~~Optimisation: des maths pour des décisions plus robustes, résilientes, responsables~~

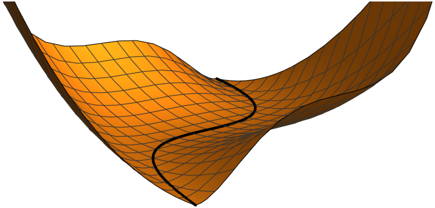
# Plaidoyer pour l'optimisation non-lisse

## illustré par l'aide à la décision robuste

Jérôme MALICK

Journées des Lauréats SMI 2023

# Mathematical Optimization ?



- branch of applied maths
  (theory, algorithms, software, modeling)
- applications everywhere
  (industry, decision-making, sciences,...)
- being revolutionized by its interactions with data
  (computational statistics, machine learning, IA)

$\boxed{\text{optim. is at the core of IA, playing a fundamental role, behind the scenes}}$

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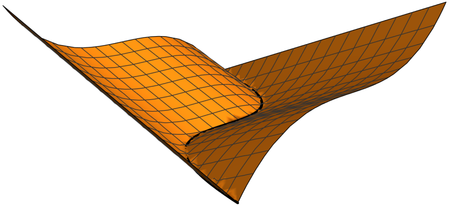

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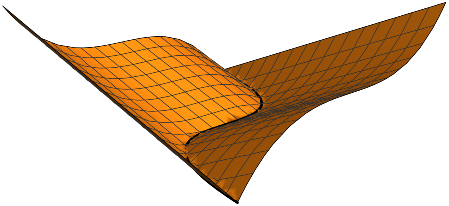
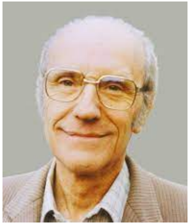


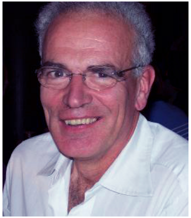
nonsmooth optim. is at the core of IA, playing a fundamental role, behind the scenes

(French) pioneers
on optimization...

worked on nonsmooth aspects



J.-J. Moreau



C. Lemaréchal

H. Attouch

# Nonsmooth objective functions are everywhere...

## Max functions

$$F(x) = \sup_{u \in U} \; h(u, x)$$

- robust optimization, stochastic optimization, decomposition methods
- Relaxations of combinatorial problems

## Nonsmooth regularization

$$F(x) = f(x) + g(x)$$

- image/signal processing, inverse problems
- sparsity-inducing regularizers in machine learning

## Nonsmooth composition

$$F(x) = g \circ c(x)$$

- risk-averse optimization, eigenvalue optimization
- deep learning: nonsmooth activation, implicit layers

## Probability functions

$$F(x) = \mathbb{P}\big(h(x, \xi) \leqslant 0\big)$$

- optimization under uncertainty, energy optimization

# So what ?...

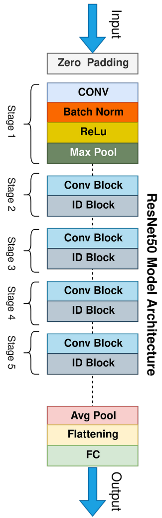
Is nonsmoothness really important ? useful ?

## Why not just ignoring it ?

- **Ex:** nonsmooth deep learning
  (with RELU, max-pooling or implicit layers)
- Just apply SGD with back-prog
- Or just apply quasi-Newton with (sub)gradients

## Why not smoothing it ?

- Smoothing by (inf-)convolution (e.g. Moreau regularization)
- Smoothings by overparameterization, ad hoc, or...



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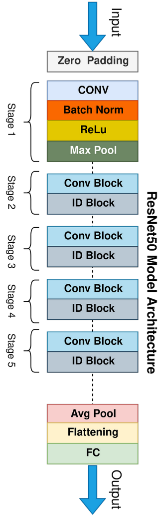
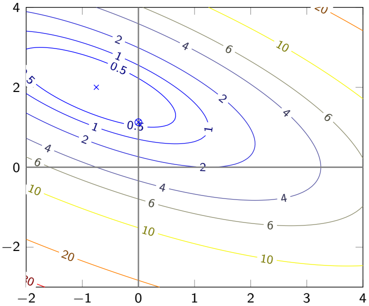



My point: nonsmoothness is relevant !

