}{V_P^3} \int_0^T \frac{\partial^2 u(x,t)}{\partial t^2} \lambda(x,t) dt$

Gradient building through the adjoint state strategy: illustration



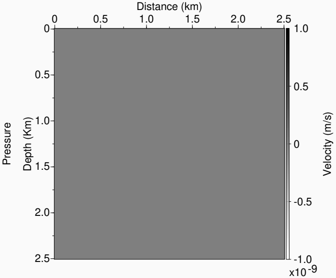
Incident wavefield

$u(x, T - t)$



Adjoint wavefield

$\lambda(x, T - t)$



Gradient

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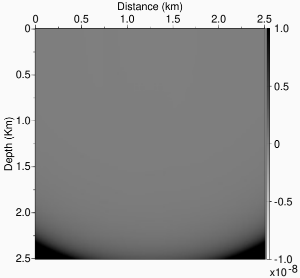
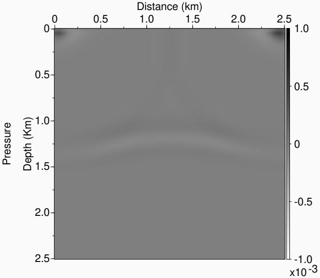
Gradient building through the adjoint state strategy: illustration



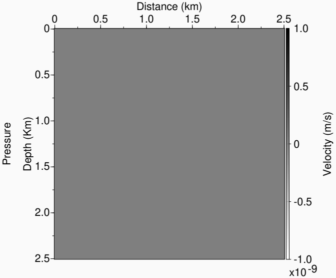
Incident wavefield

$u(x, T-t)$



Adjoint wavefield

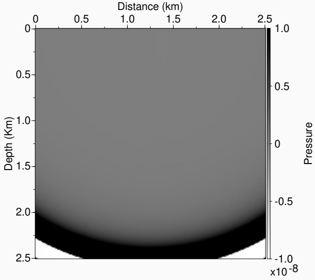
$\lambda(x, T-t)$



Gradient

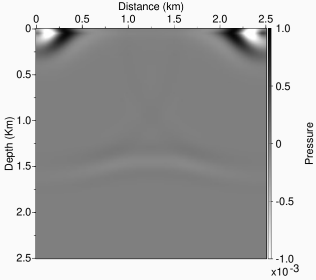
$-\frac{2}{V_P^3}\int_0^T \frac{\partial^2 u(x,t)}{\partial t^2}\lambda(x,t)dt$

Gradient building through the adjoint state strategy: illustration



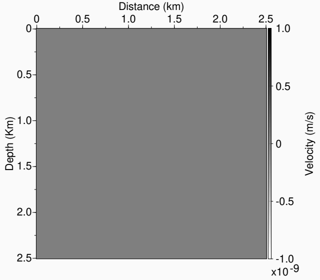
Incident wavefield

$u(x, T - t)$



Adjoint wavefield

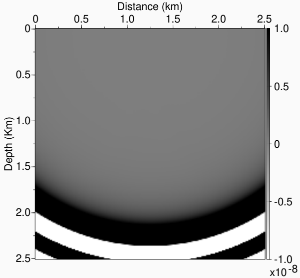
$\lambda(x, T - t)$



Gradient

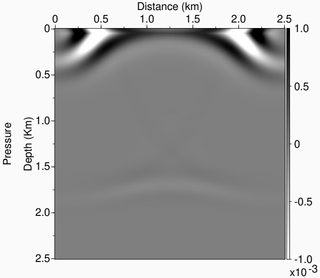
$-\frac{2}{V_P^3}\int_0^T \frac{\partial^2 u(x,t)}{\partial t^2}\lambda(x,t)dt$

Gradient building through the adjoint state strategy: illustration



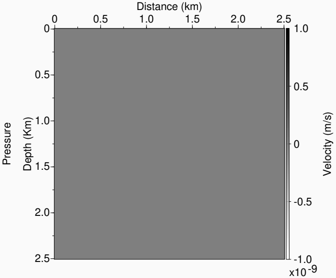
Incident wavefield

$u(x, T-t)$



Adjoint wavefield

$\lambda(x, T-t)$



Gradient

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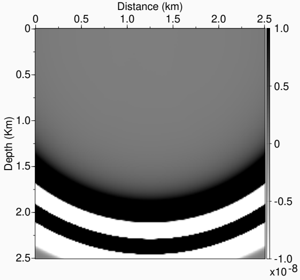
Gradient building through the adjoint state strategy: illustration



