Detection of dependence patterns with delay

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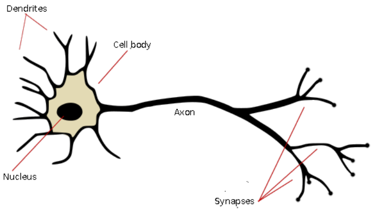
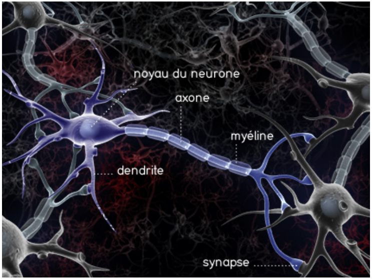
Journées de la SFdS

4 Juin 2015

Biological context



Structure of a typical neuron



Connected neurons

- Neural network: Interacting cells.
- Information transport via electric pulses: action potentials.

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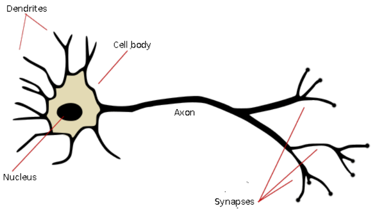

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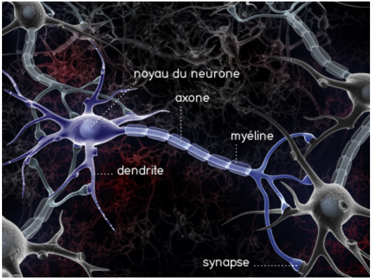

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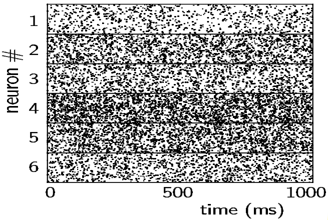
After preprocessing, we dispose of M trials of simultaneously recorded spike trains.



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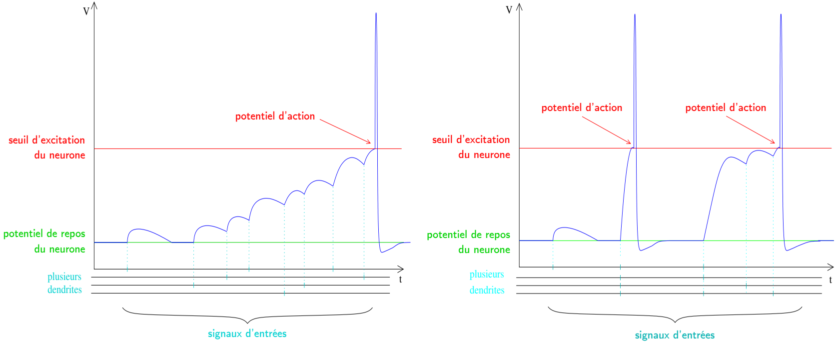
Synchronization phenomenon



Without synchronization **With synchronization**

- The synchronization phenomenon can occur during sensory-motor tasks.
- The repetition of a given task may give birth to neuronal assemblies.

Goal

Detection of synchronizations.

Statistical analysis

- Cross-correlogram (Perkel et al., '67).
- Peristimulus time histogram (PSTH, (Aertsen et al., '89)).

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UE method

- Unitary event: spike synchrony that recurs more often than expected.
- The test statistic is based on the *number of coincidences*.
  - Introduced in the PhD thesis of S. Grün ('96).
  - Applied to time discrete data.

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GAUE method for two neurons (Tuleau-Malot et al., 2014)

- Notion of coincidence transposed to the continuous time framework.
- Independence test between Poisson processes based on this new notion.

Notion of delayed coincidences

- $N_1, \ldots, N_n$ are point processes on $[a, b]$.
- $\mathscr{I} \subset \{1, \ldots, n\}$ is a set of indices.