# Example: $\ell_1$-regularized least-squares (1/2)

$$\min_{x\in\mathbb{R}^d} \quad \frac{1}{2}\|Ax - y\|^2 \; + \; \lambda\|x\|_1 \qquad \text{(LASSO)}$$

Illustration (on an instance with $d = 2$)



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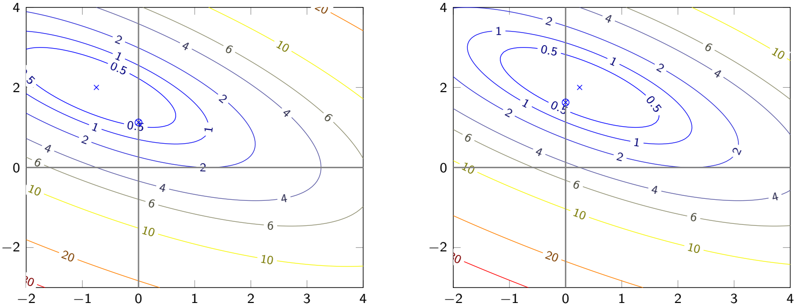


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$$\boxed{\text{Nonsmoothness traps solutions in low-dimensional manifolds}}$$

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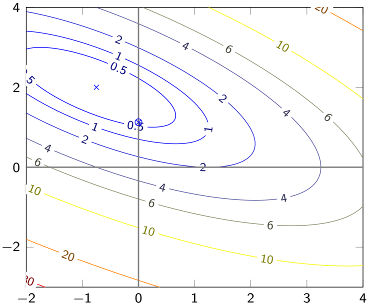
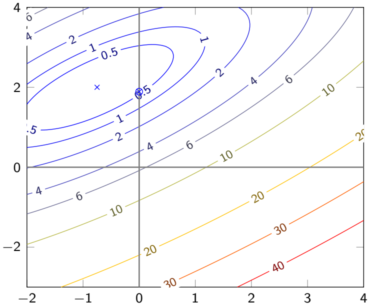

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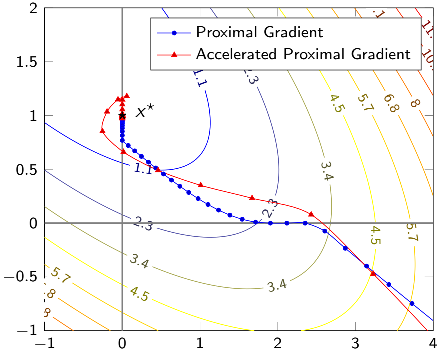
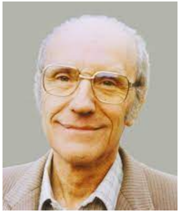



Proximal operator

[Moreau '60]



$$\operatorname{prox}_{\gamma g}(y) = \operatorname*{argmin}_z \left\{ g(z) + \frac{1}{2\gamma}\|z - y\|^2 \right\}$$

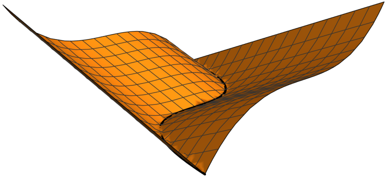
(proximal-gradient) algorithms produce iterates...

...that eventually have the same support as the optimal solution

$\boxed{\text{Nonsmoothness attracts (proximal) algorithms}}$

# Today's message

Nonsmoothness is sometimes useful
and always nice-looking



## Modest goals of this talk:

- Spotlights on 2 applications:
  - in industry : electricity generation
  - in learning : towards robustness and fairness
- High level: underline ideas, duality, models...

  No theorems ! No algorithms ! (Almost) No references !
- For more, feel free to contact me :

  `jerome.malick@cnrs.fr`

# Spotlight #1: Optimization of electricity production

In France: EDF produces electricity by $N$ production units

nuclear 63%

renewables 14%

oil/gaz/coal 12%

hydro 17%

Question : finding "optimal" daily production schedules

Day-to-day optimization of production ("unit-commitment" )

$$\binom{\text{simplified}}{\text{model}} \quad \left\{ \begin{array}{lr} \min \quad \sum_i c_i^\top x_i & \text{(production costs)} \\ \quad \sum_i x_i = d & \text{(demand constraints)} \\ \quad (x_1, \dots, x_N) \in X_1 \times \cdots \times X_N & \text{(operational constraints)} \end{array} \right.$$

Hard optimization problem: large-scale, heterogeneous, complex ($\geqslant 10^6$ variables, $\geqslant 10^6$ constraints)

Out of reach for (mixed-integer linear) solvers... But where is the nonsmoothness ?