Incident wavefield

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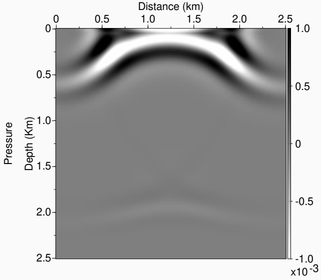
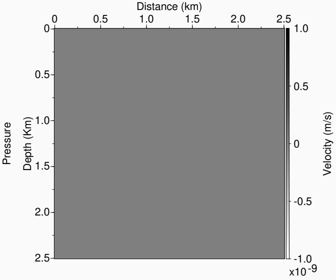


Adjoint wavefield

$\lambda(x, T-t)$



Gradient

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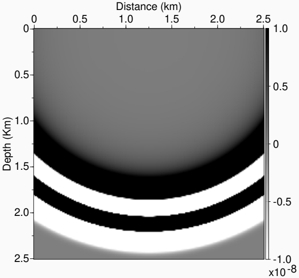
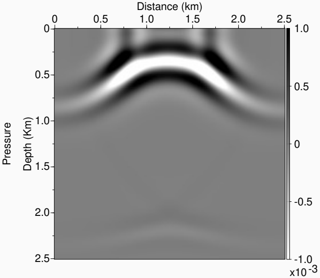
Gradient building through the adjoint state strategy: illustration



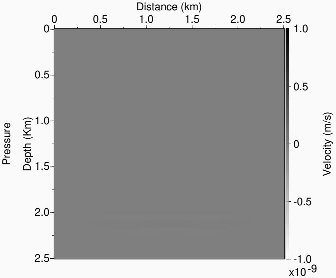
Incident wavefield

$u(x, T - t)$



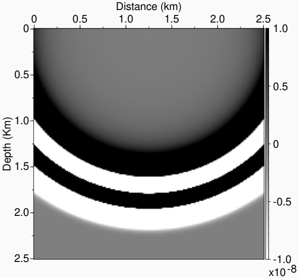
Adjoint wavefield

$\lambda(x, T - t)$



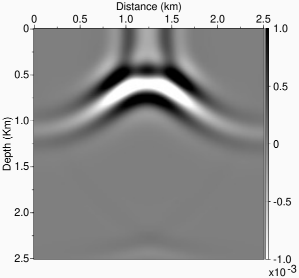
Gradient

$-\frac{2}{V_P^3}\int_0^T \frac{\partial^2 u(x,t)}{\partial t^2}\lambda(x,t)dt$



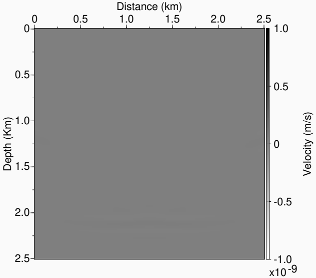
Incident wavefield

$u(x, T - t)$



Adjoint wavefield

$\lambda(x, T - t)$



Gradient

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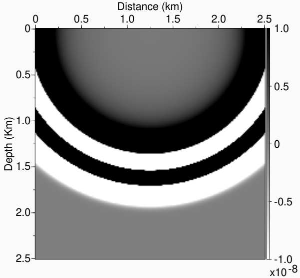
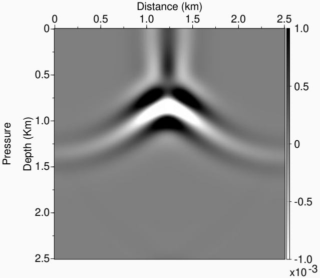


