Geophysical imaging

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Inversion

Outline

Inversion

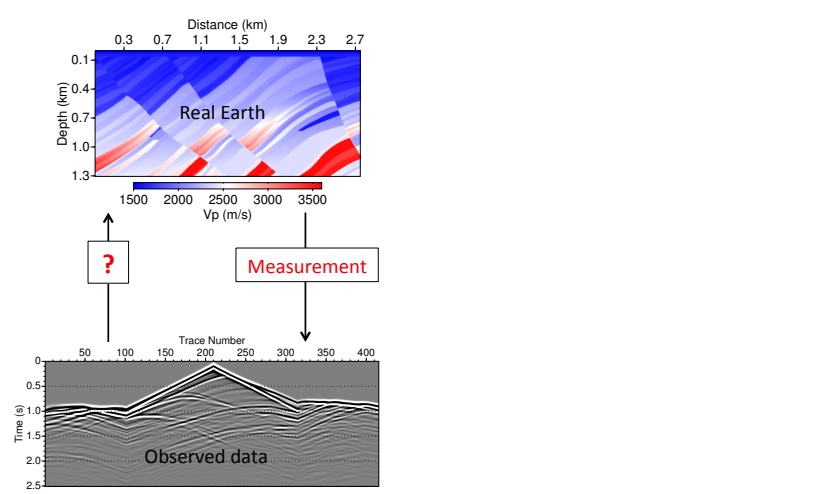
Full waveform inversion principle

Recap where we are

Illustration on a simple test: reconstructing a positive Gaussian velocity anomaly

Principle of full waveform inversion

FWI loop



Principle of full waveform inversion

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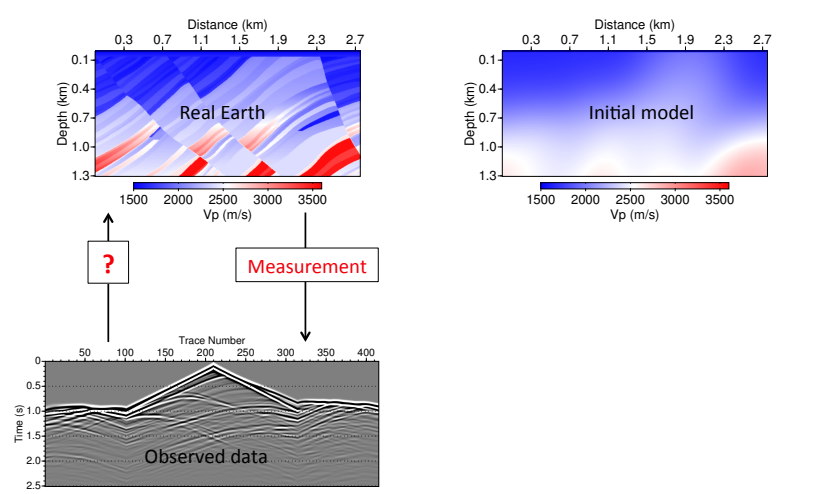

Principle of full waveform inversion

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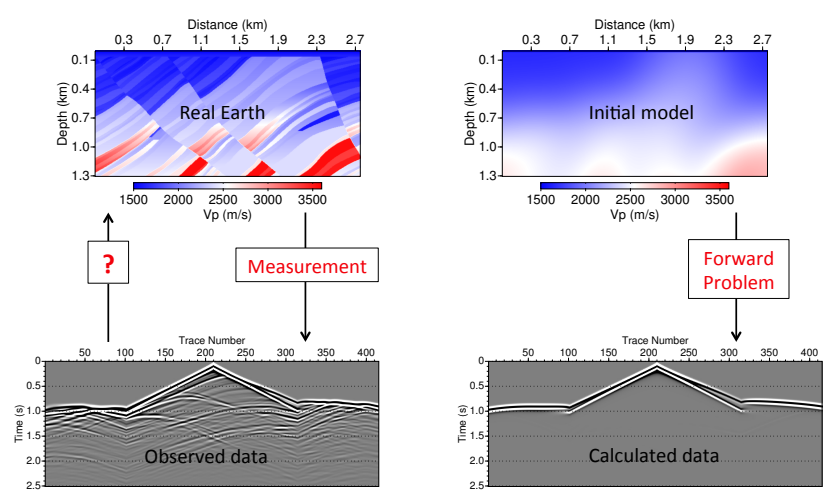