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# Lagrangian decomposition

- Dual function (concave)

$$\theta(u) = \left\{ \begin{array}{l} \min \quad \sum_{i=1}^{N} c_i^\top x_i + \sum_{t=1}^{T} u^t \Big( d^t - \sum_{i=1}^{N} x_i^t \Big) \\ (x_1, \dots, x_N) \in X_1 \times \cdots \times X_N \end{array} \right.$$

- Dualizing the coupling constraint makes it decomposable by units

$$\theta(u) = d^\top u + \sum_{i=1}^{N} \theta_i(u)$$

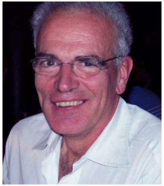
$$\theta_i(u) = \left\{ \begin{array}{l} \min \quad (c_i - u)^\top x_i \\ \quad x_i \in X_i \end{array} \right.$$

- Nonsmooth algorithm: inexact prox. bundle [Lemaréchal '75... '95]

Nonsmooth optimization algorithm

shadow prices ↓ ↑ decentralized productions




C. Lemarechal


S. Charousset


A. Renaud

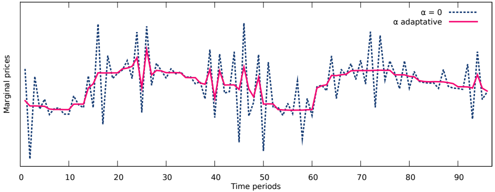
- Research in the 1990's
- In action in early 2000's
- Save money and $CO2$ !

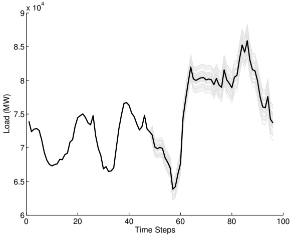
# On the shoulders of giants

## My contributions on this topic

- Acceleration of the bundle method (using coarse linearizations) [Malick, Oliveira, Zaourar '15]
- (Level) asynchronous bundle algorithm [Iutzeler, Malick, Oliveira '18]
- Denoising dual solutions (by TV-regularization) [Zaourar, Malick '13]
- Introducing weather uncertainty in the model
  - robust version of the problem + bundle method [van Ackooij, Lebbe, Malick '16]
  - 2-stage stochastic version + double decomposition algorithm [van Ackooij, Malick '15]

...handling uncertainty adds extra nonsmoothness ☺



# Spotlight #2: towards robust, resilient, responsible decisions

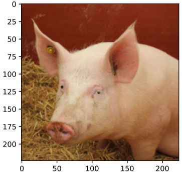
Spectacular success of deep learning, in many fields/applications... E.g. in generation

**Ex:** picture generated with `https://stablediffusionweb.com` in oct. 2023.

input: "towards robust, resilient, responsible decisions"

# Example 2.1: Don't forget how fragile deep learning can be !

Flying pigs (notebooks of NeurIPS 2018, tutorial on robustness)




pig (99%)

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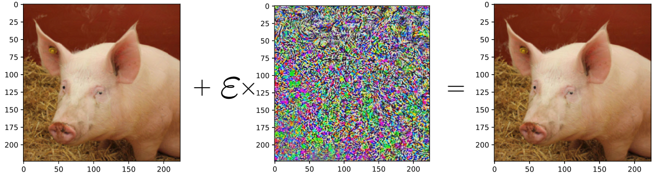


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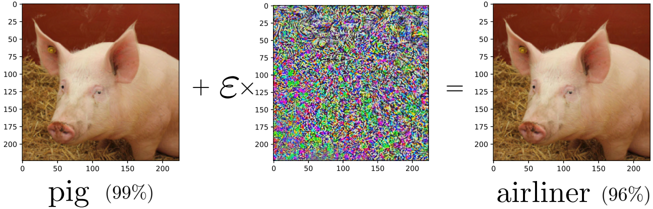


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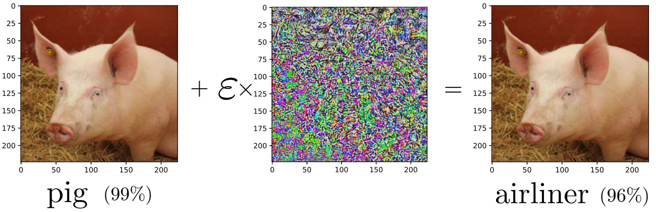


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Attacks against self-driving cars [@ CVPR '18]

