

Geophysical imaging: Mathematics for imaging the subsurface

L. Métivier¹

Thursday 24th September, 2026



LABORATOIRE
JEAN KUNTZMANN
MATHÉMATIQUES APPLIQUÉES - INFORMATIQUE



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Lecture notes and (some of the) slides: are available on my webpage.

https://membres-ljk.imag.fr/Ludovic.Metivier/webpage_LMetivier.html

Main outline: 12 sessions

- Introduction/Context/Motivation (1 session: 24/09/2026)
- Full Waveform Modeling (6 sessions: 01/10/2026, 08/10/2026, **14/10/2026**, 05/11/2026, 12/11/2026, 19/11/2026)
- Full Waveform Inversion (5 sessions: 03/12/2026, 10/12/2026, 17/12/2026, 07/01/2027, 14/01/2027)

Room CO10, Thursday, 2:00pm - 3:30pm, **except 14/10/2026 on Wednesday 8:15am - 9:45am**

Final exam: 2 hours in-class exam, no notes, no electronic devices

Objectives of the course: present some key concepts about the mathematics used for subsurface imaging

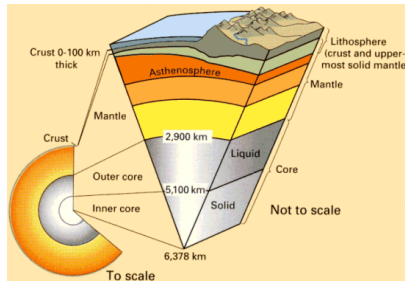
- **Why** and **how** to image the subsurface
- Mechanical **waves** in an **elastic** medium
- **Hyperbolic** partial differential equations
- **Absorbing** boundary conditions for wave propagation problems
- **Discretization** of wave propagation problems
- **Inverse** data fitting problems
- **Diffraction imaging**
- **High performance computing**

Geophysical imaging: to do what?

Seismic data

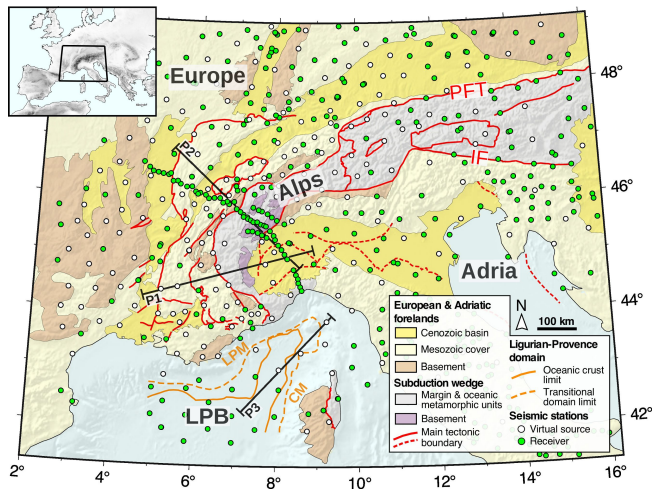
A first glance at seismic inversion methods

- mantle convection, mantle/core boundary
- planet formation
- geomagnetic field generation through convection in the outer core
- plate tectonic



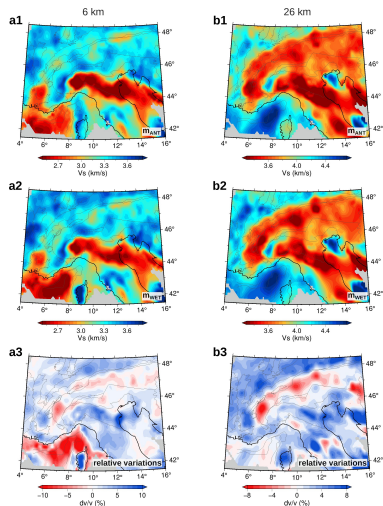
"The layer at the core mantle boundary may serve as the source of material for mantle plumes that give rise to hot spots, which are important in plate tectonics. The thermal properties of this layer might also influence the outward transport of heat from the Earth's core; in turn this could affect the intricate processes that generate the Earth's magnetic field."

Regional imaging at the Alps scale

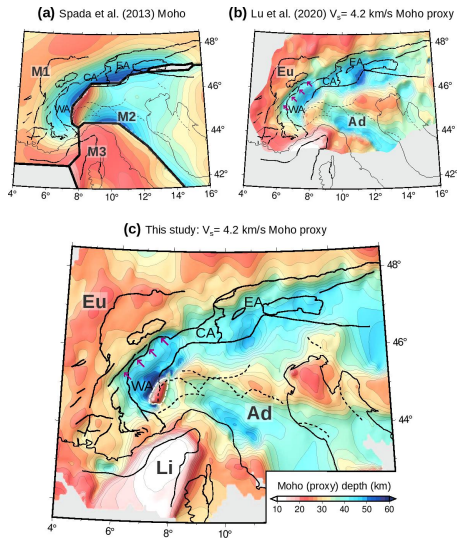


Geological and tectonic setting, locations of seismic stations (white circles: virtual sources; green circles: receivers).

LPB: Ligurian-Provence basin, LPM: Ligurian-Provence margin, CM: Corsican margin (Nouibat et al., 2023).

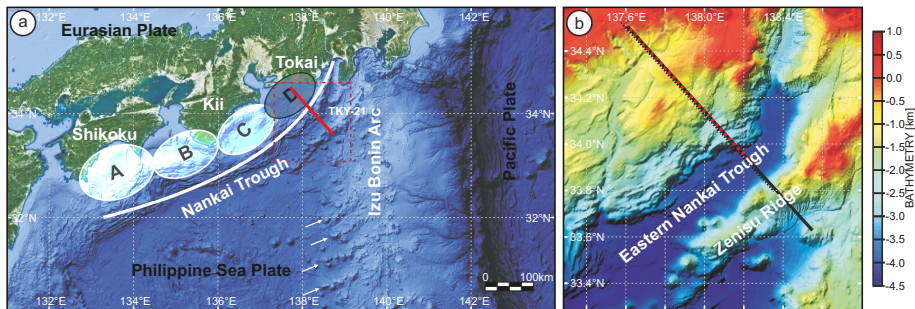


Depth slices of the initial V_S (a1-b1), final V_S (a2-b2) and perturbation (a3-b3)

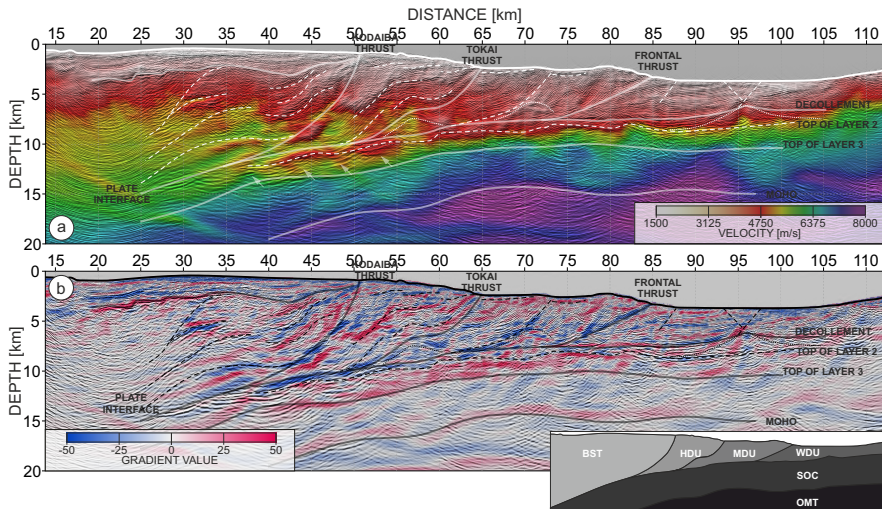


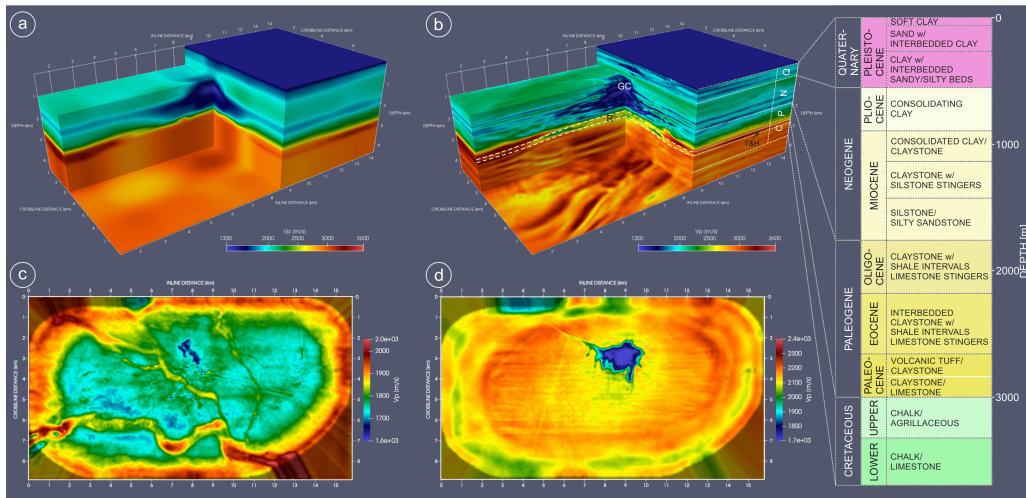
Understanding active zones, preventing seismic hazard

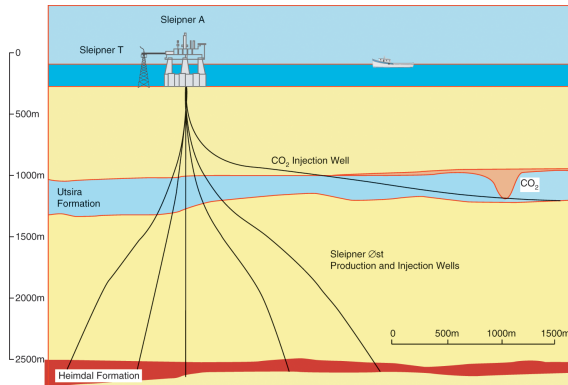
- active zones = active faults, subduction zones, volcanic area
- example: Nankai Trough



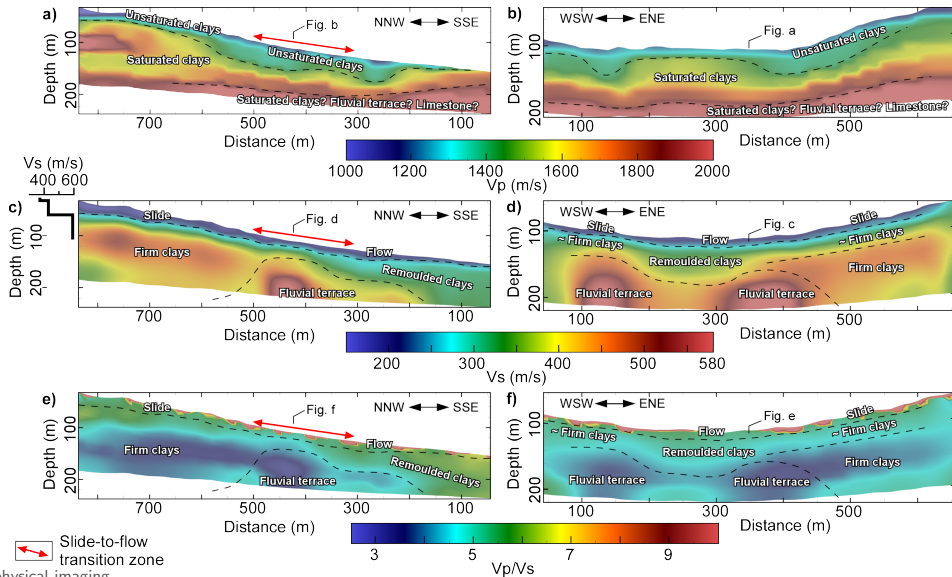
Partitioning of the Nankai Trough into four segments as described by Ando (1975). Region D was left unruptured during the most recent sequence of two large earthquakes (1944 Tonankai and 1946 Nankaido). Figure taken from [Gorszczyk et al. \(2019\)](#).