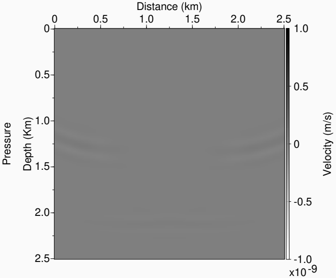
Incident wavefield

$u(x, T - t)$



Adjoint wavefield

$\lambda(x, T - t)$



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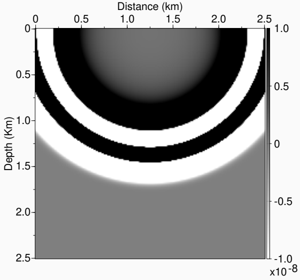
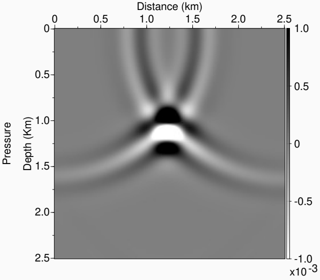
$-\frac{2}{V_P^3} \int_0^T \frac{\partial^2 u(x,t)}{\partial t^2} \lambda(x, t) dt$

Gradient building through the adjoint state strategy: illustration



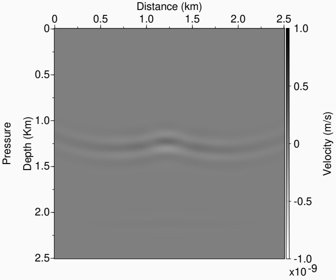
Incident wavefield

$u(x, T - t)$



Adjoint wavefield

$\lambda(x, T - t)$



Gradient

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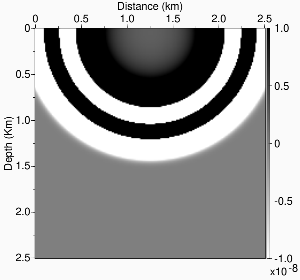
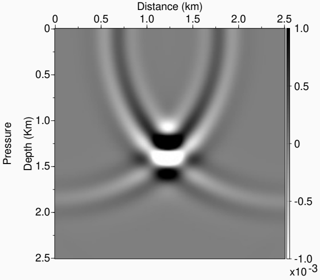


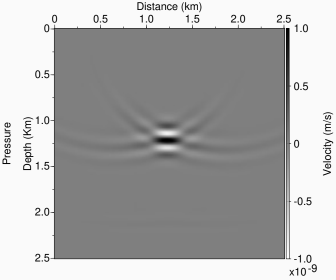
Incident wavefield

$u(x, T - t)$



Adjoint wavefield

$\lambda(x, T - t)$



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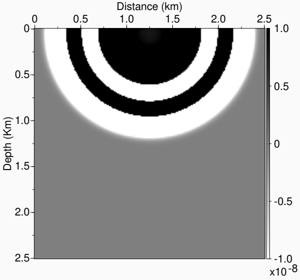
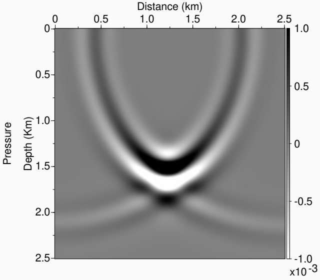


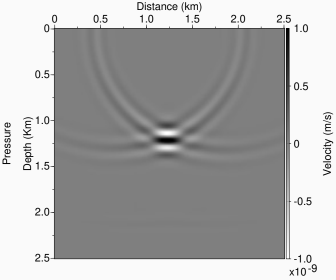
Incident wavefield

$u(x, T-t)$



Adjoint wavefield

$\lambda(x, T-t)$



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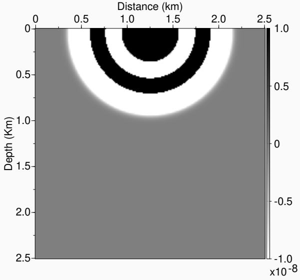
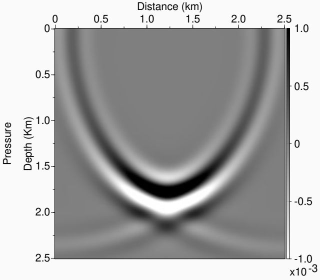
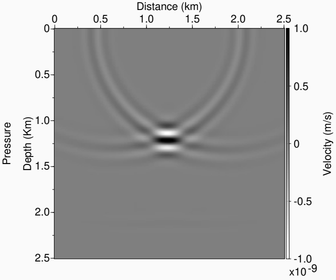