Principle of full waveform inversion

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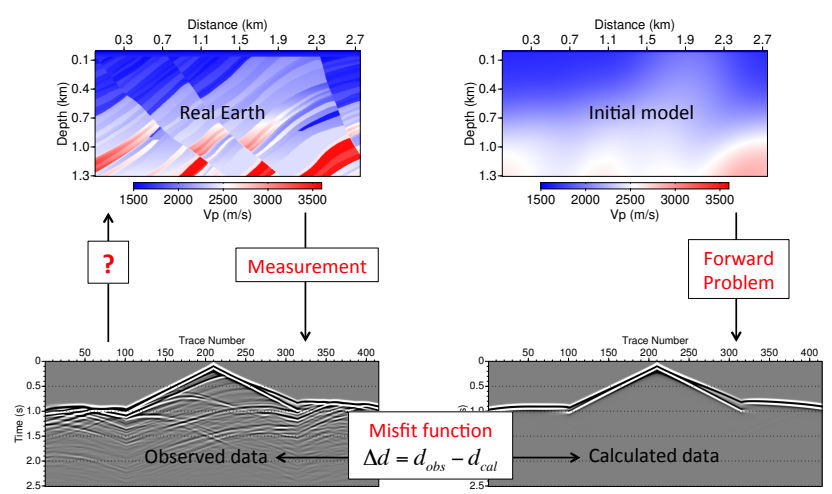
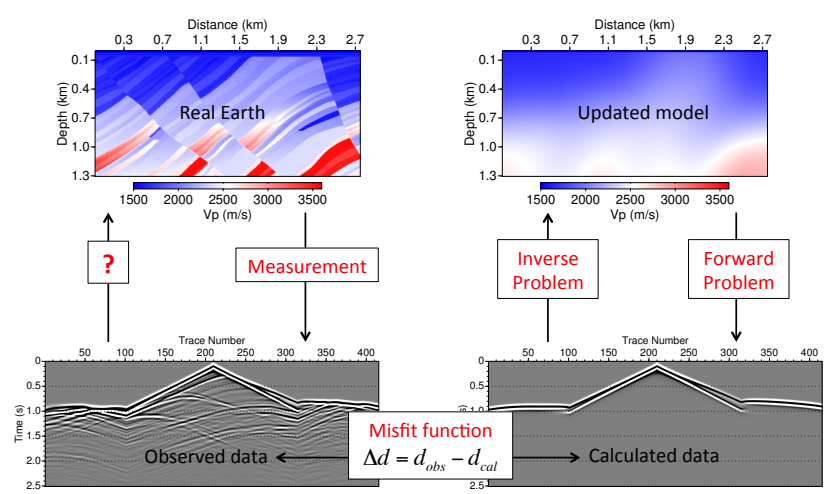


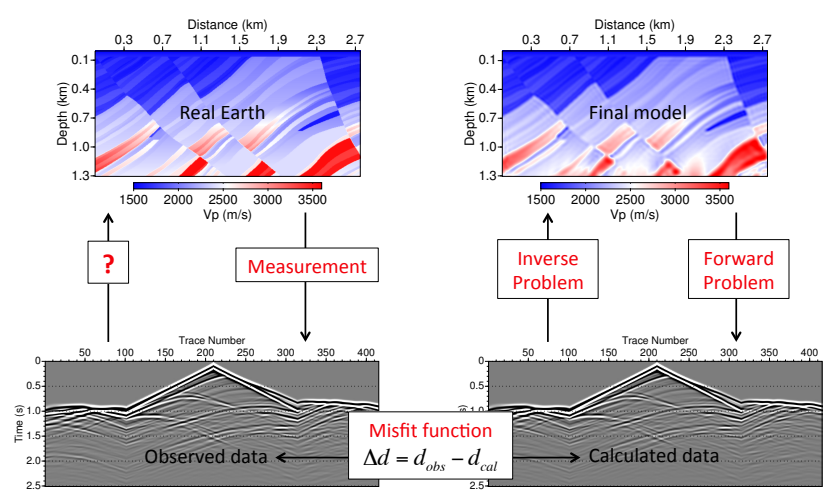
Principle of full waveform inversion

FWI loop



Principle of full waveform inversion

FWI loop



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Mathematical formulation of the FWI problem

- PDE constrained optimization problem

$$\begin{aligned} &\min_m \frac{1}{2}\sum_{s=1}^{N_s} \|d_{cal,s} - d_{obs,s}\|^2, \\ \text{subject to} \quad &\forall s = 1,\dots,N_s, \quad d_{cal,s} = Ru_s, \\ &\forall s = 1,\dots,N_s, \quad A(m)u_s = \varphi_s, \end{aligned} \tag{1}$$

- Reduced space approach

$$\min_m \frac{1}{2}\|RA(m)^{-1}\varphi - d_{obs}\|^2. \tag{2}$$

Solution through local optimization

- We use local optimization methods, designed to solve general unconstrained minimization problems

$$\min_m f(m), \tag{3}$$

- These methods consist in iterating from an initial model m_0 such that

$$m_{k+1} = m_k + \alpha_k \Delta m_k, \tag{4}$$

- In (4), α_k is a linesearch parameter (positive scalar) determined so as to satisfy the Wolfe conditions to guarantee the convergence towards the closest local minimum of $f(m)$ from m_0.
- In (4), Δm_k is a descent direction, computed, under quasi-Newton schemes such as l-BFGS (**?**) as

$$\Delta m_k = -Q_k \nabla f(m_k) \tag{5}$$

where Q_k is an approximation of the inverse Hessian operator $H(m_k)^{-1}$. Here $\nabla f(m_k)$ is the gradient of the misfit function $f(m)$ and $H(m_k)$ is the Hessian operator associated to $f(m_k)$ (matrix of second-order derivatives of $f(m_k)$).