Schematic view of the CO2 storage injection in Sleipner (Norway) ([Eiken, 2019](#)). Sleipner is a pilot site, which has made possible to test and evaluate the interest of CO2 storage technology since 1996.



- mining
- geothermal energy
- geotechnical engineering
- archaeology
- medical imaging with ultra-sound
- medical imaging with elastography
- ...

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These applications go across scales from the global Earth to human tissues

- Deepest drilling in the world: 12, 2 km $\simeq$ **0.2 % of the Earth's radius**
- Location: Kola Peninsula, Russia
- Drilling duration: 1970-1989 *i.e.* 19 years



Localization of the Kola peninsula on Google Earth (left). Picture of the Kola Superdeep Borehole drilling site(right).

Drawback of drilling:

- direct measurement can destroy the target: archeology and geotechnical engineering belong to this category of applications.
- local information only: the subsurface, especially the crust, can not be accurately represented as a layered medium.
- highly technical and complex operation: thus expensive and risky
- for regional scale and global scale imaging, the depth of investigation of a drilling operation is far from being sufficient.

Starting point: subsurface rheology has an impact on the propagation of waves.

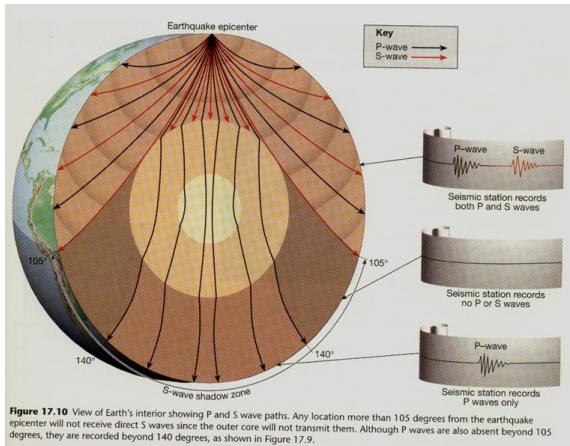
- Electromagnetic waves. In this case, the subsurface structure/rheology variations affect the mean **permittivity and conductivity** of the subsurface, which have an effect on the propagation of electromagnetic waves.
- Mechanical (elastic) waves. In this case, the subsurface structure/rheology variations affect the mean **velocity, density, anisotropy, and attenuation** of the subsurface, which have an effect on the propagation of mechanical (elastic) waves.

Principle: from the observation of electromagnetic or elastic waves, infer the mean electromagnetic or mechanical properties of the subsurface

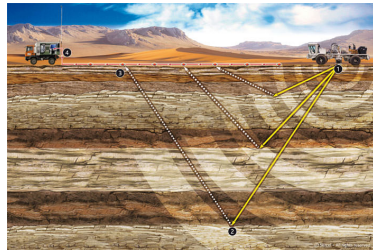
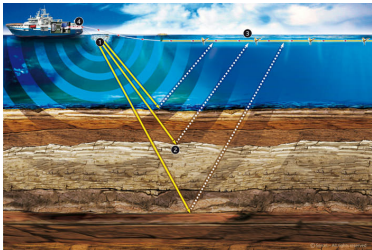
Geophysical imaging: to do what?

Seismic data

A first glance at seismic inversion methods



Global tomography sketch: an earthquake acts as a source which propagates elastic waves. These waves are recorded by seismic stations spread at different locations at the surface.



Controlled source acquisition sketch, in a marine environment (left) in a land environment (right)

In terms of mathematics, the seismic data is thus a collection of time functions $d(t)$ associated with a source s at position x_s and a receiver r at position x_r . We will denote it as

$$d_{r,s}(t), \tag{1}$$

in the following, or equivalently

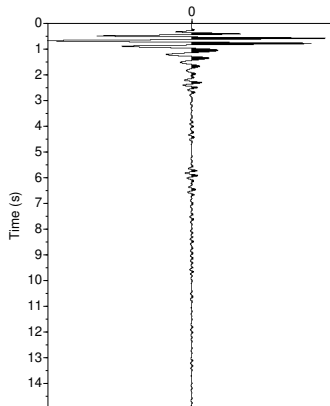
$$d(x_s, x_r, t), \tag{2}$$

or

$$d_s(x_r, t), \tag{3}$$

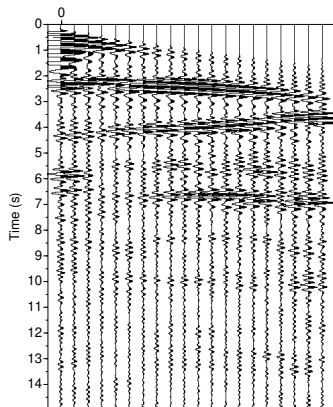
depending on the context. A single function $d_{r,s}(t)$ will be referred to as a *seismic trace* in the following.

A typical example of a seismic trace



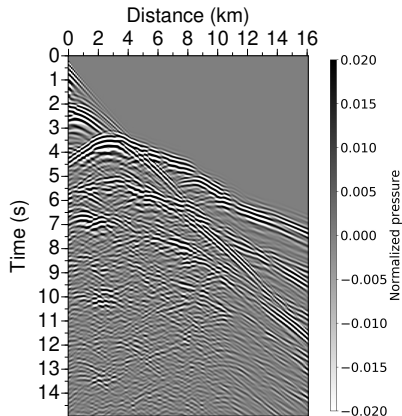
Seismic trace $d(t)$ as a function of time. We can identify a first wave packet of larger amplitude and later wave packets with smaller amplitude.

Instead of analyzing the data trace by trace: look simultaneously at several traces



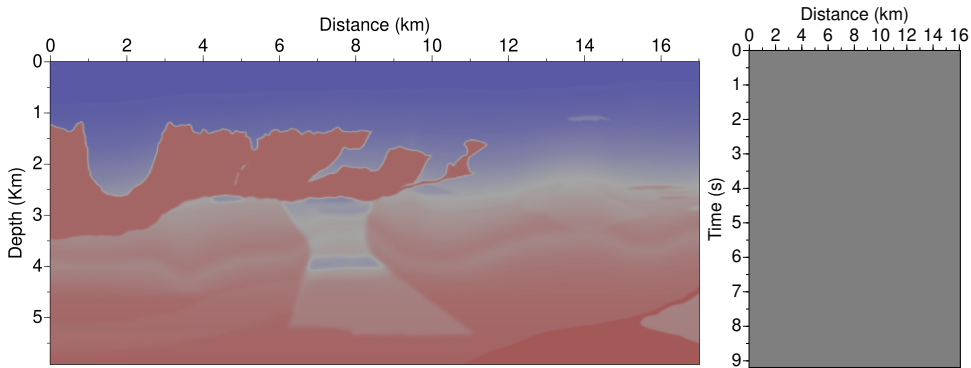
20 seismic traces $d_r(t)$ as a function of time, depending on the receiver/source distance, also referred to as *offset* in the following.

Large trace number: use a 2D density plot

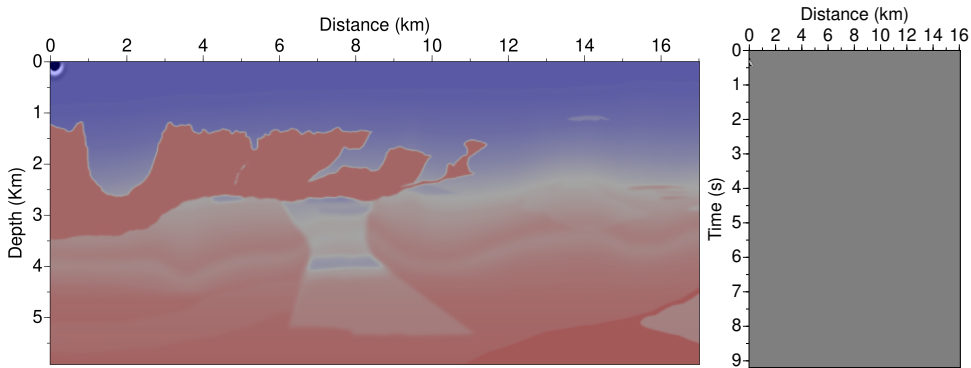


A typical seismogram in black and white representation. 161 traces spanning 16 km are used here. White correspond to negative values, black to positive values, while gray corresponds to 0.