Incident wavefield
$u(x, T-t)$



Adjoint wavefield
$\lambda(x, T-t)$



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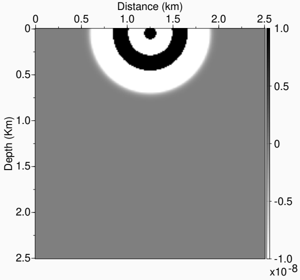
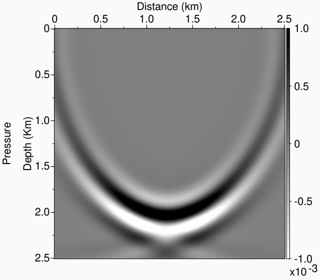
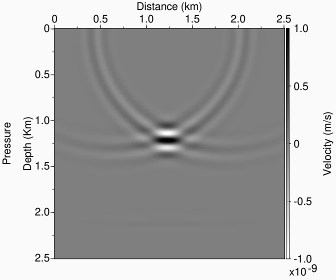


Incident wavefield
$u(x, T - t)$



Adjoint wavefield
$\lambda(x, T - t)$



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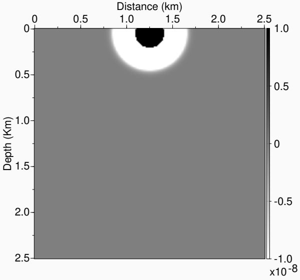
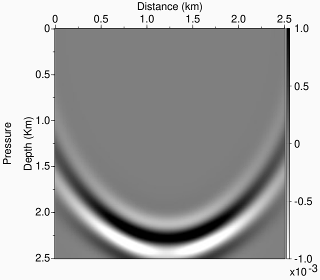
Gradient building through the adjoint state strategy: illustration



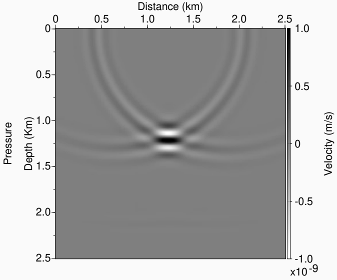
Incident wavefield

$u(x, T-t)$



Adjoint wavefield

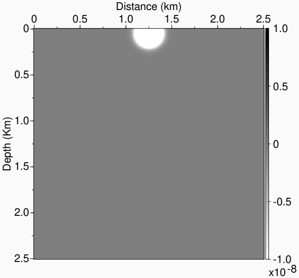
$\lambda(x, T-t)$



Gradient

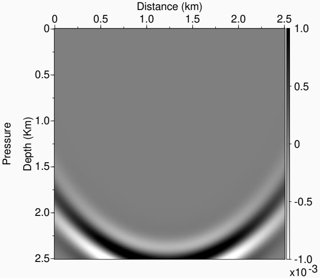
$-\frac{2}{V_P^3} \int_0^T \frac{\partial^2 u(x,t)}{\partial t^2} \lambda(x,t) dt$

Gradient building through the adjoint state strategy: illustration



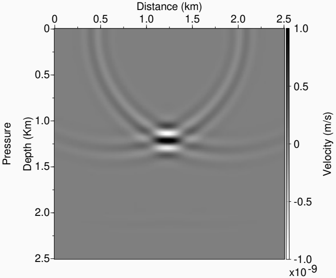
Incident wavefield

$u(x, T-t)$



Adjoint wavefield

$\lambda(x, T-t)$



Gradient

$-\frac{2}{V_P^3} \int_0^T \frac{\partial^2 u(x,t)}{\partial t^2} \lambda(x,t) dt$

Gradient building through the adjoint-state strategy: comments

The method is practicable.

- It requires to solve two wave propagation problems for one gradient: 1 incident field and 1 adjoint field have to be computed only

The method can be seen as a more clever way to assemble the gradient

- Instead of scanning all the points of the medium, the residuals are backpropagated to the location of the diffracting point, and correlated with the incident field.