Gradient computation

- The key quantity to compute is the gradient $\nabla f(m_k)$.
- The l-BFGS approximation Q_k of $H(m_k)^{-1}$ can be deduced from l stored values of $\nabla f(m) : \nabla f(m_k), \nabla f(m_{k-1}, \ldots, \nabla f(m_{k-l+1})$.
- The direct formula for $\nabla f(m_k)$ is

$$\nabla f(m)_i = \sum_{j=1}^{N} J_{ij}^T \Delta d_j = \sum_{j=1}^{N} J_{ji} \Delta d_j = \sum_{r=1}^{N_r} \int_0^T \frac{\partial d_{cal}}{\partial m_i}(x_r, t) \Delta d(x_r, t) \tag{6}$$

which provides a nice interpretation of what is the gradient but is not practicable for applications: gradient built as the correlation between primary scattered field and residuals, for each discrete point of the medium

Gradient computation

- In practice the gradient is computed through the adjoint state method
- Following this strategy, the gradient is obtained as

$$\nabla f(m) = \left(\frac{\partial A}{\partial m} u, \lambda \right)_T \tag{7}$$

where u is the incident field solution of the wave equation

$$A(m)u = \varphi \tag{8}$$

and λ is the adjoint field, solution of the adjoint wave equation

$$A(m)^T \lambda = -R^T \Delta d. \tag{9}$$

Gradient computation

- In the acoustic approximation, one obtains

$$-\frac{2}{V_P^3}\int_0^T \frac{\partial^2 u(x,t)}{\partial t^2}\lambda(x,t)dt \tag{10}$$

where $u(x,t)$ is the incident wavefield satisfying

$$\frac{1}{V_P^2}\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial^2 u(x,t)}{\partial x^2} = \varphi(x,t) \tag{11}$$

with homogeneous initial conditions and $\lambda(x,t)$ is the adjoint wavefield satisfying

$$\frac{1}{V_P^2}\frac{\partial^2 \lambda(x,t)}{\partial t^2} - \frac{\partial^2 \lambda(x,t)}{\partial x^2} = -\sum_{r=1}^{N_r}\left(d_{cal}(x_r,t) - d_{obs}(x_r,t)\right)\delta(x-x_r) \tag{12}$$

with homogeneous final conditions

Gradient computation

- The adjoint method requires to solve only two wave propagation problems for one gradient: 1 incident field and 1 adjoint field have to be computed only
- The method can be seen as a more clever way to assemble the gradient: instead of scanning all the points of the medium, the residuals are backpropagated to the location of the diffracting point, and
- The construction pattern is
  - from the source to the diffracting point
  - from the receivers to the diffracting point

  The gradient is thus assembled globally for each discrete point, instead of being built for each discrete point independently.

Resolution analysis

- In the frame of plane wave analysis, with incident and adjoint fields considered as monochromatic fields

$$u(\mathbf{x}, \omega) = e^{ik_0 \mathbf{p_s} \cdot \mathbf{x}}, \quad \lambda(\mathbf{x}, \omega) = e^{ik_0 \mathbf{p_r} \cdot \mathbf{x}}, \tag{13}$$

where $\mathbf{x} = (x, z) \in \mathbb{R}^2$ and k_0 is given by

$$k_0 = \frac{2\pi\omega}{c_0}. \tag{14}$$

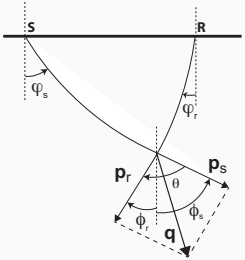
the gradient is given by

$$\nabla f \simeq e^{ik_0 \mathbf{p_s} \cdot \mathbf{x}} e^{ik_0 \mathbf{p_r} \cdot \mathbf{x}} = e^{ik0(\mathbf{p_s} + \mathbf{p_r}) \cdot \mathbf{x}} \tag{15}$$

Resolution analysis

- The wavenumber content of the gradient is thus $\mathbf{k} = k0\,(\mathbf{p_s} + \mathbf{p_r})$, such that

$$\mathbf{k} = \frac{4\pi\omega}{c_0}\cos\left(\frac{\theta}{2}\right)(\cos\phi, \sin\phi) \tag{16}$$



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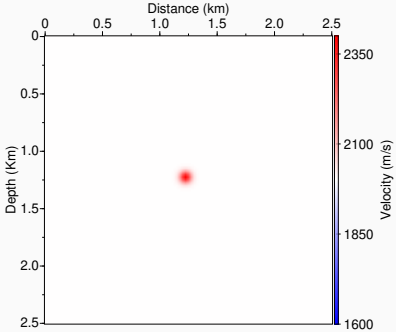
Inversion

- Full waveform inversion principle
- Recap where we are
- Illustration on a simple test: reconstructing a positive Gaussian velocity anomaly

Case study

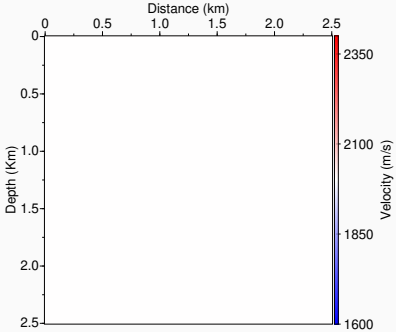
- We consider a homogeneous medium with a positive Gaussian velocity anomaly in the center
- We assume we know the background and we want to recover the Gaussian anomaly
  Exact velocity model to recover



Case study

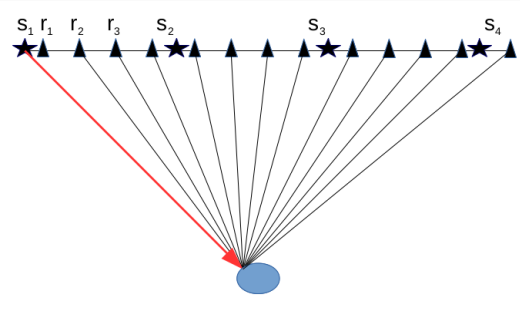
- We consider a homogeneous medium with a positive Gaussian velocity anomaly in the center
- We assume we know the background and we want to recover the Gaussian anomaly