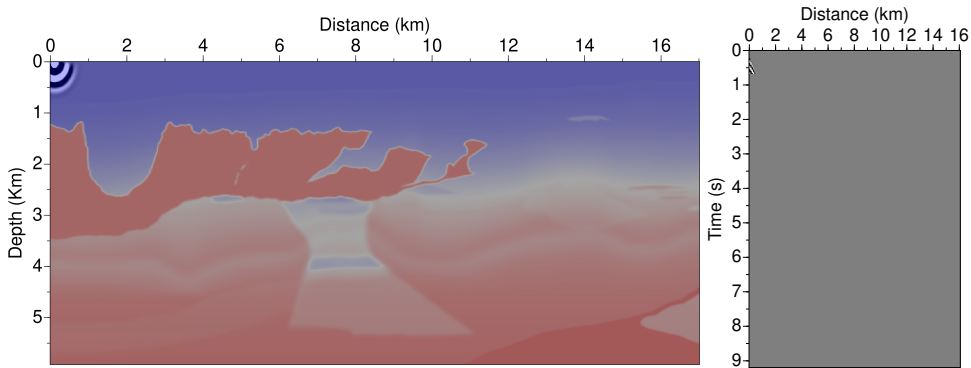
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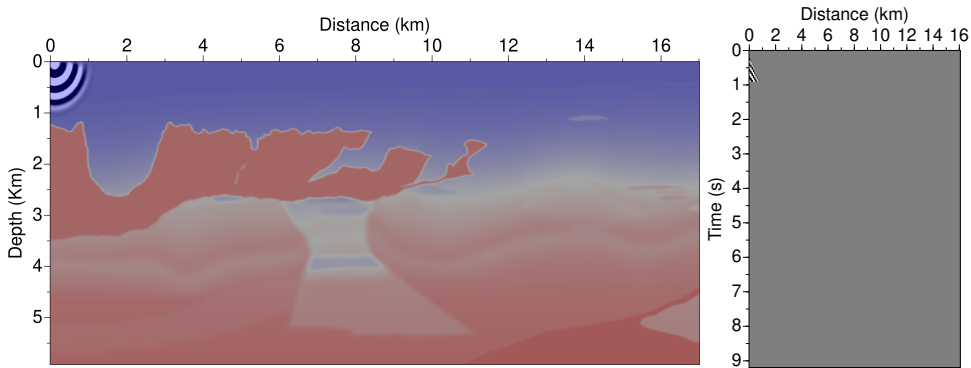
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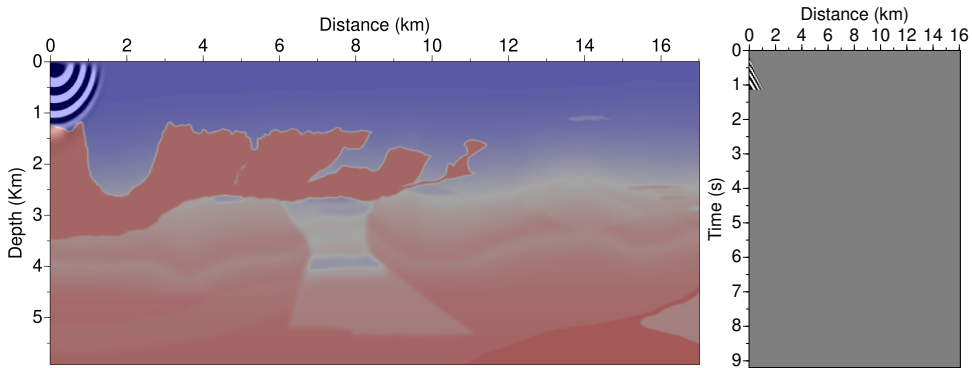
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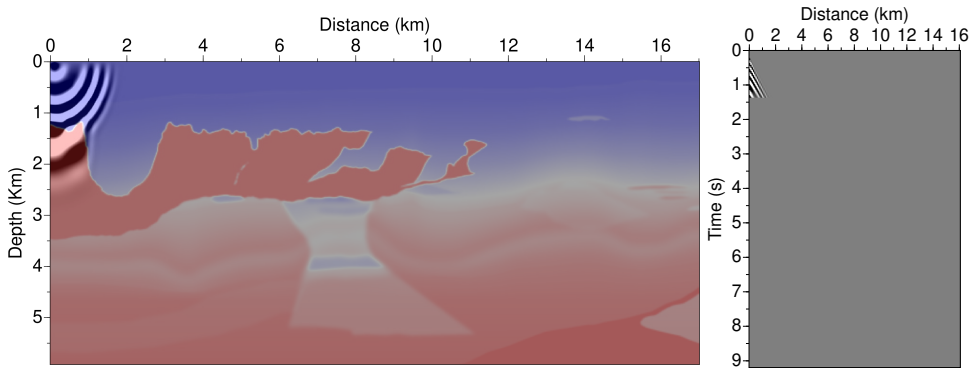
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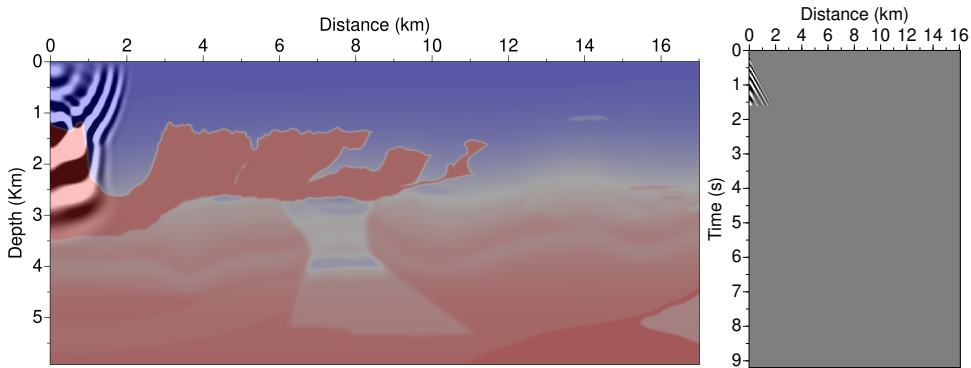
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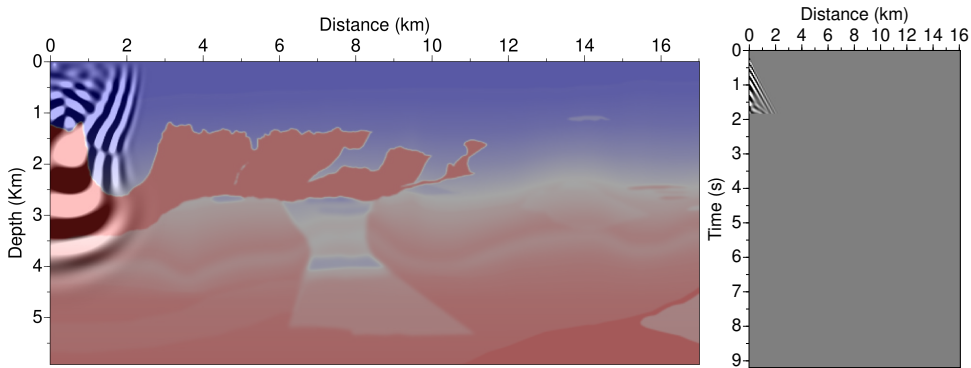


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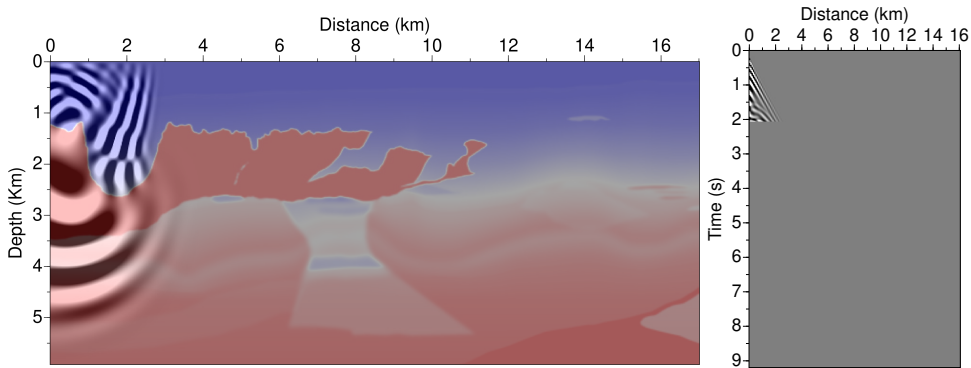


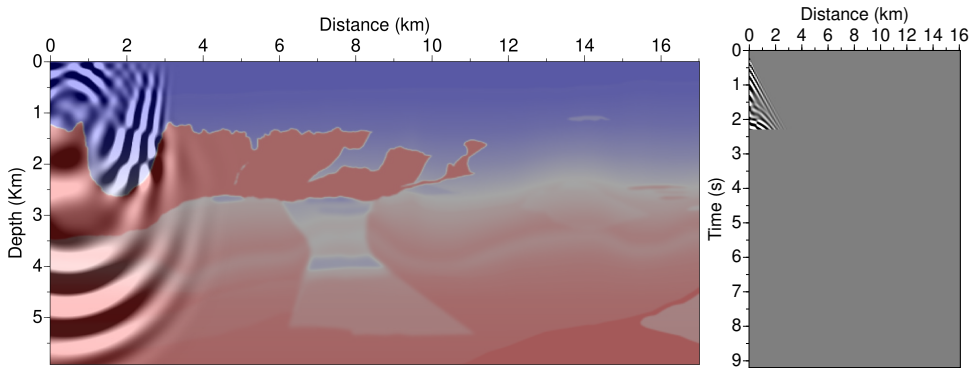
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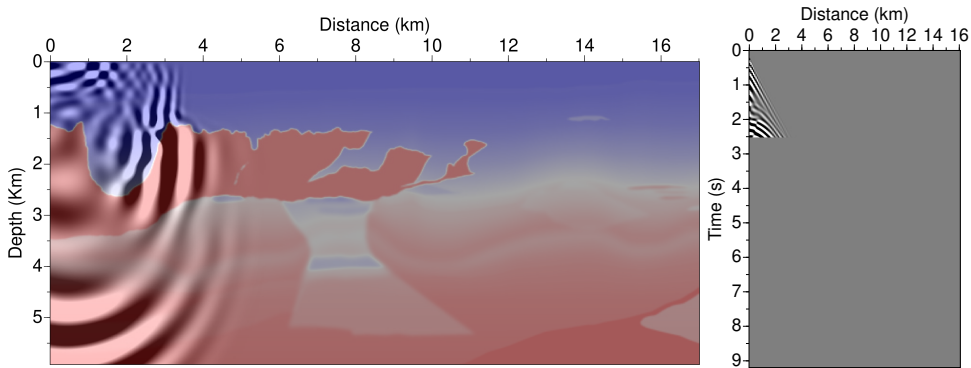


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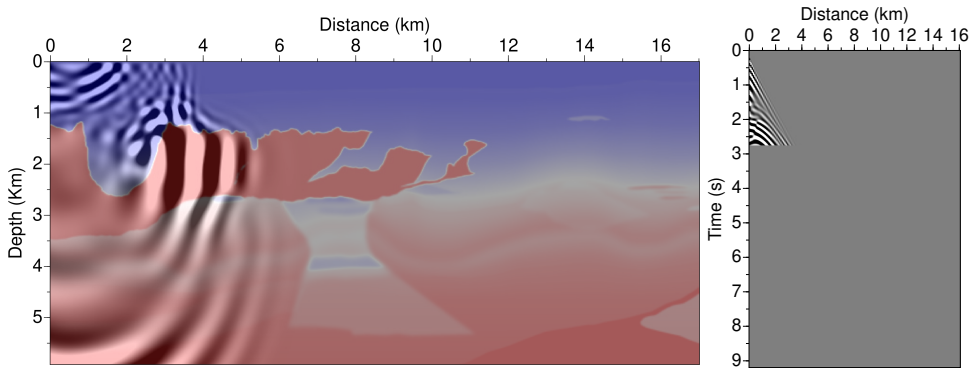




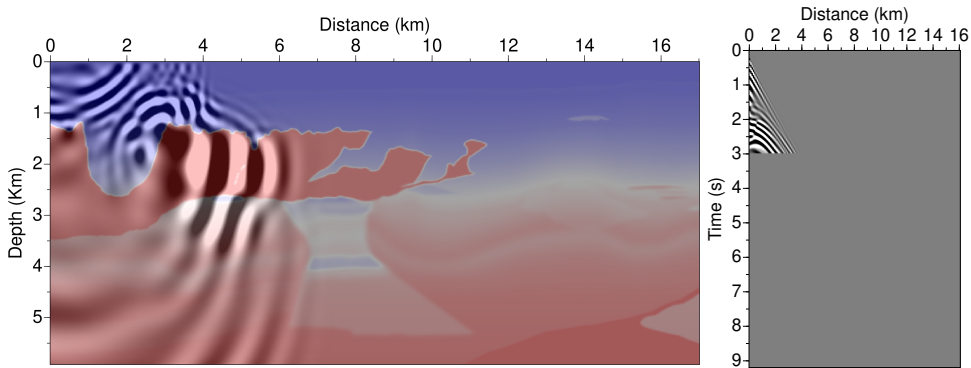
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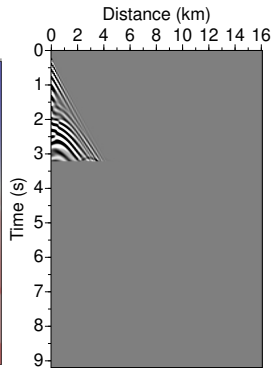
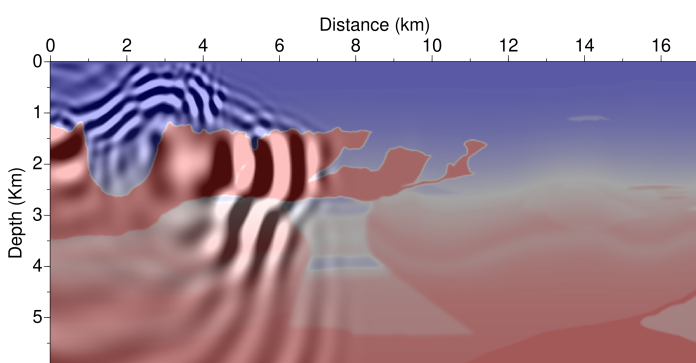
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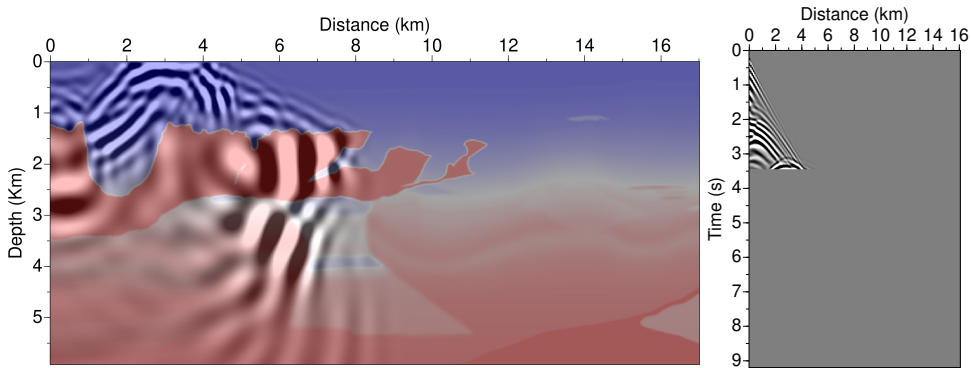
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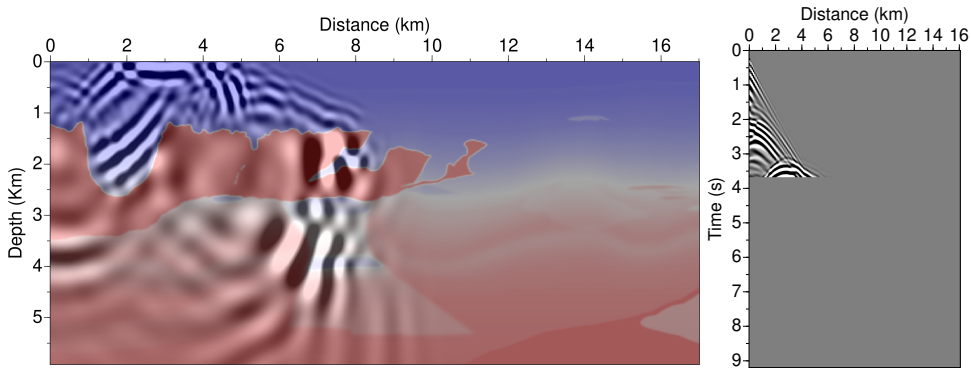


