COURSE 1
— MAP101 —
e-training

Myanmar Universities
UGA students - France

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Session 1

Exercise 1

With the previous instructions, construct the following arrays in the shortest way. Save your script as `exoArrayConst3.sce`

\[
M_1 = \begin{bmatrix}
1 & 1 & 2 & 2 & 3 & 3 \\
1 & 1 & 2 & 2 & 3 & 3 \\
2 & 2 & 3 & 3 & 4 & 4 \\
2 & 2 & 3 & 3 & 4 & 4 \\
3 & 3 & 4 & 4 & 5 & 5 \\
3 & 3 & 4 & 4 & 5 & 5
\end{bmatrix}
\]

\[
M_2 = \begin{bmatrix}
1 & 0 & 2 & 0 & 4 & 0 \\
0 & 2 & 0 & 4 & 0 & 8 \\
2 & 0 & 4 & 0 & 8 & 0 \\
0 & 4 & 0 & 8 & 0 & 16 \\
4 & 0 & 8 & 0 & 16 & 0 \\
0 & 8 & 0 & 16 & 0 & 32
\end{bmatrix}
\]

Solution to exercise

```plaintext
// File exoArrayConst3.sce
// Construction of simple matrices

// matrix M1
A = ones(2,2); M1 = [A, 2*A, 3*A ; 2*A, 3*A, 4*A ; 3*A, 4*A, 5*A]

// matrix M2
// solution 1 :
B = diag([1,2]); M2 = [B, 2*B, 4*B ; 2*B, 4*B, 8*B ; 4*B, 8*B, 16*B]

// solution 2 :
n = 5; u = 0:n; v = 2^u;
M2 = diag(v) + diag(v(2:n),2) + diag(v(2:n),-2) ... + diag(v(3:n-1),4) + diag(v(3:n-1),-4)
```
Exercise 2

Last names of people in France.
My name is Luc W. Biard and more specifically Luc is my firstname, W. for William is my middle name (that I never use...), and Biard is my last name or family name. I inherited this family name from my father and he himself inherited it from his father,...

1. Is my mother’s last name Biard ?

2. Is my brother’s last name Biard ?

3. Can a man change his last name or get a new last name in France ?

4. Can a woman change her last name or get a new last name in France ?

5. Can I be sure that all my nephews and nieces have the same last name as me ?

And in Myanmar...?
Explain in a few words how people’s names are chosen. Is there a “common denominator” in a family ?

Exercise 3

Write a Scilab script named exoLoop3.sce that performs the following calculations.

1) Determine the sum of the cubes of all positive integers less than or equal to a given integer \( N \).

\[
S_3(N) = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + N^3 = \sum_{k=1}^{N} k^3
\]

2) Determine the sum of the reciprocals (the inverses) of the squares of odd numbers

\[
SR_2(N) = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots + \frac{1}{(2N+1)^2} = \sum_{k=0}^{N} \frac{1}{(2k+1)^2}
\]

3) Determine the sum of the reciprocals (the inverses) of the factorials

\[
SRF(N) = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{N!} = \sum_{k=0}^{N} \frac{1}{k!}
\]

4) Assuming that

\[
SR_2(\infty) = \sum_{k=0}^{+\infty} \frac{1}{(2k+1)^2} = \frac{\pi^2}{8} \quad \text{and} \quad SRF(\infty) = \sum_{k=0}^{+\infty} \frac{1}{k!} = e = \exp(1)
\]

determine the smallest integers \( n_1 \) and \( n_2 \) such that

a) \(| SR_2(\infty) - SR_2(n_1) | < 10^{-5} \)

b) \(| SRF(\infty) - SRF(n_2) | < 10^{-10} \)

5) Compare the rate of convergence of the two series \( SR_2(n) \) and \( SRF(n) \).
Solution to exercise 3

// File exoLoop3.sce
// Sum evaluations with conditional and unconditional loops

// 1) Sum of the cubes
// 2) Sum of the reciprocals of the squares of odd numbers
N = 15;
S3 = 0;
SR2 = 0;
for k = 0 : N
    S3 = S3 + k^3;
    SR2 = SR2 + 1 / ((2*k+1)^2);
end
mprintf("S3(%d) = %.15f\n",N,S3)
mprintf("SR2(%d) = %.15f\n",N,SR2)

// 3) Sum of the inverses of factorials
N = 15;
SRF = 1;
fact = 1;
for k = 1 : N
    fact = fact * k;
    SRF = SRF + 1 / fact;
end
mprintf("SRF(%d) = %.15f\n",N,SRF)

// 4) Conditionnal loops
// a) determination of n1
SR2infinite = (%pi)^2 / 8;
SR2 = 0;
k = 0;
while abs(SR2infinite - SR2) > 10^(-5)
    SR2 = SR2 + 1 / ((2*k+1)^2);
    k = k + 1;
end
mprintf("SR2infinite - SR2(%d) = %.15f\n",k-1,SR2infinite-SR2)