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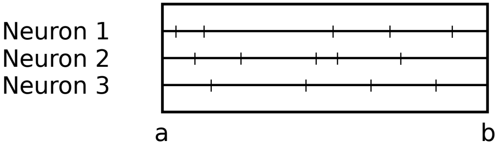
Definition

The *delayed coincidence count* of delay $\delta < (b-a)/2$ is

$$X_{\mathscr{J}} := \int_{[a,b]^J} \mathbf{1}_{\left|\max_{i \in \{\mathbf{1},\dots,J\}} x_i - \min_{i \in \{\mathbf{1},\dots,J\}} x_i\right| \leq \delta} N_{i_1}(dx_1) \dots N_{i_J}(dx_J).$$



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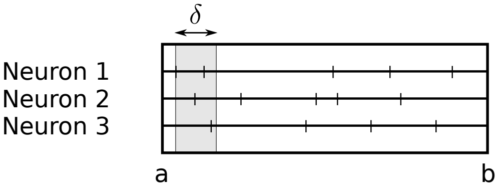

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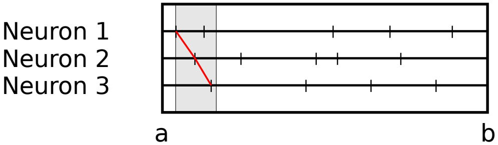

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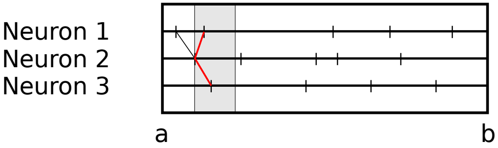

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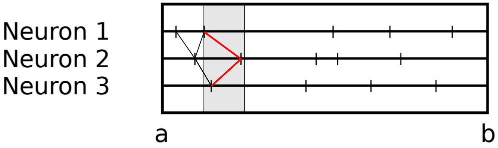

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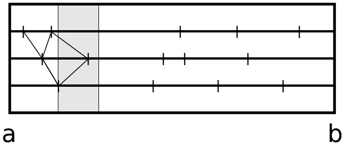
Neuron 3



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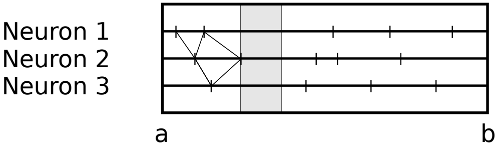

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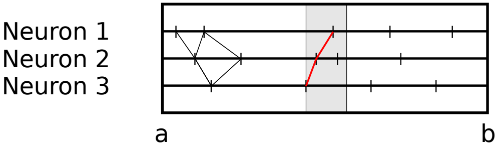

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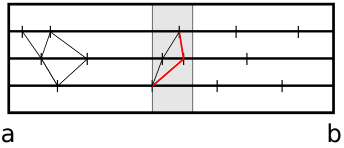

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- $N_1, \ldots, N_n$ are point processes on $[a, b]$.
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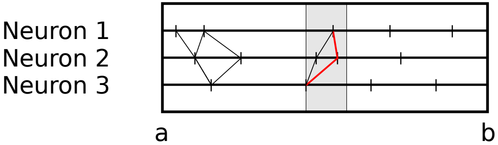
Definition

The *general coincidence count* is

$$X_{\mathscr{J}} := \int_{[a,b]^J} c(x_1, \ldots, x_J)\ N_{i_1}(dx_1) \ldots N_{i_J}(dx_J).$$



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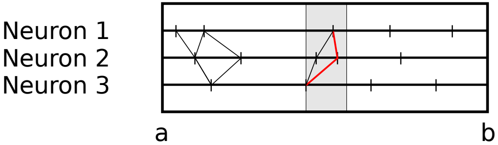

Goal: Test $\mathscr{H}_0$ against $\mathscr{H}_1$

$$\begin{cases} \mathscr{H}_0 : & \text{The processes } N_j,\ j \in \mathscr{J} \text{ are independent;} \\ \mathscr{H}_1 : & \text{The processes } N_j,\ j \in \mathscr{J} \text{ are not independent.} \end{cases}$$

Asymptotic properties

Let $(N_1^{(k)}, \ldots, N_n^{(k)})_{1 \leq k \leq M}$ denote a M-sample. We compare two estimates.