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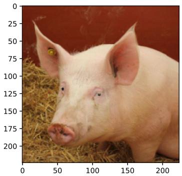
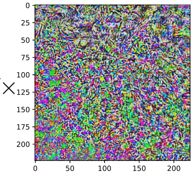



$+\ \mathcal{E}\times$

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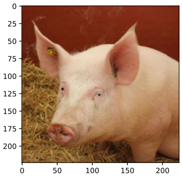


pig (99%) airliner (96%)

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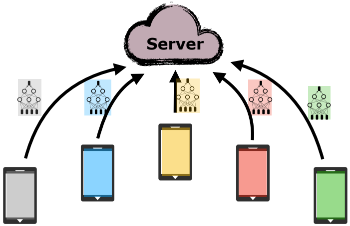
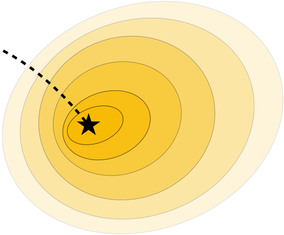
Attacks against self-driving cars [@ ICLR '19]



# Example 2.2: ML may perform poorly for some people

**Example:** Global model is trained on *average distribution* across clients (ERM)



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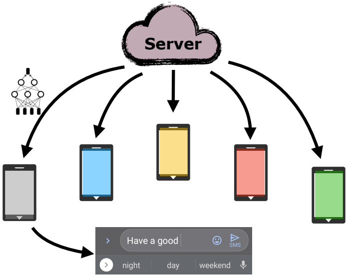


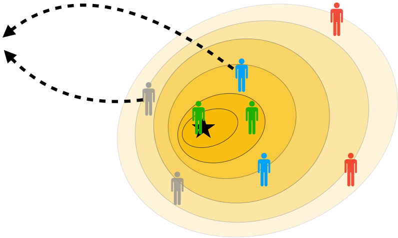

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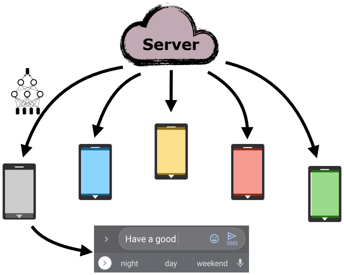


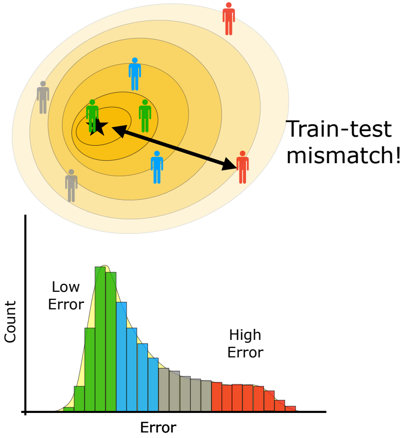

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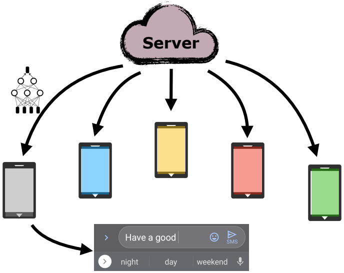


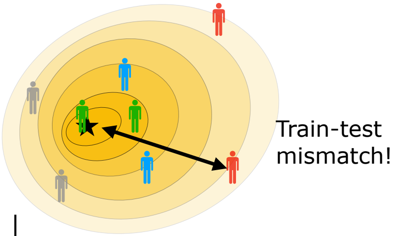


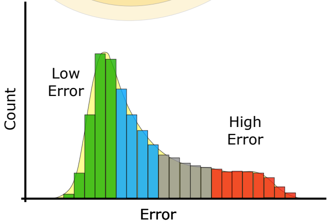


Fairness issues, e.g.

**Amazon : l'intelligence artificielle qui n'aimait pas les femmes**

Fil info

09:55

09:55

Municipales à Paris:

09:40

En Inde, des applicat

Accélérer le recrutement en faisant analyser les CV par une IA : l'idée semblait prometteuse à Amazon. Mais elle s'est mise à sous-noter les femmes candidates à des postes tech.