  Initial velocity model



First configuration: surface acquisition, 5 Hz Ricker source

- We first consider a surface acquisition



- The source signal is a Ricker wavelet centered on 5 Hz
- We perform 10 l-BFGS iterations

First configuration: surface acquisition, 5 Hz Ricker source

- Optimization toolbox output

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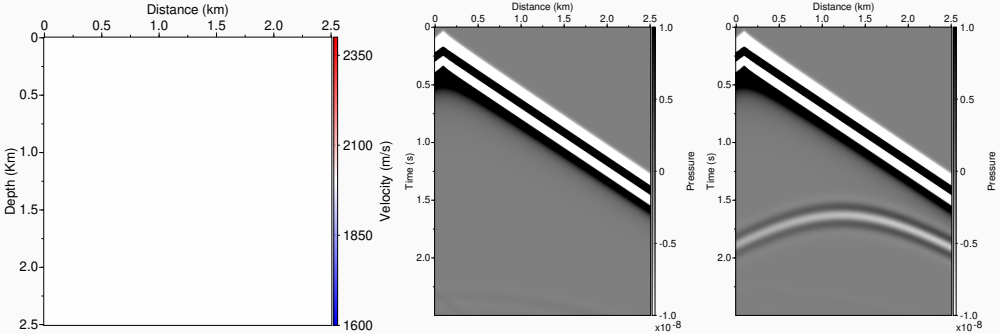
```
************************************************************************
                    l-BFGS ALGORITHM
************************************************************************
     Convergence criterion  :    1.00E-08
     Niter_max              :       10
     Initial cost is        :    1.04E+05
     Initial norm_grad is   :    5.61E+02
************************************************************************
   Niter      fk         ||gk||       fk/f0       alpha       nls     ngrad
     0    1.04E+05    5.61E+02    1.00E+00    1.00E+00       0       0
     1    1.04E+05    7.93E+02    9.97E-01    1.00E+00       0       1
     2    8.54E+04    1.93E+03    8.22E-01    2.50E-01       2       4
     3    5.58E+04    1.48E+03    5.37E-01    2.50E-01       0       5
     4    3.95E+04    1.16E+03    3.80E-01    2.50E-01       0       6
     5    2.93E+04    8.91E+02    2.81E-01    2.50E-01       0       7
     6    2.29E+04    6.91E+02    2.20E-01    2.50E-01       0       8
     7    1.87E+04    5.21E+02    1.80E-01    2.50E-01       0       9
     8    1.59E+04    3.92E+02    1.53E-01    2.50E-01       0      10
     9    1.38E+04    2.81E+02    1.33E-01    2.50E-01       0      11
    10    1.23E+04    1.98E+02    1.18E-01    2.50E-01       0      12
************************************************************************
     STOP: MAXIMUM NUMBER OF ITERATION REACHED
************************************************************************
```

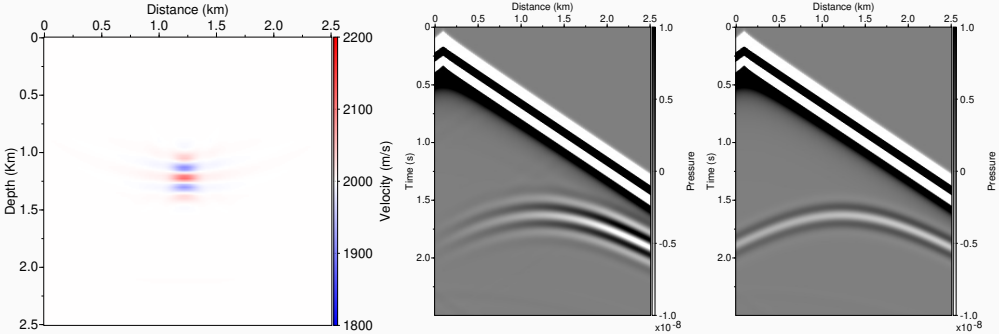
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 0



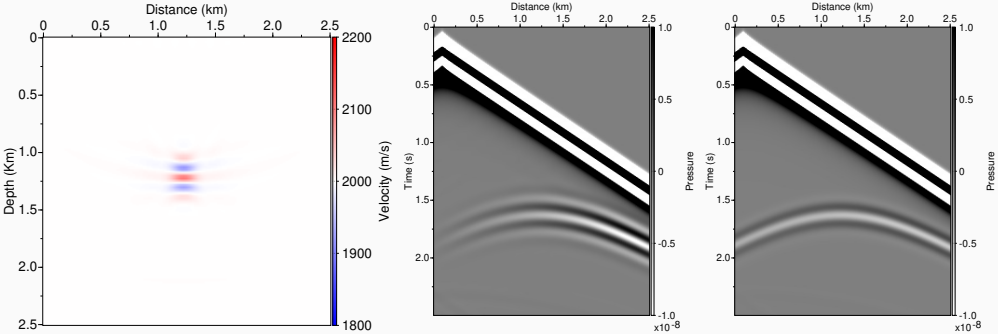
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 1



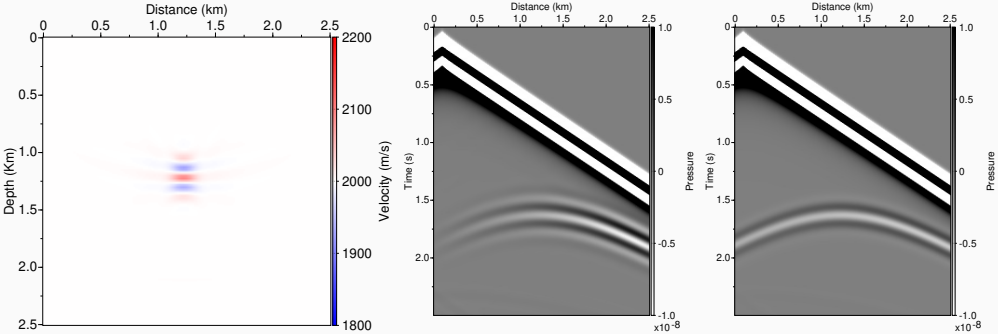
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 2