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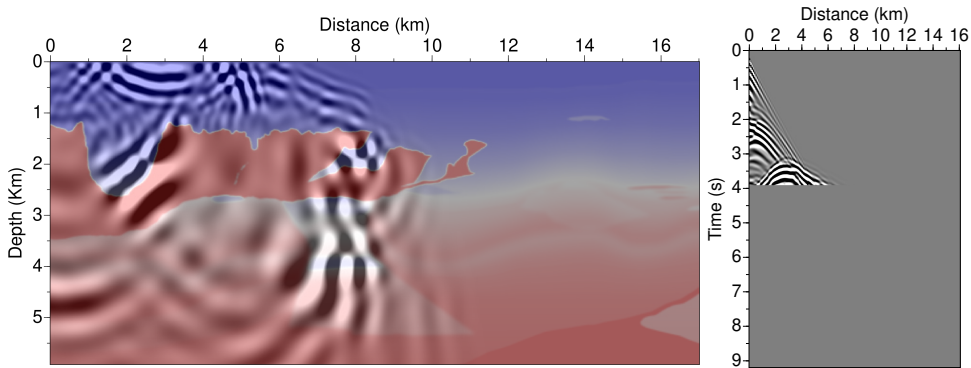


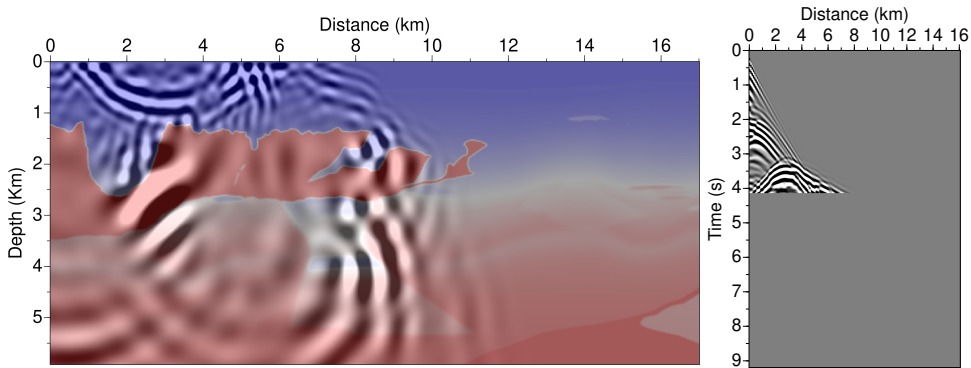
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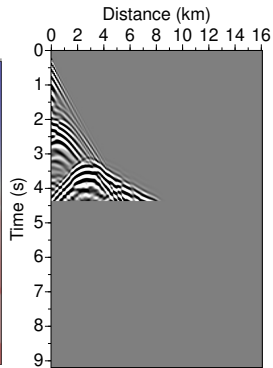
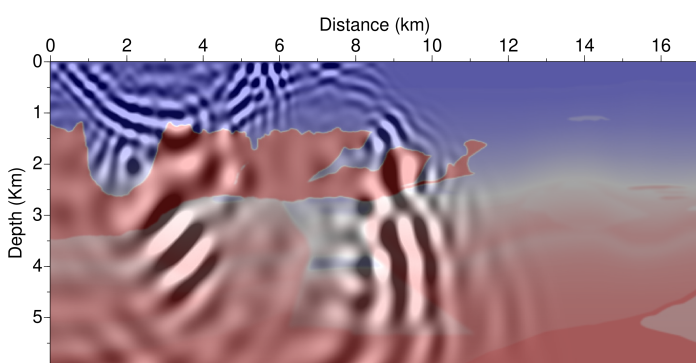


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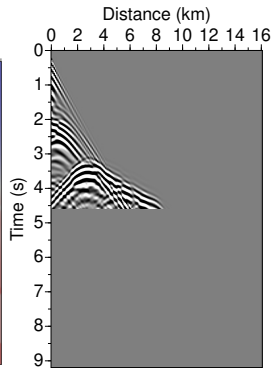
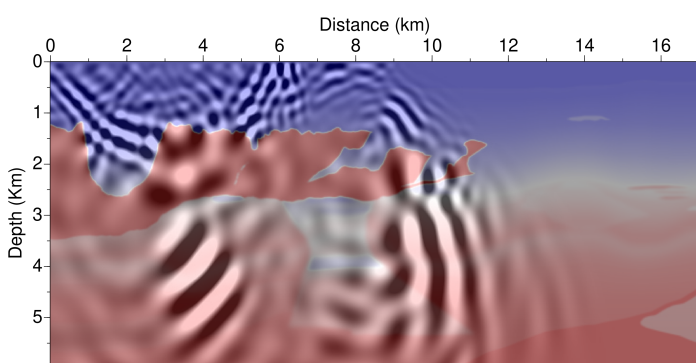




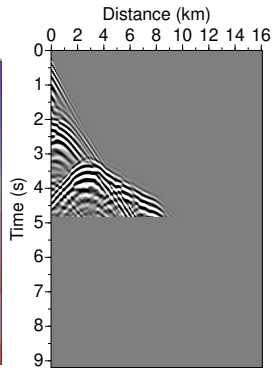
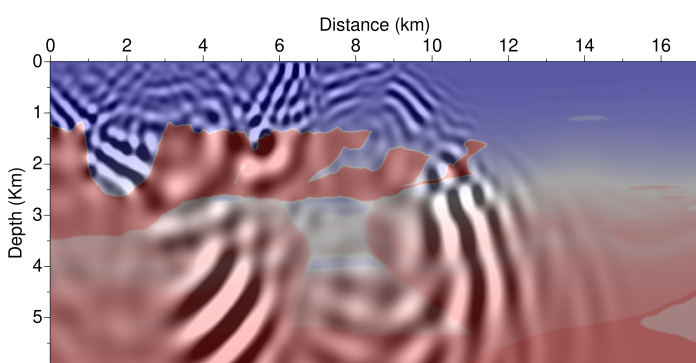
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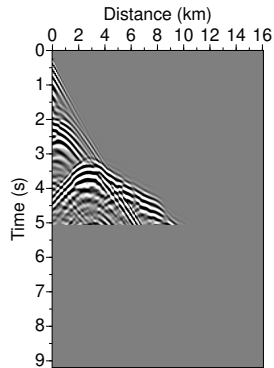
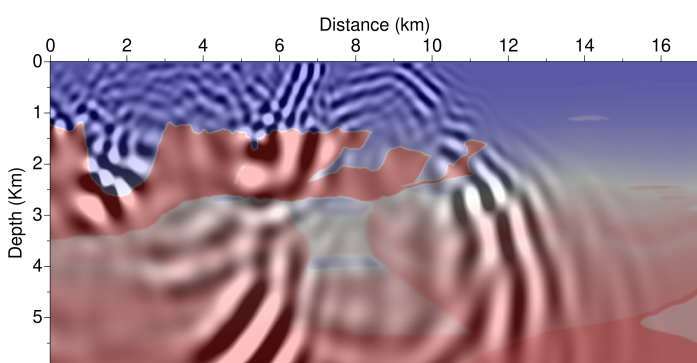


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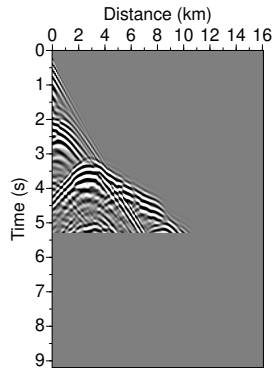
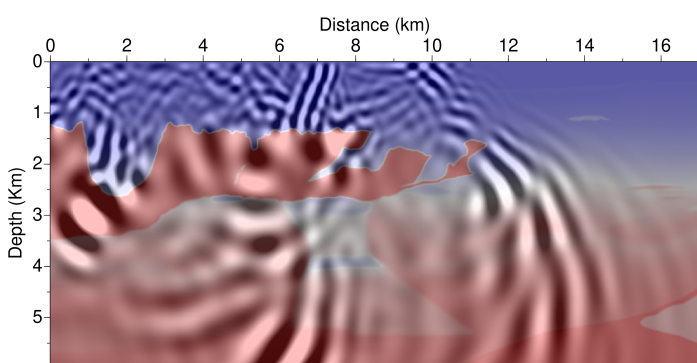


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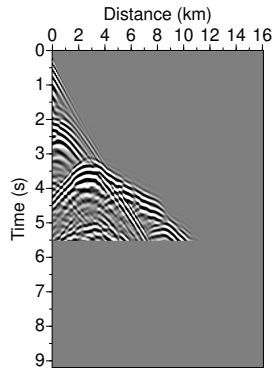
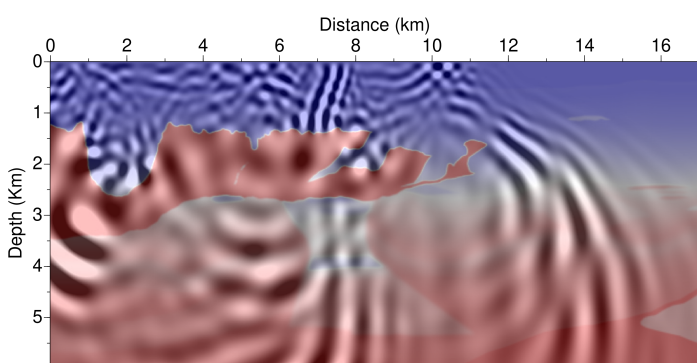


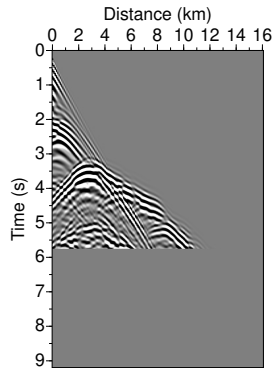
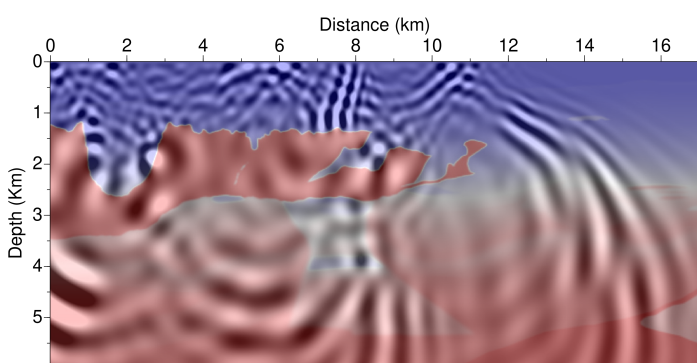