**Machine Bias**

There's software used across the country to predict future criminals. And it's biased against blacks.

by Julia Angwin, Jeff Larson, Surya Mattu and Lauren Kirchner, ProPublica

May 23, 2016

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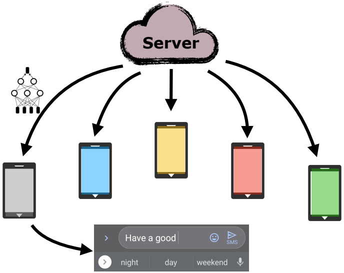


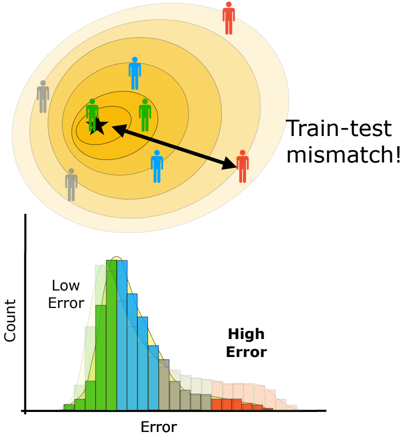


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# Optimization set-up

- Training data: $\xi_1, \ldots, \xi_N$

  e.g. in supervised learning: labeled data $\xi_i = (a_i, y_i)$ feature, label

- Train model: $f(x, \cdot)$ the loss function with $x$ the parameter/decision $(\omega, \beta, \theta, \ldots)$

  e.g. least-square regression: $f\big(x, (a, y)\big) = (x^\top a - y)^2$

- Compute $x$ via empirical risk minimization (a.k.a SAA)

$$\min_x \frac{1}{N} \sum_{i=1}^{N} f(x, \xi_i) = \mathbb{E}_{\widehat{\mathbb{P}}_N}[f(x, \xi)] \qquad \text{with } \widehat{\mathbb{P}}_N = \frac{1}{N} \sum_{i=1}^{N} \delta_{\xi_i}$$

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- Prediction with $x$ for different data $\xi$
  - Adversarial attacks (e.g. flying pigs, driving cakes...)
  - Presence of bias, e.g. heterogeneous data
  - Distributional shifts: $\mathbb{P}_{\text{train}} \neq \mathbb{P}_{\text{test}}$

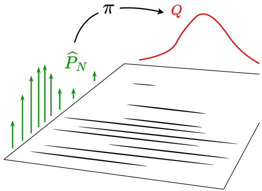
- Solution: take possible variations into account during training

...and nonsmoothness comes into play ☺

# (Wasserstein) Distributionally Robust Optimization

Rather than $\min\limits_{x} \mathbb{E}_{\widehat{\mathbb{P}}_N}[f(x,\xi)]$ solve instead $\min\limits_{x} \max\limits_{\mathbb{Q}\in\mathcal{U}} \mathbb{E}_{\mathbb{Q}}[f(x,\xi)]$

with $\mathcal{U}$ a neighborhood of $\widehat{\mathbb{P}}_N$




**Wasserstein** balls as ambiguity sets

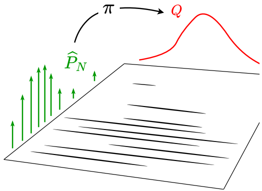
$$\mathcal{U} = \{\ \mathbb{Q} : W(\widehat{\mathbb{P}}_N, \mathbb{Q}) \leqslant \rho\ \}$$

$$W(\widehat{\mathbb{P}}_N, \mathbb{Q}) = \min_{\boldsymbol{\pi}} \left\{ \mathbb{E}_{\boldsymbol{\pi}}[c(\xi,\xi')] : [\boldsymbol{\pi}]_1 = \widehat{\mathbb{P}}_N, [\boldsymbol{\pi}]_2 = \mathbb{Q} \right\}$$

# (Wasserstein) Distributionally Robust Optimization

Rather than $\min_{x} \mathbb{E}_{\widehat{\mathbb{P}}_N}[f(x,\xi)]$ solve instead $\min_{x} \max_{\mathbb{Q}\in\mathcal{U}} \mathbb{E}_{\mathbb{Q}}[f(x,\xi)]$

with $\mathcal{U}$ a neighborhood of $\widehat{\mathbb{P}}_N$




**Wasserstein** balls as ambiguity sets

$$\mathcal{U} = \{\ \mathbb{Q} : W(\widehat{\mathbb{P}}_N, \mathbb{Q}) \leqslant \rho\ \}$$