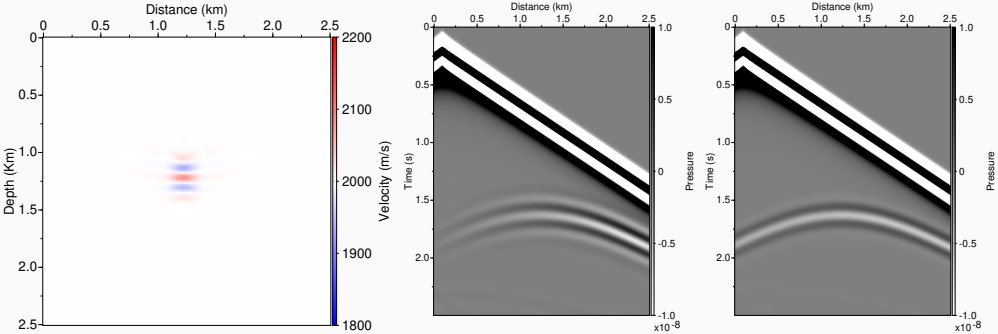
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 3



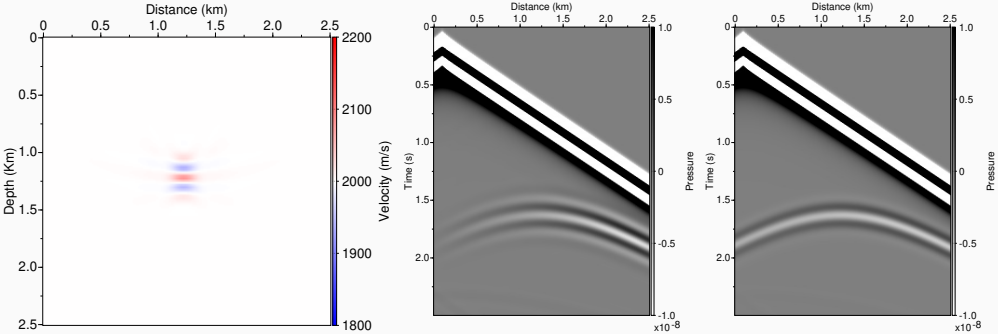
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 4



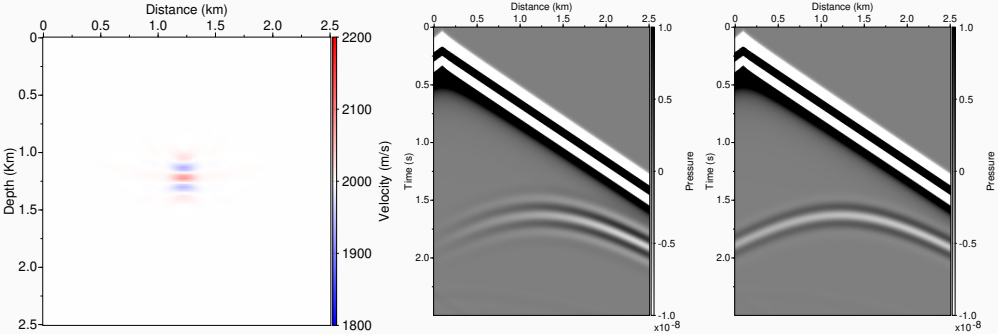
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 5



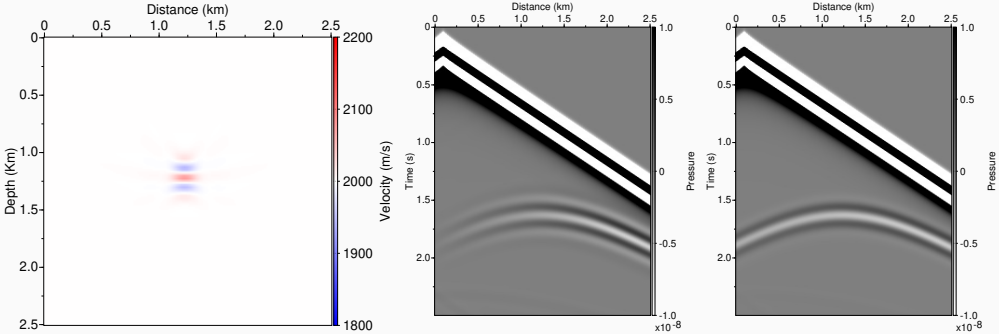
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 6



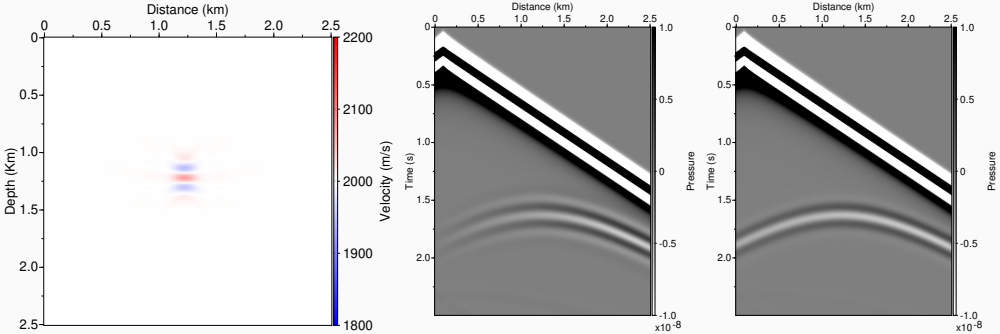
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 7