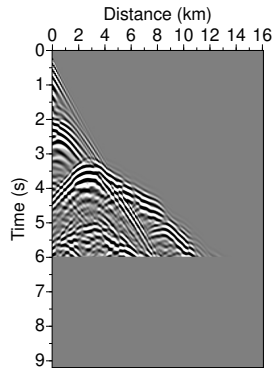
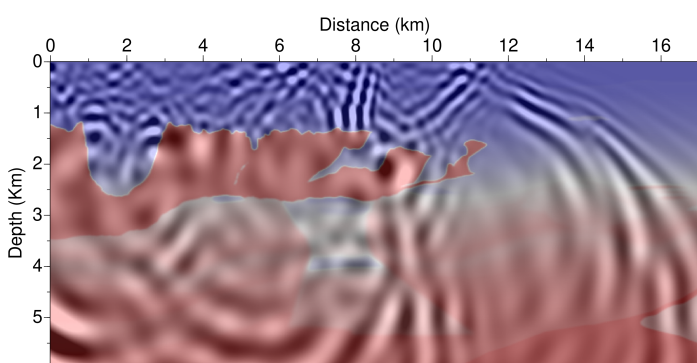
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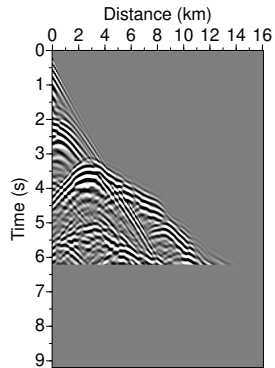
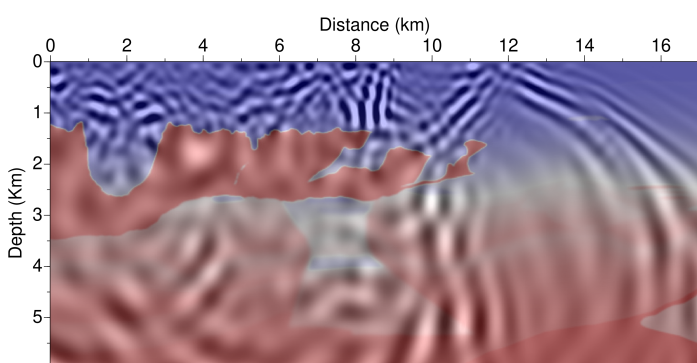


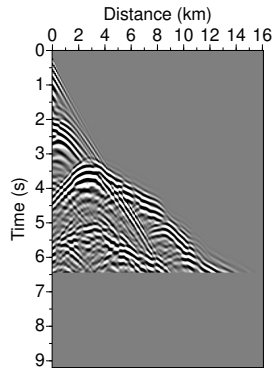
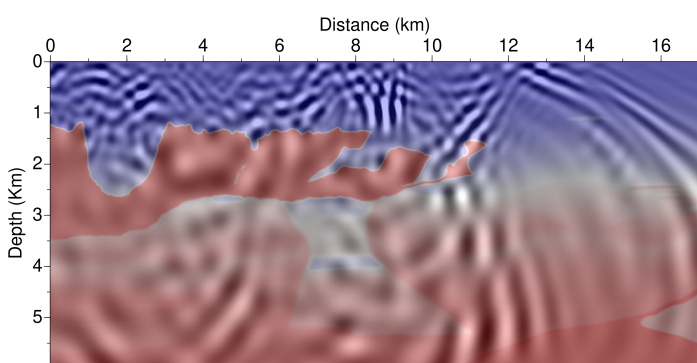
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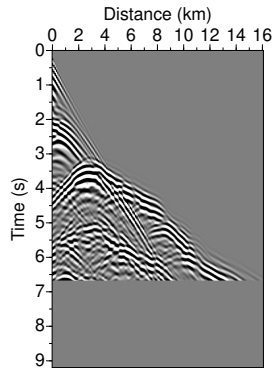
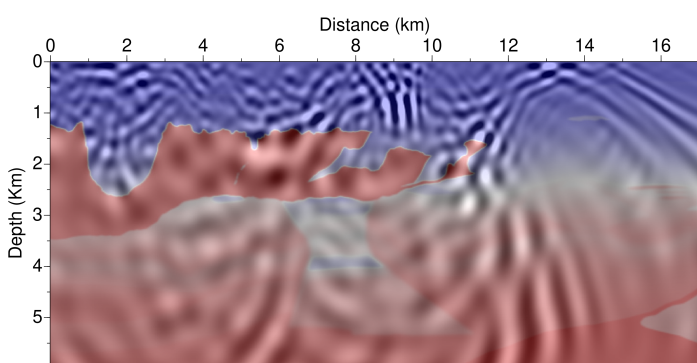


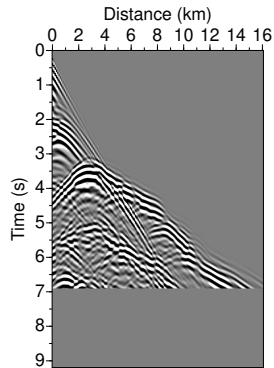
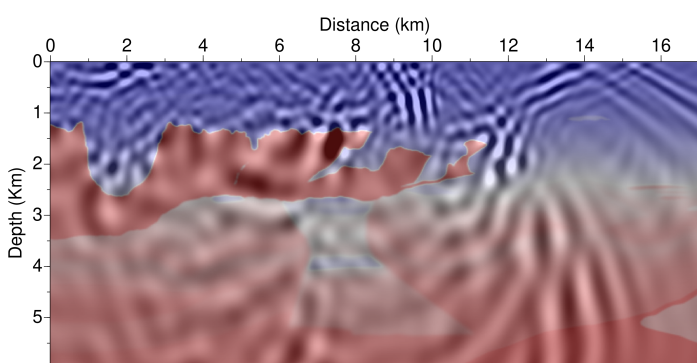


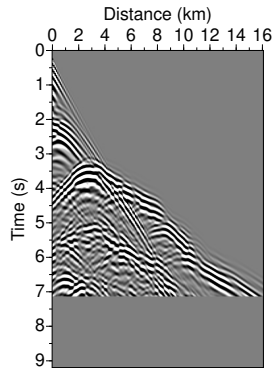
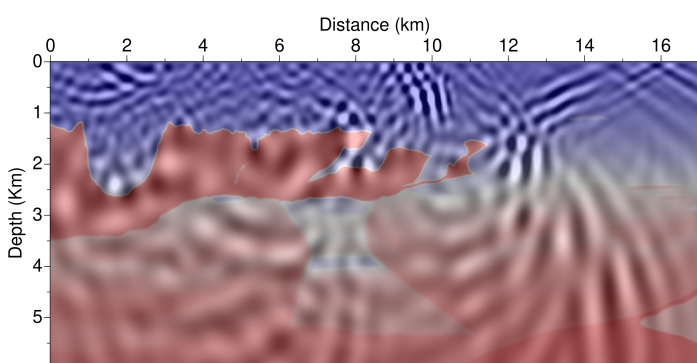


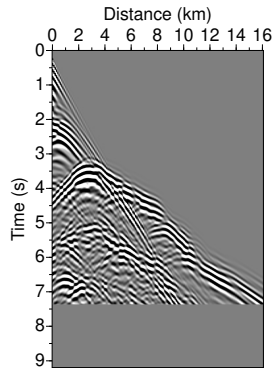
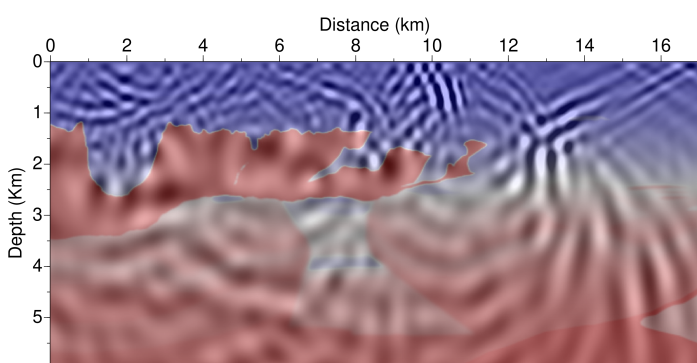




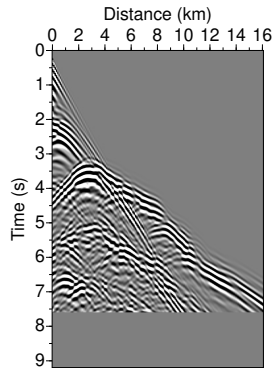
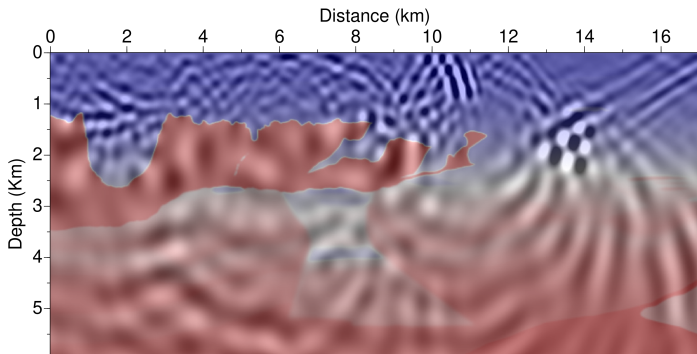


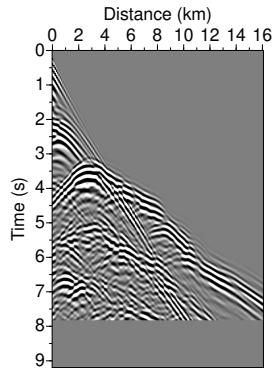
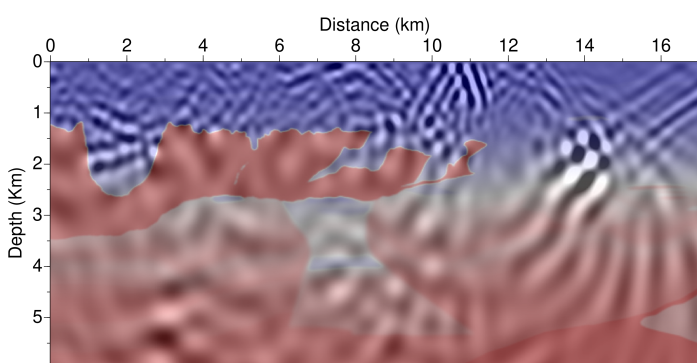


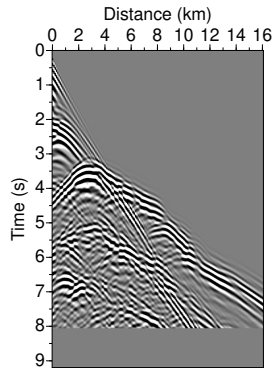
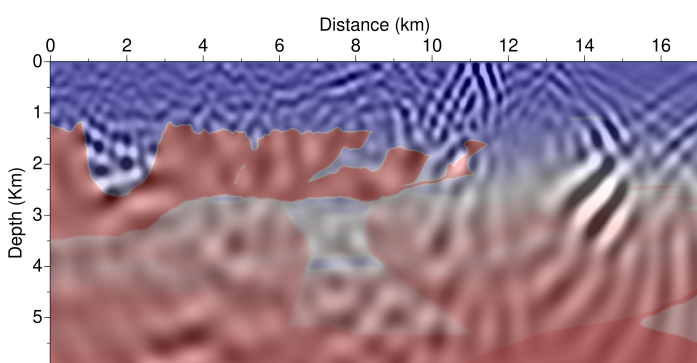