The construction pattern is

- from the source to the diffracting point
- from the receivers to the diffracting point

The gradient is thus assembled globally for each discrete point, instead of being built for each discrete point independently.

Gradient building: summation on sources

Remember the misfit function in FWI is a built as a summation over sources

$$f(m) = \sum_{s=1}^{N_s} f_s(m) \tag{9}$$

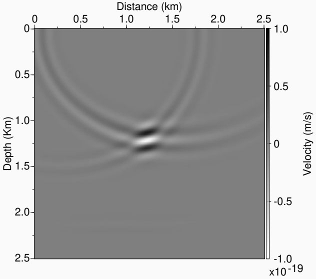
where

$$f_s(m) = \sum_{r=1}^{N_r} \int_0^T d_{cal,s}[m](x_r,t) - d_{obs,s}(x_r,t)dt. \tag{10}$$

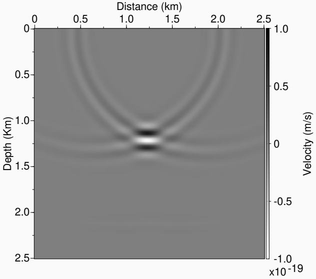
Therefore, the gradient of the misfit function is also built as a summation over sources

$$\nabla f(m) = \sum_{s=1}^{N_s} \nabla f_s(m) \tag{11}$$

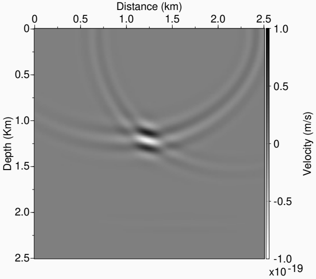
Gradient building: summation on sources



Source at $x = 0.625$ km

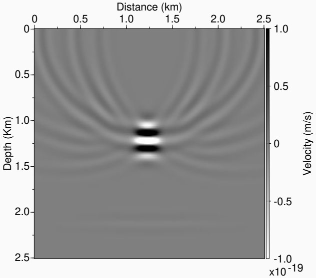


Source at $x = 1.25$ km



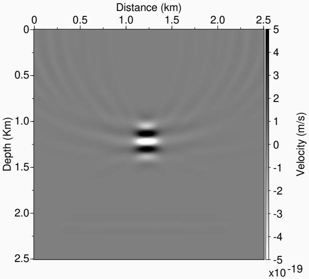
Source at $x = 1.875$ km

Gradient building: summation on sources



Sum of the 3 gradients

Gradient building: summation on sources



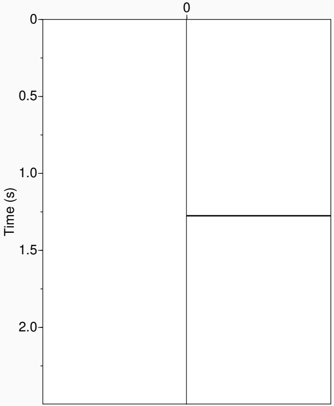
Sum with 11 source positions: x_S=125 m, 375 m, 625 m, 850 m, 1125 m, 1250 m, 1375 m, 1625 m, 1875 m, 2125 m

Gradient building: summation on sources

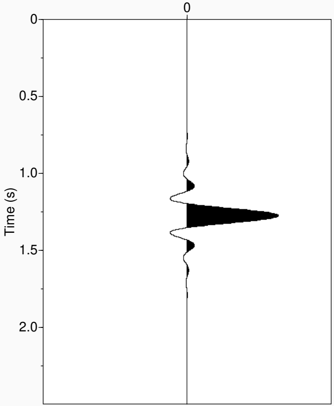
- The summation over sources is also called “stacking” in seismic processing.
- The effect is to emphasize the information on the perturbation and remove artifacts through constructive and destructive interferences
- We obtained finally in the gradient a “finite-frequency view” of the Gaussian anomaly in the exact model

Gradient building: summation on sources

- The oscillations around the anomaly are typical of band pass filtered of a step function or a Dirac function



Dirac function in time



Band-pass filtered Dirac

References

Thomsen, L. A. (1986). Weak elastic anisotropy. *Geophysics*, 51:1954–1966.