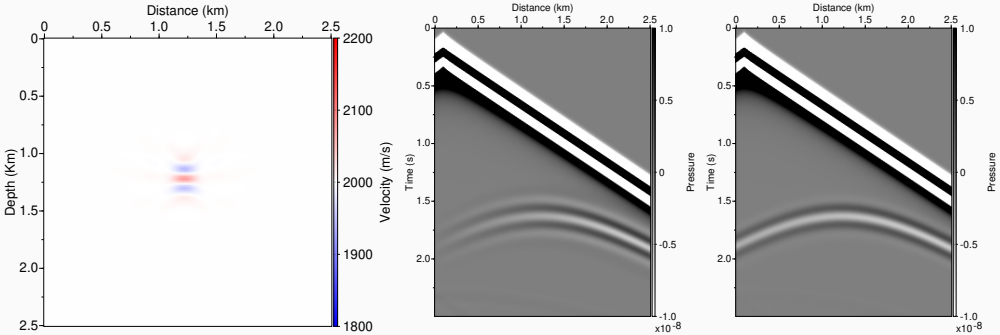
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 8



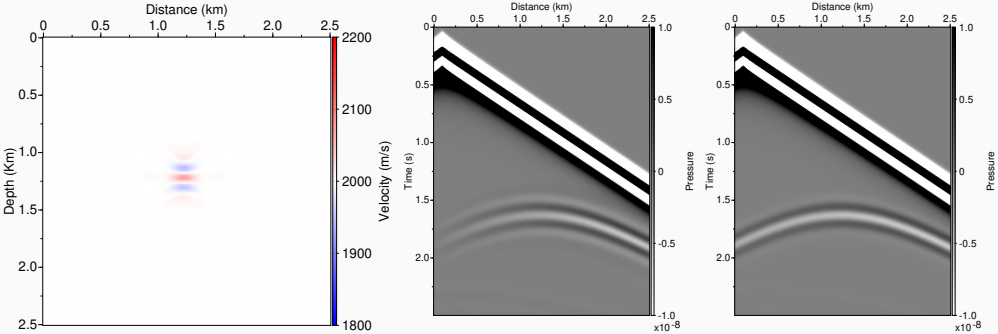
First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 9

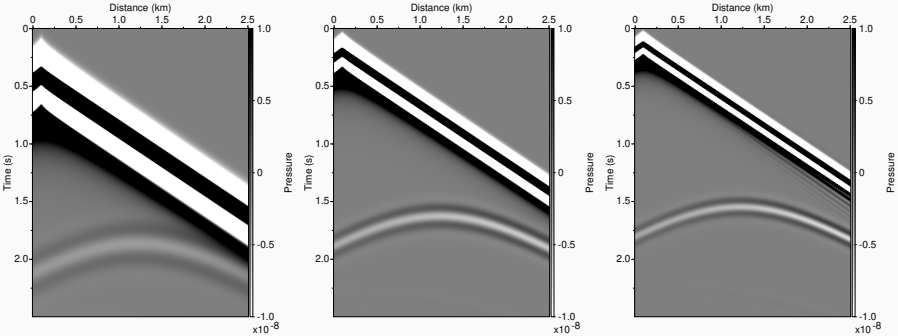


First configuration: surface acquisition, 5 Hz Ricker source

Inversion results: iteration 10

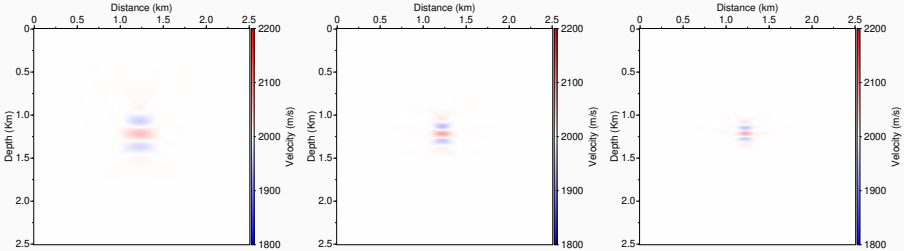


First configuration: surface acquisition, comparison of different frequencies



Surface data with Ricker source at 2.5 Hz (left), 5 Hz (center), 7.5 Hz (right)

First configuration: surface acquisition, comparison of different frequencies

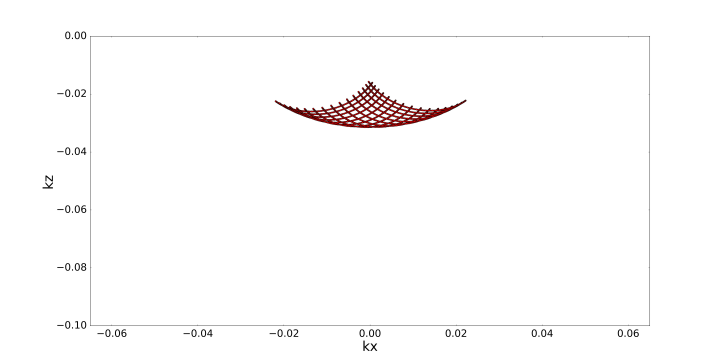


Surface data with Ricker source at 2.5 Hz (left), 5 Hz (center), 7.5 Hz (right)