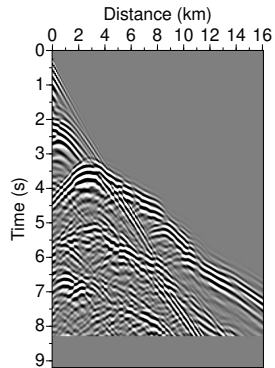
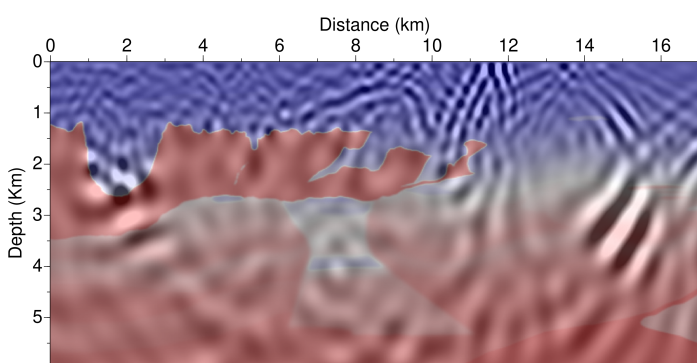


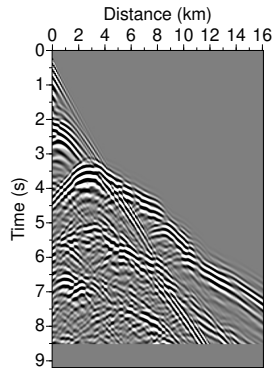
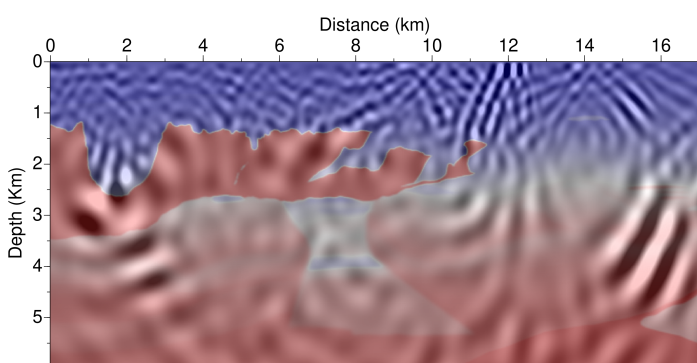
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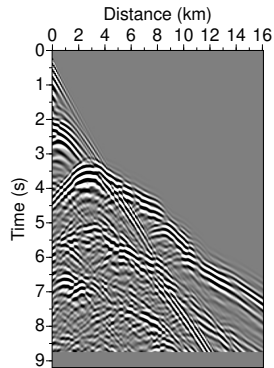
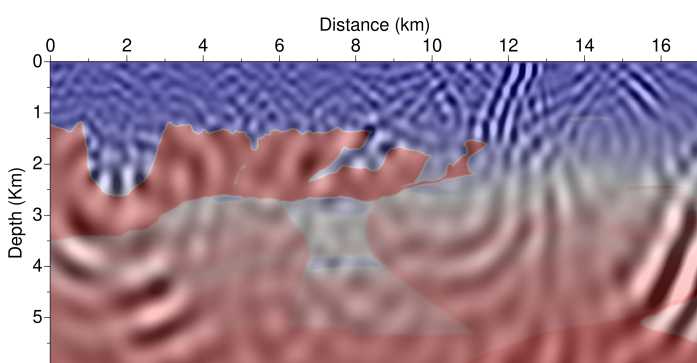


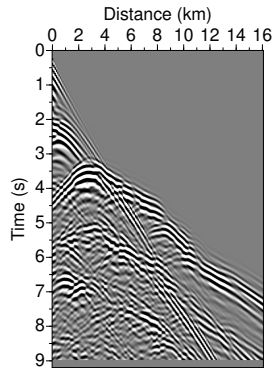
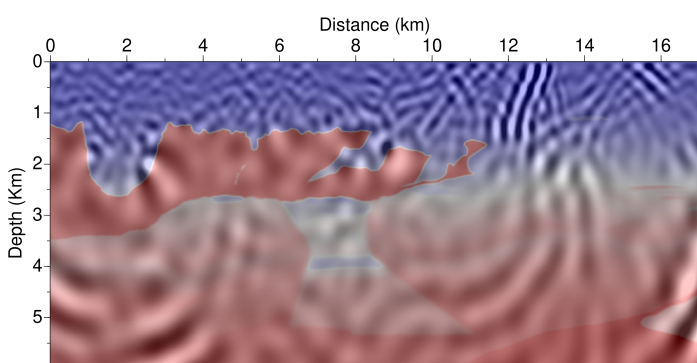


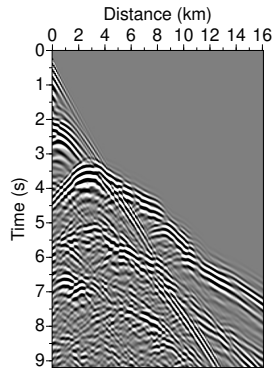
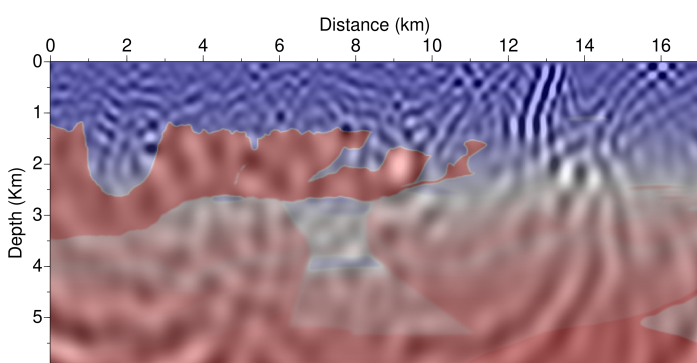




Where does this seismogram come from?



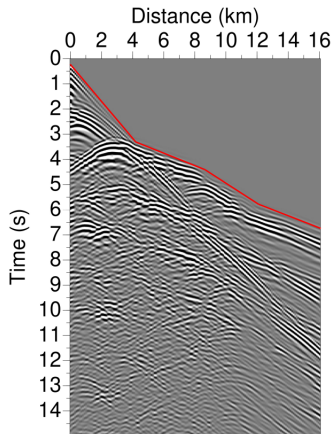




Geophysical imaging: to do what?

Seismic data

A first glance at seismic inversion methods



Same seismogram as in previous Figure with first-arrival travel time denoted by the red line.

Inverse tomography problem

$$d_{obs} = t_{obs}(x_s, x_r), \quad m = v_P \quad (4)$$

where $t_{obs}(x_s, x_r)$ denotes the picked travel times from source s to receiver r , and v_P is the pressure wave velocity.

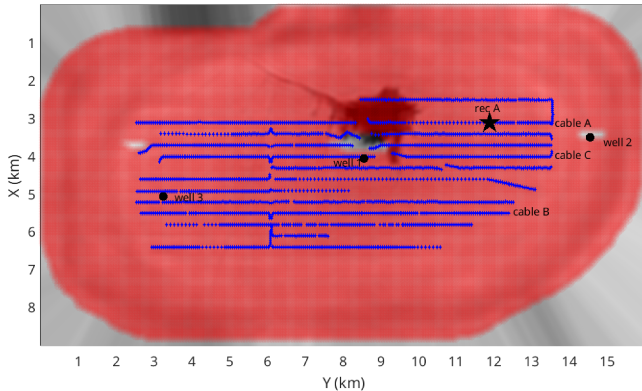
Least-squares first-arrival travel time tomography

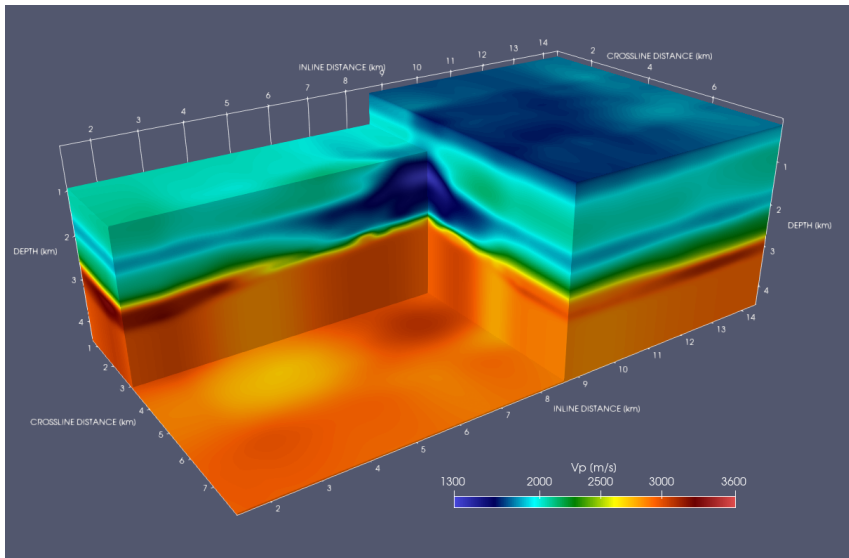
$$\min_{v_P} \frac{1}{2} \|t_{cal} - t_{obs}\|^2 + \eta R(v_P), \quad t_{cal} = g(v_P). \quad (5)$$

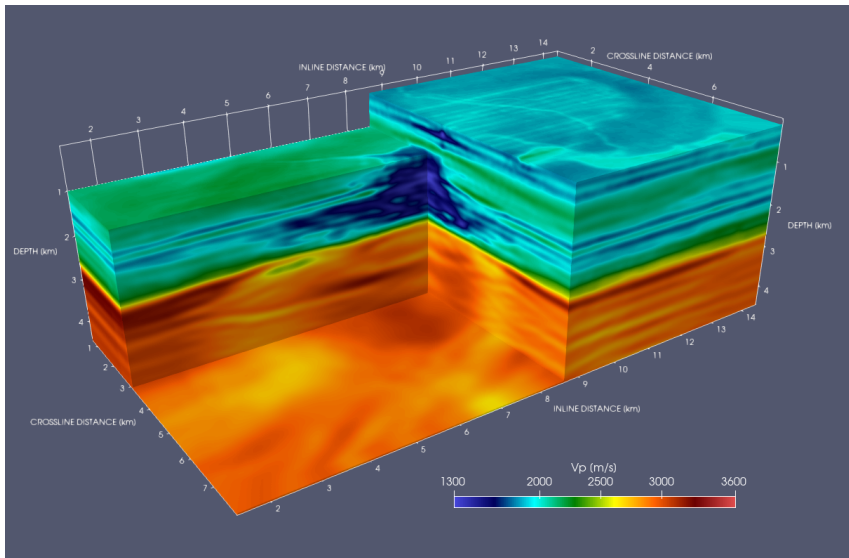
Idea: replace the forward modeling operator $g(m)$ by a full wave modeling solver, and to compare the resulting synthetic data to the full observed data $d_{obs}(x_s, x_r, t)$. The FWI problem is thus formulated as

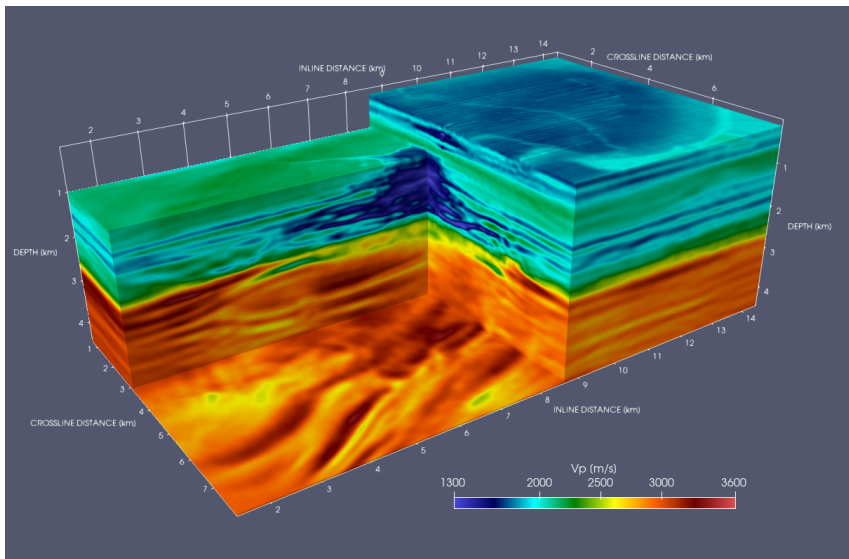
$$\min_m \frac{1}{2} \|d_{cal} - d_{obs}\|^2 + \eta R(m), \quad d_{cal} = g(m) \quad (6)$$