- CLT $\Rightarrow \sqrt{M}\frac{\overline{m} - \mathbb{E}[X_{\mathscr{I}}]}{\sqrt{\mathbb{V}\mathrm{ar}(X_{\mathscr{I}})}} \xrightarrow{M \to \infty} \mathscr{N}(0,1)$, where $\overline{m} = \frac{1}{M}\sum_{k=1}^{M} X_{\mathscr{I}}^{(k)}$.

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- If $N_1,\dots,N_n$ are Poisson processes with intensities $\lambda_1,\dots,\lambda_n$, then

$$\begin{cases}\mathbb{E}\left[X_{\mathscr{I}}\right]=m_0((\lambda_i)_i)\\ \mathbb{V}\mathrm{ar}\left(X_{\mathscr{I}}\right)=v_0((\lambda_i)_i)\end{cases} \qquad \text{under } \mathscr{H}_0.$$

- Let us denote

$$\hat{\lambda}_i := \frac{1}{M}\sum_{k=1}^M \frac{N_i^{(k)}([a,b])}{b-a} \quad \text{and} \quad \begin{cases}\hat{m}_0 = m_0((\hat{\lambda}_i)_i)\\ \hat{v}_0 = v_0((\hat{\lambda}_i)_i).\end{cases}$$

- Plug-in step (delta method + Slutsky) $\Rightarrow \sqrt{M}\frac{\overline{m}-\hat{m}_0}{\sqrt{\hat{\sigma}^2}} \xrightarrow[\mathscr{H}_0]{M\to\infty} \mathscr{N}(0,1)$ where

$$\hat{\sigma}^2 = \hat{v}_0 - (b-a)^{-1}\hat{m}_0^2\left(\sum_{j\in\mathscr{I}}\hat{\lambda}_j^{-1}\right).$$

Our independence test

Definition

Denote z_α the α-quantile of the standard Gaussian distribution. Then the symmetric test Δ_α rejects $\mathscr{H}_0$ when $\bar{m}$ and $\hat{m}_0$ are too different, that is when

$$\left| \sqrt{M} \frac{(\bar{m} - \hat{m}_0)}{\sqrt{\hat{\sigma}^2}} \right| > z_{1-\alpha/2}.$$

Theorem

If $N_1, \ldots, N_n$ are homogeneous Poisson processes, the test Δ_α is of asymptotic level α.

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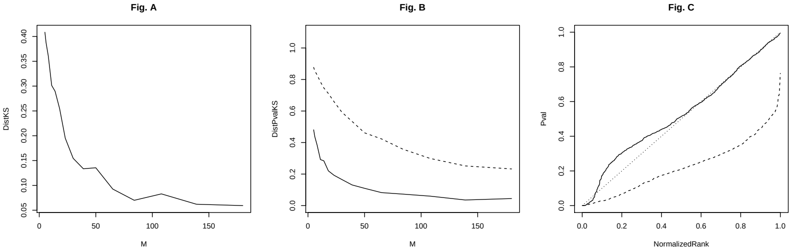
Simulation procedure

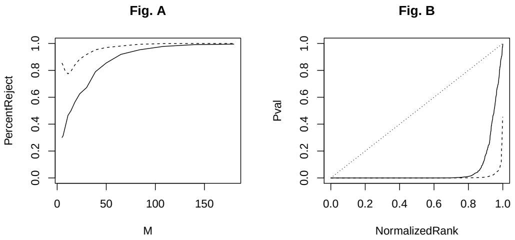
1. Generate a set of random parameters $(b-a, (\lambda_i)_i)$ according to the appropriate Framework;
2. Use this set (and $\delta = 10$ms) to generate M trials;
3. Compute the different statistics;
4. Repeat steps 1 to 3 a thousand times.