First configuration: surface acquisition, comparison of different frequencies

$$\mathbf{k} = \frac{4\pi\omega}{c_0} \cos\left(\frac{\theta}{2}\right)(\cos\phi, \sin\phi) \tag{17}$$

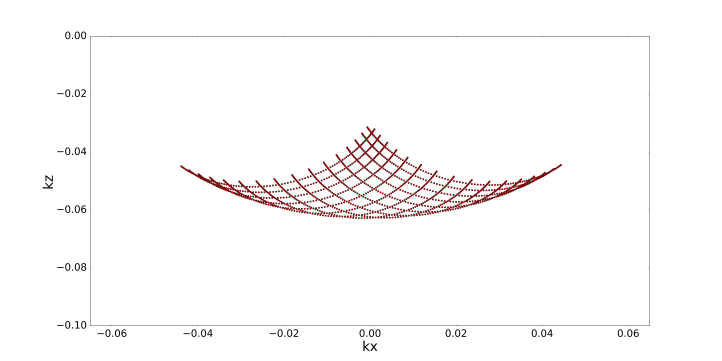
Gradient Wavenumber content analysis: 2.5 Hz surface data



First configuration: surface acquisition, comparison of different frequencies

$$\mathbf{k} = \frac{4\pi\omega}{c_0} \cos\left(\frac{\theta}{2}\right) (\cos\phi, \sin\phi) \tag{17}$$

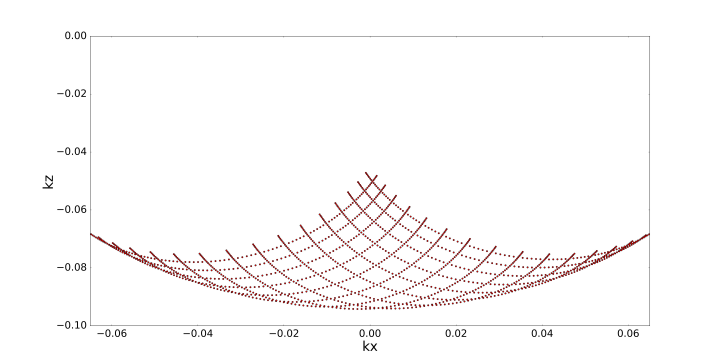
Gradient Wavenumber content analysis: 5 Hz surface data



First configuration: surface acquisition, comparison of different frequencies

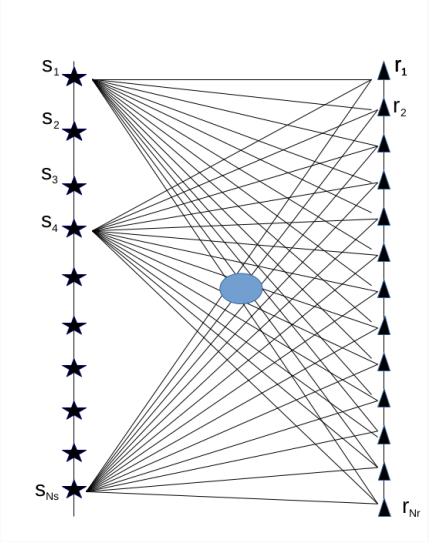
$$\mathbf{k} = \frac{4\pi\omega}{c_0} \cos\left(\frac{\theta}{2}\right)(\cos\phi, \sin\phi) \tag{17}$$

Gradient Wavenumber content analysis: 7.5 Hz surface data



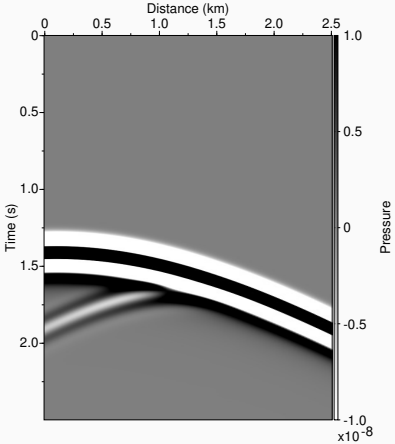
Second configuration: transmission acquisition

Acquisition setup



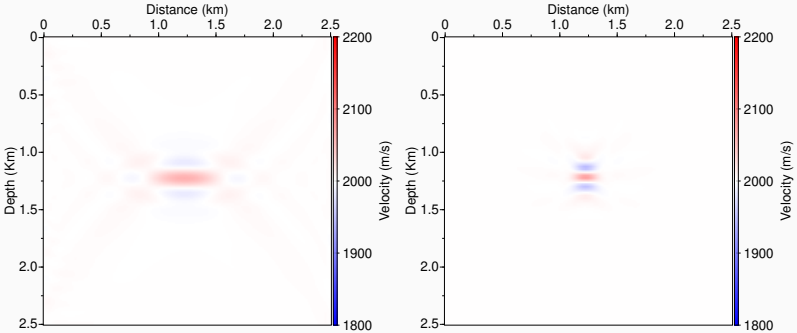
Second configuration: transmission acquisition

Data



Second configuration: transmission acquisition

Inverted model compared with surface acquisition:



Second configuration: transmission acquisition

Gradient Wavenumber content analysis comparison:

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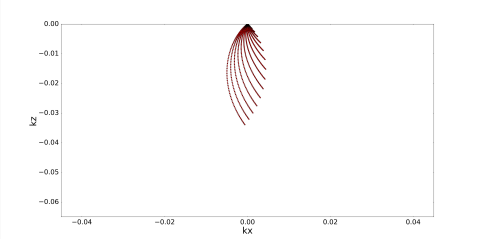
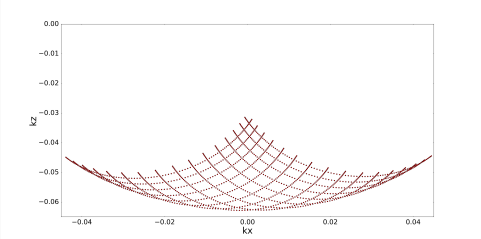
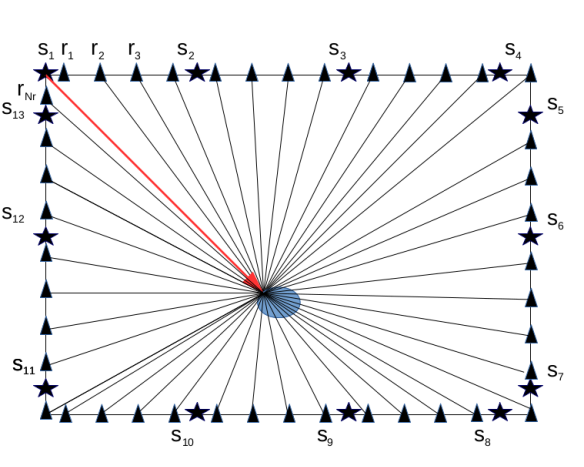




Left: transmission acquisition, right: surface acquisition. Central frequency is the same, fixed at 5 Hz.

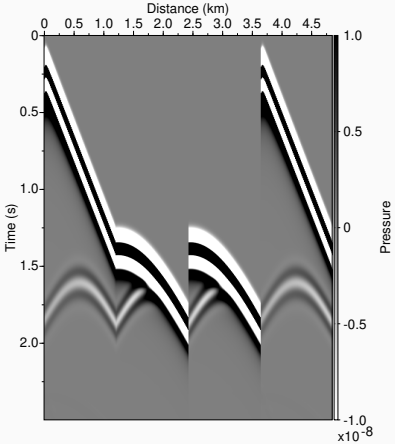
Third configuration: full acquisition

Acquisition setup



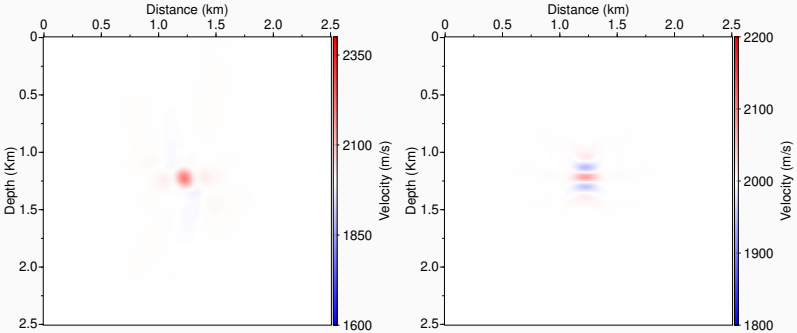
Third configuration: full acquisition

Data



Third configuration: full acquisition

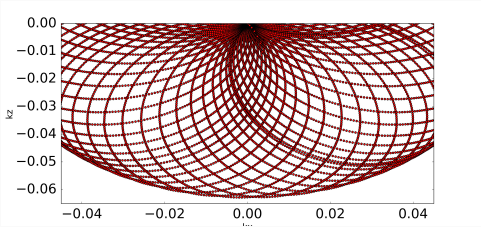
Inverted model compared with surface acquisition:

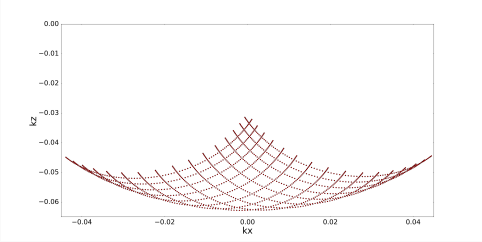


Third configuration: full acquisition

Gradient Wavenumber content analysis comparison:

$$\mathbf{k} = \frac{4\pi\omega}{c_0} \cos\left(\frac{\theta}{2}\right)(\cos\phi, \sin\phi) \tag{19}$$

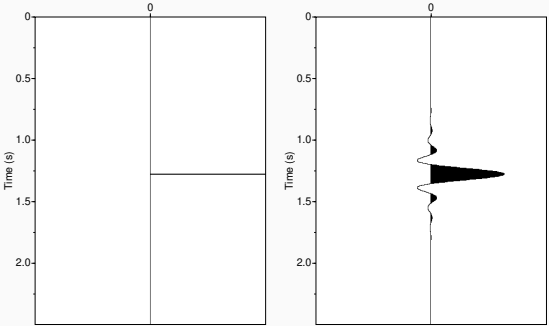




Left: full acquisition, right: surface acquisition. Central frequency is the same, fixed at 5 Hz.

Interpretation

- Seismic imaging with sparse acquisition and finite frequency data can allow to retrieve only a bandpass version of the mechanical parameters
- Oscillations around the Gaussian anomaly are typical of finite frequency reconstruction/imaging
- Illustration: Dirac function and its bandpass filtered version



References