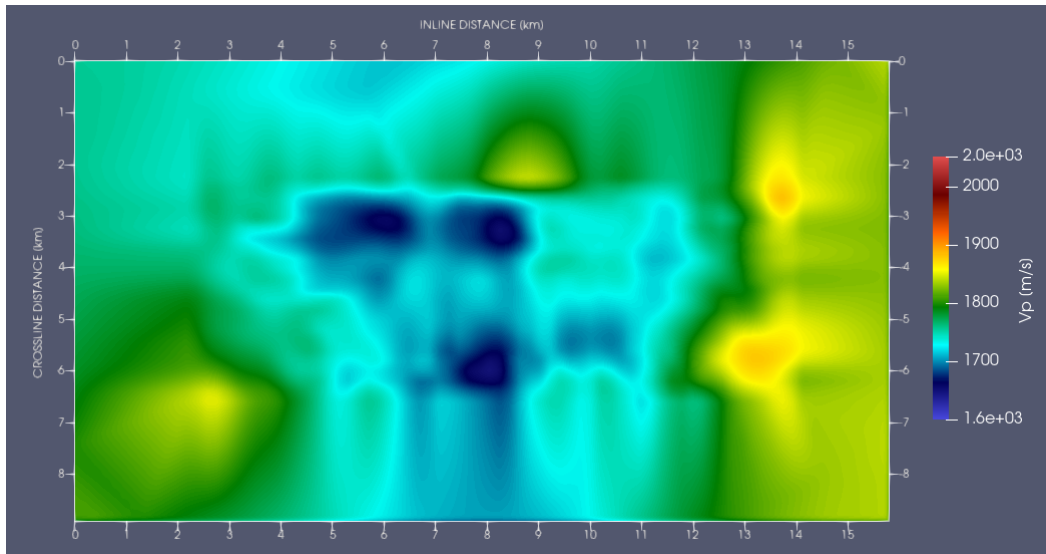
- Reciprocity: 50,000 explosive sources (airguns)+ 2048 hydrophones along the cables $\Rightarrow$ 2048 explosive source + 50,000 hydrophones

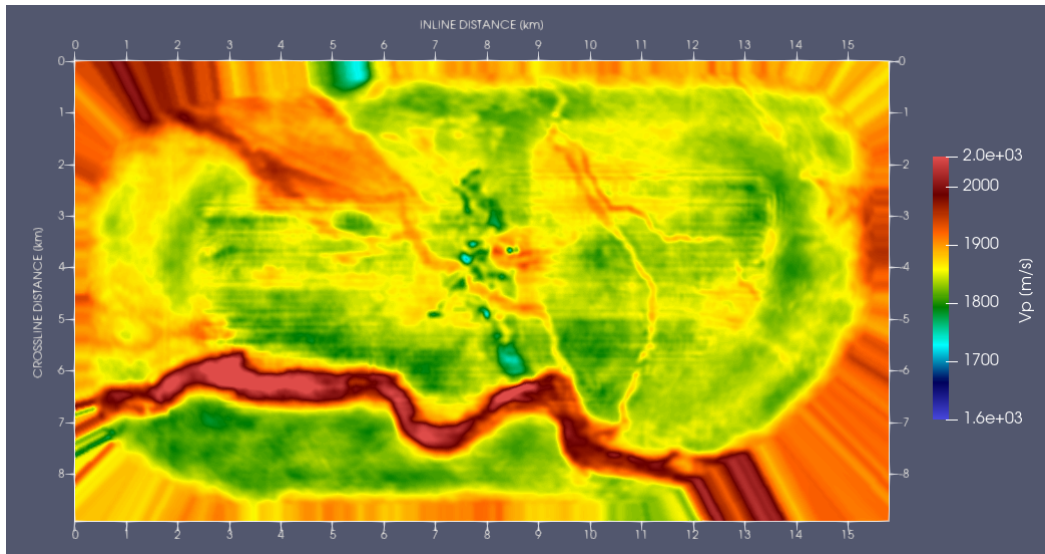


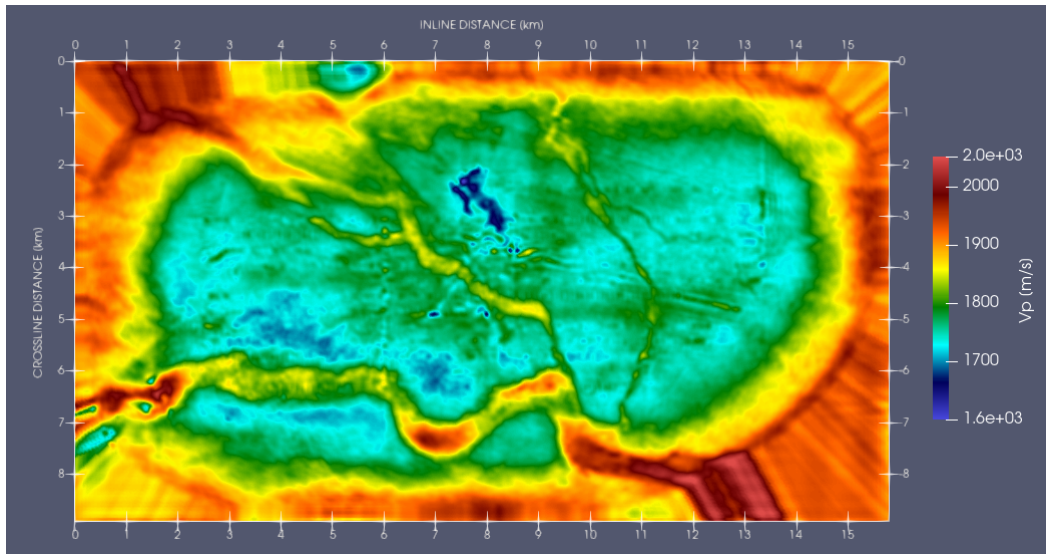


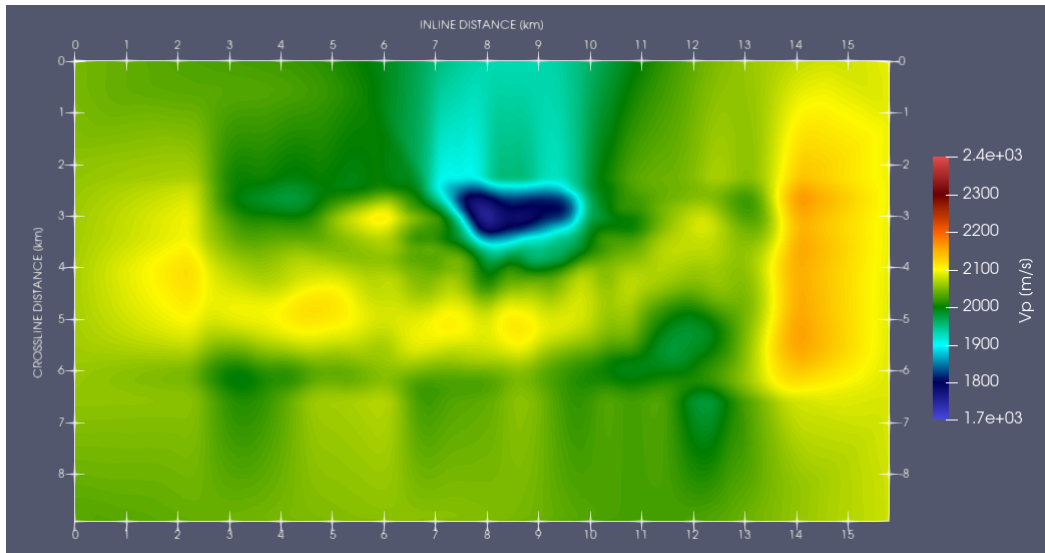




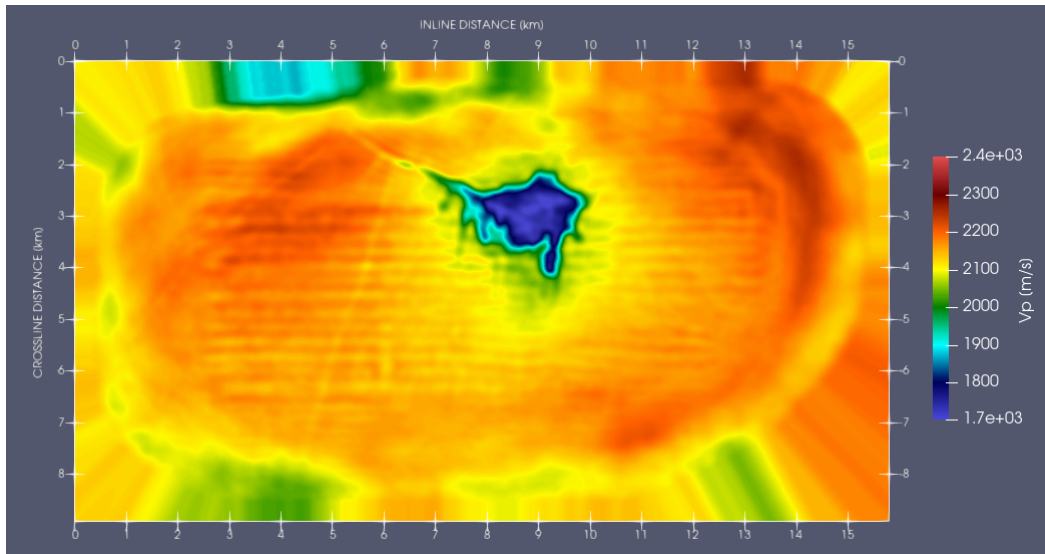
Tomo V_P , shallow depth slice

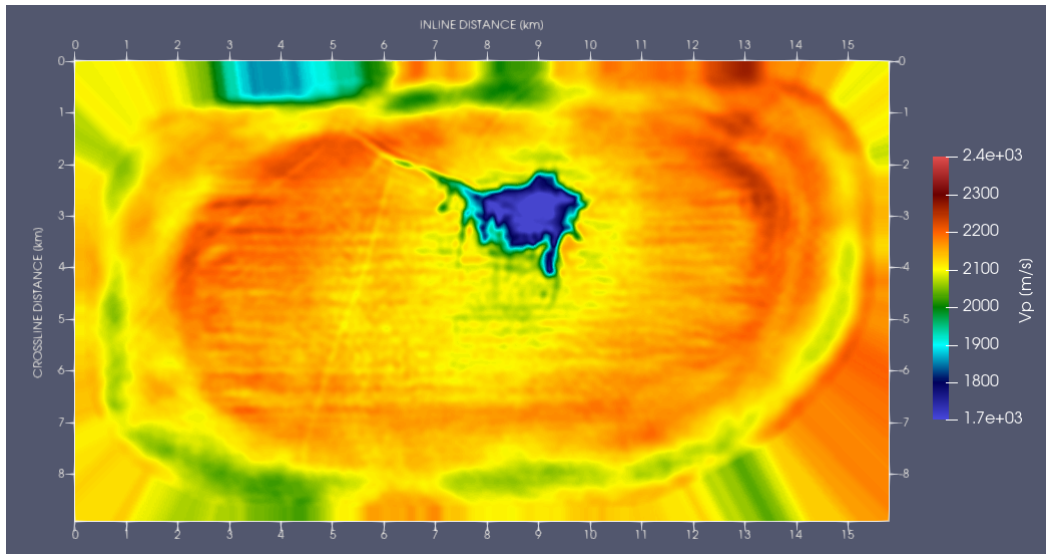
10 Hz V_P , shallow depth slice

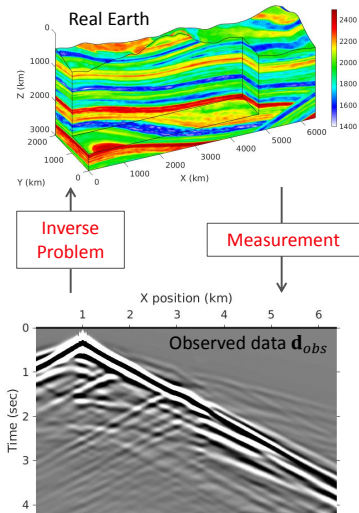


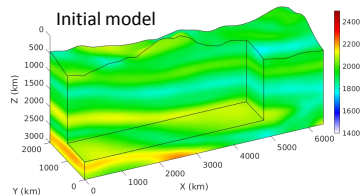
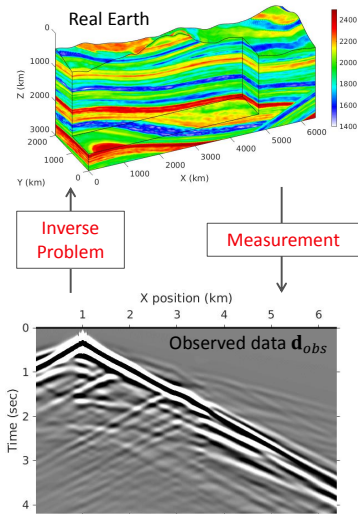
Tomo V_P , deeper depth slice