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**WDRO objective function** for given $x$, $\widehat{\mathbb{P}}_N$, $\rho$

$$\begin{cases} \max_{\mathbb{Q}} \ \mathbb{E}_{\mathbb{Q}}[f(x,\xi)] \\ W(\widehat{\mathbb{P}}_N, \mathbb{Q}) \leqslant \rho \end{cases} \Leftrightarrow \begin{cases} \max_{\mathbb{Q},\boldsymbol{\pi}} \ \mathbb{E}_{\mathbb{Q}}[f(x,\xi)] \\ [\boldsymbol{\pi}]_1 = \widehat{\mathbb{P}}_N, [\boldsymbol{\pi}]_2 = \mathbb{Q} \\ \min_{\boldsymbol{\pi}} \mathbb{E}_{\boldsymbol{\pi}}[c(\xi,\xi')] \leqslant \rho \end{cases}$$

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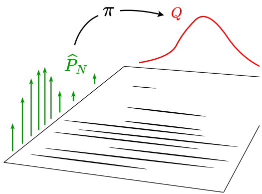


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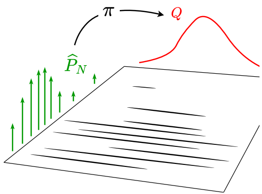


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$$\Leftrightarrow \ \min_{\lambda \geqslant 0} \ \lambda\rho + \mathbb{E}_{\widehat{\mathbb{P}}_N}\left[ \max_{\xi'} \{ f(x,\xi') - \lambda c(\xi,\xi') \} \right]$$

...(finite dimension) nonsmooth... computable in some (specific) cases [Kuhn *et al.* '18]

...actually many more [Vincent, Azizian, Iutzeler, Malick '24]

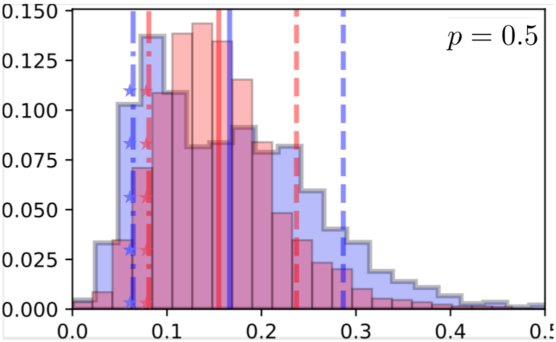
# Illustration: (W)DRO reshapes test histograms

Classification task, federated learning context [Laguel, Pillutla, Harchaoui, Malick '23]

ConvNet with EMNIST dataset (1730 users, 179 images/users)

Histogram over users of test misclassification error: standard vs. robust
(dashed lines: 10%/90%-quantiles)

Recall the goal:

# Main current topic in my group

## Our work

- Applications in federated learning [Laguel, Pillutla, Harchaoui, Malick '23]
- (abstract, entropic) regularizations of WDRO [Azizian, Iutzeler, Malick '22]
- Statistical guarantees [Azizian, Iutzeler, Malick '23] [Le, Malick '24]
- Numerical work for an easy-to-use toolbox `skWDRO` [Vincent, Azizian, Iutzeler, Malick '24]

Try it out !

robustify our model with `skWDRO` !

scikitlearn interface + pytorch wrapper

# Conclusion

## Main take-aways

- Nonsmooth optimization rocks
- Electricity managment optimization is huge

  Handling size (and uncertainty) leads to nonsmooth optimization
- Deep learning works very well... unless it does not

  Handling robustness leads to nonsmooth optimization
- More work is needed resilience, fairness...

# Conclusion

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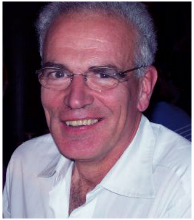
- More work is needed resilience, fairness...

Merci à vous

pour votre confiance

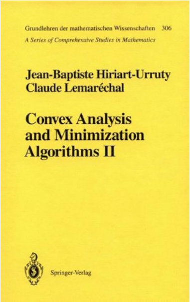
et pour votre écoute aujourd'hui

# Remark: smooth but stiff problems



J.-B. Hiriart-Urruty C. Lemaréchal

"There is no clear cut between functions that are smooth and functions that are not.
In-between there is a rather fuzzy boundary of stiff functions"




In sharp contrast with smoothing-like approaches:

- Toy example from the book (Section VIII.3.3): for a smooth problem, run usual algorithms

  nonsmooth methods (prox/level-bundle) $\gg$ smooth methods (gradient, conj. grad., q-Newton)

- Real-life example in energy optimization :
  - problem of managment of reservoirs : smooth
  - state-of-the-art algos to solve it : nonsmooth