Level



- $n = 4$ neurons. $\mathscr{I} = \{1, 2, 3, 4\}$;
- $b - a \sim \mathscr{U}([0.2, 0.4\text{s}])$;
- Independent intensities. $\lambda_i \sim \mathscr{U}([8, 20\text{Hz}])$;
- $M = 50$ (Figure C).

Power



- Add an injection process $\tilde{N}$. Intensity: 0.3Hz.
- $\alpha = 0.05$.
- $M = 50$ (Figure B).

Hawkes processes ('71)

- More realistic than Poisson processes (Goodness of fit tests, Reynaud-Bouret et al., '14).

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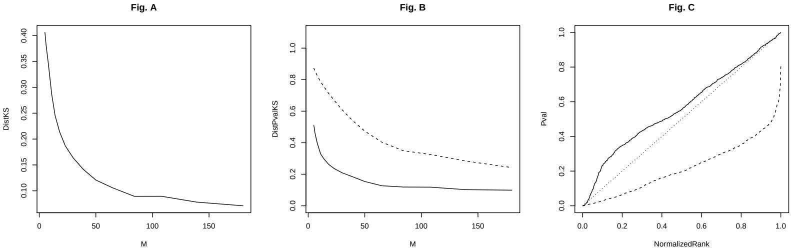
- More realistic than Poisson processes (Goodness of fit tests, Reynaud-Bouret et al., '14).
- Form of the intensity:

$$\lambda_t^j = \max\left(0, \mu_j + \sum_{i=1}^{n} \int_{s<t} h_{ij}(t-s)\, N^i(ds)\right).$$

- spontaneous rate $\mu_j \geq 0$.
- interaction function h_{ij}: influence of neuron i over neuron j.
  - Either excitatory or inhibitory phenomena.
  - Strict refractory period. ($h_{ii} << 0$)

Level



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- $b - a \sim \mathscr{U}([0.2, 0.4s])$;
- Independent spontaneous intensities. $\mu_i \sim \mathscr{U}([8, 20Hz])$;
- Auto-interaction functions h_{ii} to model refractory period of 3ms.
- $M = 50$ (Figure C).

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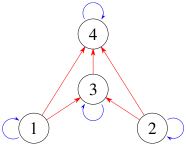
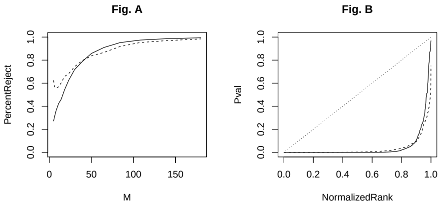
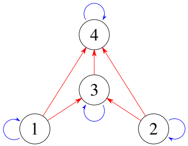
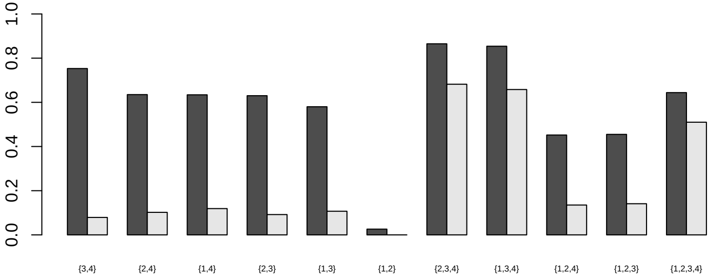




- Add interaction functions according to the graph. Range: 5ms.
- $\alpha = 0.05$.
- $M = 50$ (Figure B).

Simulations





Rate of dependence detection

Overview

- Independence test over any subset of n neurons.
- Theoretical results on Poisson processes. Remains reliable on Hawkes processes.
- Multiple testing over the subsets.

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- Outlook:
  - Find the asymptotic for Hawkes processes.
  - R package.