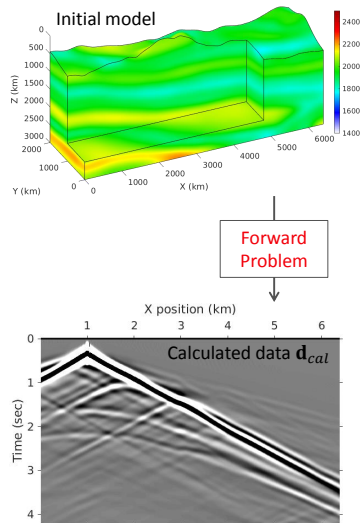
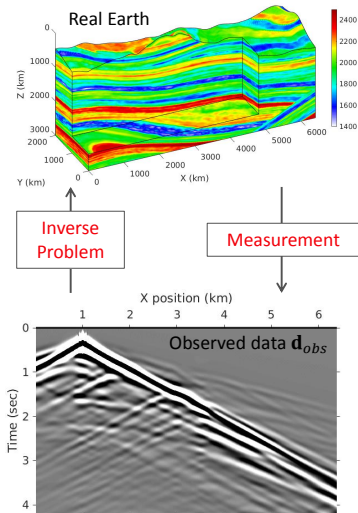
10 Hz V_P , deeper depth slice



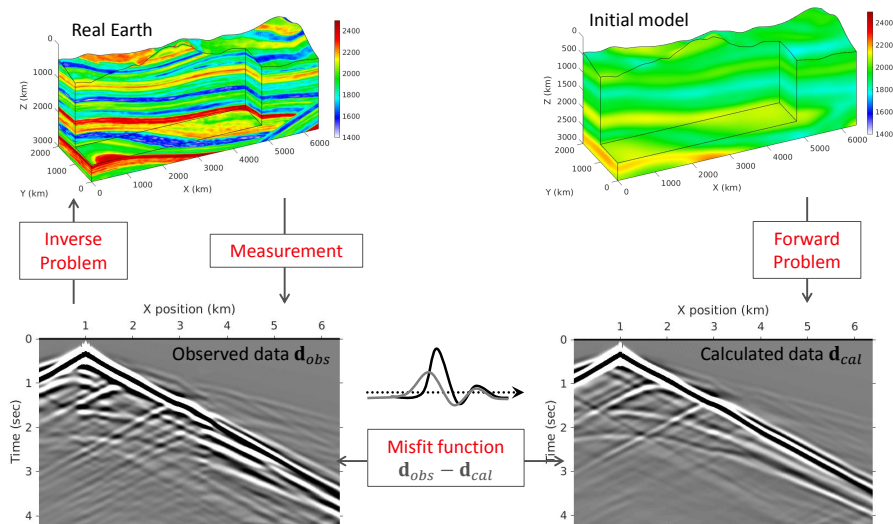


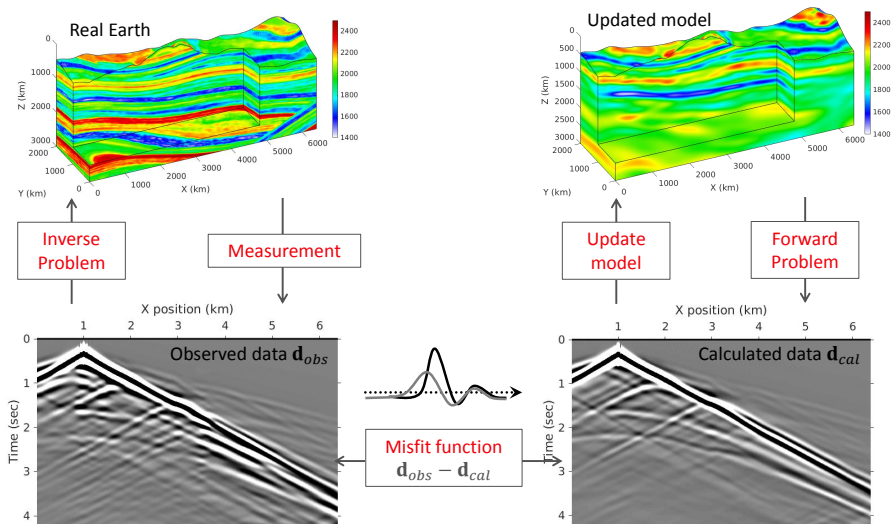


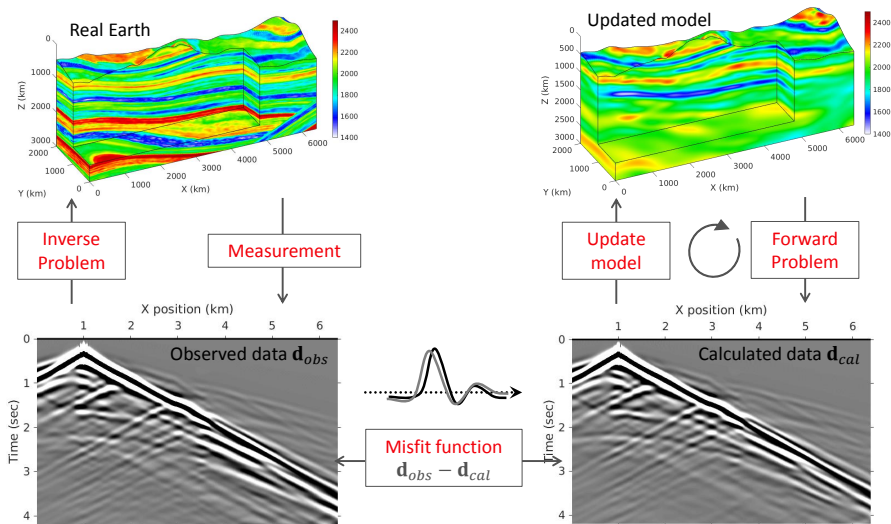


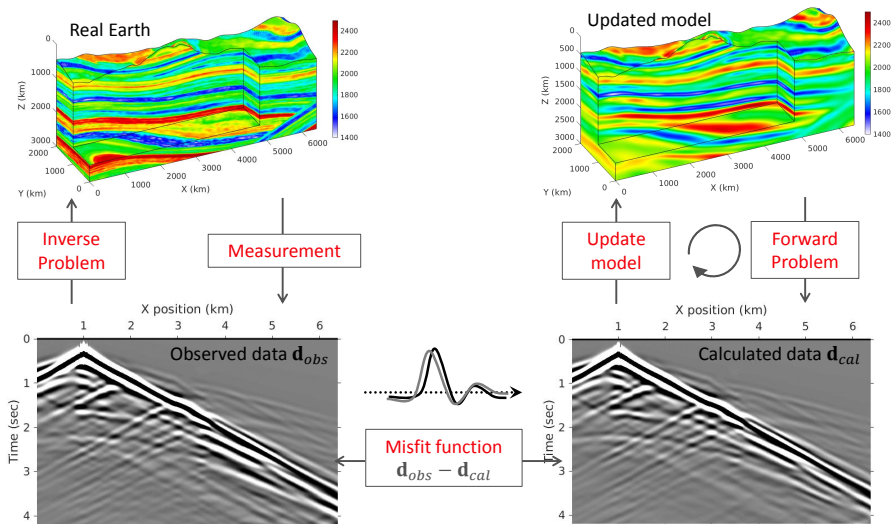


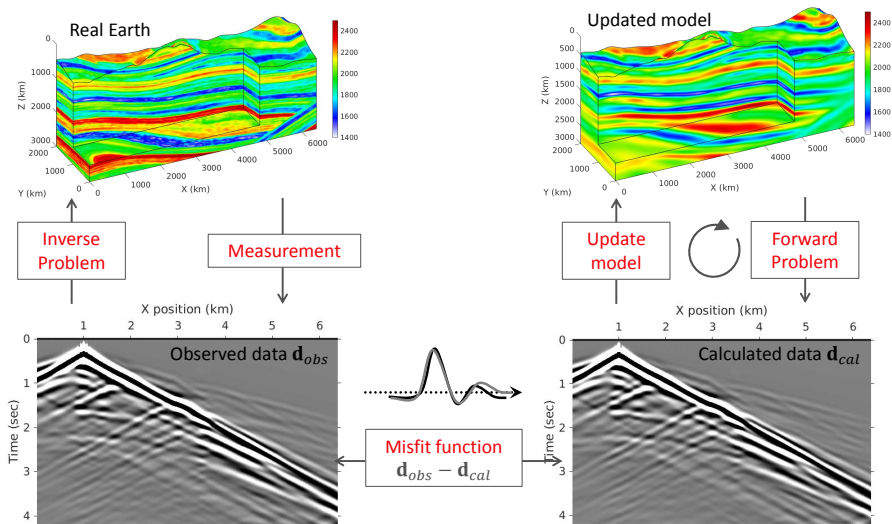
Full waveform inversion principle











$$\min_m J(m), \quad J(m) = \frac{1}{2} \|d_{cal}[m] - d_{obs}\|^2 = \frac{1}{2} \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} \int_0^T |d_{cal,s}[m](x_r, t) - d_{obs,s}(x_r, t)|^2 dt \quad (7)$$

$$\min_m J(m), \quad J(m) = \frac{1}{2} \|d_{cal}[m] - d_{obs}\|^2 = \frac{1}{2} \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} \int_0^T |d_{cal,s}[m](x_r, t) - d_{obs,s}(x_r, t)|^2 dt \quad (7)$$

where

$$d_{cal,s}[m](x_r, t) = Ru_s[m] = \int_{\Omega} \delta(x - x_r) u_s[m](x, t) dx, \quad (8)$$

$$\min_m J(m), \quad J(m) = \frac{1}{2} \|d_{cal}[m] - d_{obs}\|^2 = \frac{1}{2} \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} \int_0^T |d_{cal,s}[m](x_r, t) - d_{obs,s}(x_r, t)|^2 dt \quad (7)$$

where

$$d_{cal,s}[m](x_r, t) = Ru_s[m] = \int_{\Omega} \delta(x - x_r) u_s[m](x, t) dx, \quad (8)$$

and

$$A(m)u_s = b_s \quad (9)$$

with $A(m)$ a wave propagation operator, for instance acoustic approximation,

$$A(m)u = \partial_{tt}u - \rho c^2 \operatorname{div} \left(\frac{1}{\rho} \nabla u \right), \quad m = [\rho \ c] \quad (10)$$

or elastic approximation

$$A(m)u = \begin{cases} \rho \partial_{tt}u - \operatorname{div} \sigma, \\ \sigma = \frac{1}{2} C(\nabla u + \nabla u^T), \end{cases}, \quad m = [\rho \ C] \quad (11)$$

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