Nonsmoothness can help, even for (difficult) smooth problems

# Two-stage stochastic unit-commitment

W. van Ackooij

- The schedule $x$ is sent to the grid-operator (RTE) before being activated and before observing uncertainty
- In real time, a new production schedule can be sent at certain times
- At time $\tau$, we have the observed load $\xi_1, \dots, \xi_\tau$ and the current best forecast $\xi_{\tau+1}, \dots, \xi_T$
- We propose a stochastic 2-stage problem:

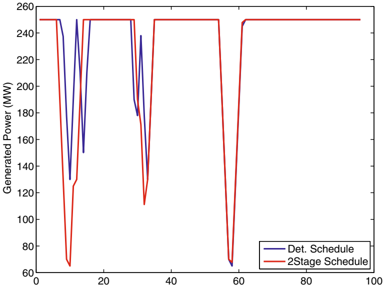
$$\boxed{\left\{\begin{array}{ll} \min & c^\top x + \mathbb{E}[c(x,\xi)] \\ & x \in X, \quad \sum_i x_i = d \end{array}\right.} \qquad \text{where } c(x,\xi) = \left\{\begin{array}{ll} \min & c^\top y \\ & y \in X, \quad \sum_i y_i = \xi \\ & y \text{ coincides with } x \text{ on } 1,\dots,\tau \end{array}\right.$$

  - 2nd stage model: same as 1st stage but with smaller horizon
  - fine operational modeling vs difficult to compute
  - complexity of $c(x,\xi)$ only allows for simple modeling of randomness

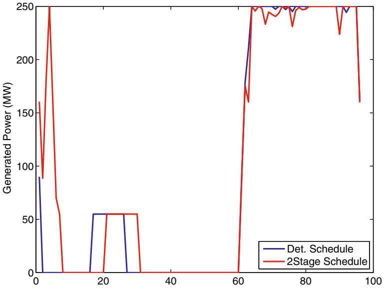
- New algo: double decomposition (by units and scenarios) using the same ingredients

# Numerical illustration for stochastic unit-commitment

- On a 2013 EDF instance (medium-size)
  - deterministic problem : 50k continuous variables, 27k binary variables, 815k constraints
  - stochastic version (50 scenarios) : 1,200k continuous var., 700k binary var., 20,000k constraints
- Our method allows to solve it ☺ (in reasonable time)
- Observation: generation transferred from cheap/inflexible to expensive/flexible
- Example: production schedules for 2 units: determinist vs stochastic




cheap/inflexible unit (nuclear)

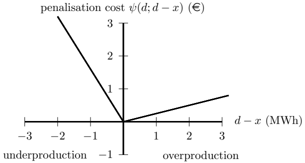



expensive/flexible unit (gaz)

# Robust unit-commitment

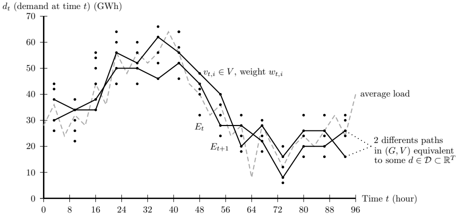
A simple robust approach
(VanAckooij Lebbe Malick '15)

- get rid of bound constraint
- penalize instead the worst gap




$$\boxed{\begin{cases} \min\limits_{x \in X} & c^\top x + \max\limits_{\xi \in \Xi} \sum_{t=1}^{T} \psi\left(\sum_i x_i^t - \xi^t\right) \end{cases}}$$

Complex model of uncertainty set $\Xi$ (vs $\Xi$ finite or $\Xi = [d_{\min}, d_{\max}]^T$)




The model of Minoux 2012

- is finite but of high cardinality
- expresses temporal dependencies
- preserves a fast computability

# WDRO objective to be minimized

Dual WDRO is nonsmooth (which complicates resolution [Kuhn *et al.* '18])

$$R_\rho(f) = \min_{\lambda \geqslant 0} \; \lambda\rho + \mathbb{E}_{\mathbb{P}}[\max_{\xi'} \{f(\xi') - \lambda\|\xi - \xi'\|^2\}]$$

What about smoothing ? Smoothed counterpart

$$R_\rho^\varepsilon(f) = \min_{\lambda \geqslant 0} \; \lambda\rho + \varepsilon\,\mathbb{E}_{\mathbb{P}} \log\left(\mathbb{E}_{\xi' \sim \pi_0(\cdot|\xi)} e^{\frac{f(\xi') - \lambda\|\xi - \xi'\|^2}{\varepsilon}}\right)$$

(Nice interpretation as entropy-regularized WDRO)