// b) determination of n2
SRFinfinite = %e;
SRF = 1;
fact = 1;
k = 1;
while abs(SRFinfinite - SRF) > 10^(-10)
    fact = fact * k;
    SRF = SRF + 1 / fact;
    k = k + 1;
end
mprintf("SRFinfinite - SRF(%d) = %.15f\n",k-1,SRFinfinite-SRF)

// 5) Clear :
// convergence of the series SR2(n) is slow compared with that of SRF(n)
Exercise 4

Plot the graph of the function $f$ defined by $f(x) = \frac{1}{x}$ ($x \neq 0$), on the interval $[-6, 6]$ so as to get the right figure below. Name your Scilab script `exoPlotOneOverX.sce` and be aware of the behavior of the function $f$ in the neighborhood of $x = 0$. As an example, the following script leads to left figure below (which is not correct).

```scilab
// file exoPlotOneOverX.sce

// definition of f
deff("y = f(x)" , "y = 1 ./ x;" );

// W R O N G VERSION :
x = linspace(-6,6,200);
y = f(x);
plot(x,y,"b-");

// W A N T E D VERSION :
// TO BE COMPLETED . . .
```

Wrong... Better!
Solution to exercise

```matlab
// file exoPlotOneOverX.sce

// definition of f
deff("y = f(x)", "y = 1 ./ x;"); // space between 1 and ./

// WRONG VERSION :
x = linspace(-6,6,200);
y = f(x);
plot(x,y,"b-");

// WANTED VERSION :
eps = 10^(-6);

// plot on [-6, 0-eps]
x = linspace(-6,0-eps,400);
y = f(x);
plot(x,y,"b-");

// plot on [0+eps, 6]
x = linspace(0+eps,6,400);
y = f(x);
plot(x,y,"b-");

axes = gca(); // get current axes
replot([-6 -6 6 6]); // modify the bounds
xtitle("function f(x) = 1/x"); // title
axes.x_location = "origin"; // x-axes through origin
axes.y_location = "origin"; // y-axes through origin
axes.box = "off"; // cancel bounding box
set(axes,"isoview","on") // normalization
```
Exercise 5 (Some remarks about windows paths and scilab)

1. Create in your desktop a folder with name MyLocalWorkingDirectory

2. Create the following Scilab script named Hello.sce and save it in the folder MyLocalWorkingDirectory

// Hello.sce
// Just a short example...
str0 = "|-------------------------------------------|"
str1 = "| HELLO, I AM LEARNING SCILAB |
str2 = "| The file Hello.sce is in the folder |
str3 = "| << MyLocalWorkingDirectory >> |
str4 = "| and this folder is located on my desktop |
mprintf("\n%s\n%s\n%s\n%s\n%s\n%s", str0,str1,str2,str3,str4,str0)

3. In the Console of Scilab, run the following instruction
   --> exec("Hello.sce",-1)
The following message should be displayed
   exec: Cannot open file Hello.sce.

4. From the menu File in the Console, run the item Current Working Directory
   As an example, on my laptop, I get the message

   C:\Users\Luke\Documents

   So, can we understand why Scilab “Cannot open file Hello.sce” at previous step?

5. Open the folder MyLocalWorkingDirectory and get its address (top of the window)
   As an example, on my laptop this address is:

   C:\Users\Luke\Desktop\MyLocalWorkingDirectory

6. In the Console run the instruction
   --> exec("C:\Users\Luke\Desktop\MyLocalWorkingDirectory\Hello.sce",-1)
   and it should work...

7. Then, from the menu File in the Console, run the item Browse for new and set the folder MyLocalWorkingDirectory as your Current Working Directory.

8. Now, in the Console of Scilab, run the instruction
   --> exec("Hello.sce",-1)
   and it should work...

9. Conclusion?
Exercise 6

Determine the decimal value (base 10) of the following positive integers, which are coded in base 2 (binary coding) or 16 (hexadecimal coding).

\[ n_1 = \overline{1111}_2 = \]
\[ n_2 = \overline{1111}_{16} = \]
\[ n_3 = \overline{01010001}_2 = \]
\[ n_4 = \overline{1ABC}_{16} = \]

Solution to exercise 6

\[ n_1 = \overline{1111}_2 = (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = \overline{15}_{10} = 15 \]
\[ n_2 = \overline{1111}_{16} = (1 \times 16^3) + (1 \times 16^2) + (1 \times 16^1) + (1 \times 16^0) = \overline{4369}_{10} = 4369 \]
\[ n_3 = \overline{01010001}_2 = \ldots = \overline{81}_{10} = 81 \]
\[ n_4 = \overline{1ABC}_{16} = (1 \times 16^3) + (10 \times 16^2) + (11 \times 16^1) + (12 \times 16^0) = \overline{6844}_{10} = 6844 \]