**Theorem (approximation bounds for WDRO [Azizian, Iutzeler, M. '21])**

*Under mild assumptions* (*non-degeneracy, lipschitz*)*, if the support of* $\mathbb{P}$ *is contained in a compact convex set* $\Xi \subset \mathbb{R}^d$*, then*

$$0 \;\leqslant\; R_\rho(f) - R_\rho^\varepsilon(f) \;\leqslant\; \left(C\,\varepsilon \,\log\frac{1}{\varepsilon}\right)d$$

# Entropic regularization: OT vs. WDRO

KL (Kullback-Leiber) divergence: $\mathrm{KL}(\mu|\nu) = \begin{cases} \int \log \frac{\mathrm{d}\mu}{\mathrm{d}\nu}\, \mathrm{d}\mu & \mu \ll \nu \\ +\infty & \text{otherwise} \end{cases}$

**OT**: Sinkhorn distance, very popular from [Cuturi '13]

$$\min_{\boldsymbol{\pi}} \left\{ \mathbb{E}_{\boldsymbol{\pi}}[\|\xi - \xi'\|^2] + \varepsilon\, \mathrm{KL}(\boldsymbol{\pi}|\pi_0) : \boldsymbol{\pi} \text{ with marginals } [\boldsymbol{\pi}]_1 = \mathbb{P} \text{ and } [\boldsymbol{\pi}]_2 = \mathbb{Q} \right\}$$

**WDRO**: entropic regularization, seemingly new [Azizian, Iutzeler, M. '21]

$$\begin{cases} \max_{\boldsymbol{\pi}} \quad \mathbb{E}_{[\boldsymbol{\pi}]_2}[f(\xi)] - \varepsilon\, \mathrm{KL}(\boldsymbol{\pi}|\pi_0) \\ \quad [\boldsymbol{\pi}]_1 = \mathbb{P} \\ \quad \mathbb{E}_{\boldsymbol{\pi}}[\|\xi - \xi'\|^2] + \delta\, \mathrm{KL}(\boldsymbol{\pi}|\pi_0) \leqslant \rho \end{cases}$$

Subtility: in **OT**, take $\pi_0 = \mathbb{P} \otimes \mathbb{Q}$

$[\boldsymbol{\pi}]_1 = \mathbb{P}, [\boldsymbol{\pi}]_2 = \mathbb{Q} \Rightarrow \boldsymbol{\pi} \ll \pi_0$

vs

but in **WDRO**, $[\pi_0]_2$ not fixed !

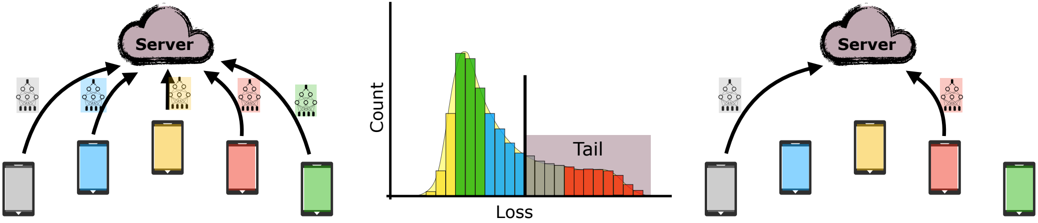
$\pi_0(\mathrm{d}\xi, \mathrm{d}\xi') \;\propto\; \mathbb{P}(\mathrm{d}\xi)\, \mathbb{I}_{\xi' \in \Xi} e^{-\frac{\|\xi - \xi'\|^2}{\sigma}}\, \mathrm{d}\xi'$

# DRO/superquantile in action in federated learning

Only step 3 differs between Standard ERM approach and our DRO approach

*Step 3 of 3: Aggregate updates contributed by **all clients***

*Step 3 of 3: Aggregate updates contributed by **tail clients** only*




DRO approach is fully compatible with secure aggregation and differential privacy [Pillutla, Laguel, M., Harchaoui '22]

# Convergence analysis

Analysis when $F_i$ are smooth (and nonconvex)

Challenges: non-smoothness of $R_\theta$, biais due to local participation,...

**Theorem ([Pillutla, Laguel, M., Harchaoui '23])**

*Suppose $F_i$ are $G$-Lipschitz and with gradients $L$-Lipshitz*

$$\mathbb{E}\|\nabla \Phi_\theta^{2L}(x_t)\|^2 \leqslant \sqrt{\frac{\Delta L G^2}{t}} + (1-\tau)^{1/3}\left(\frac{\Delta L G}{t}\right)^{2/3} + \frac{\Delta L}{t}$$

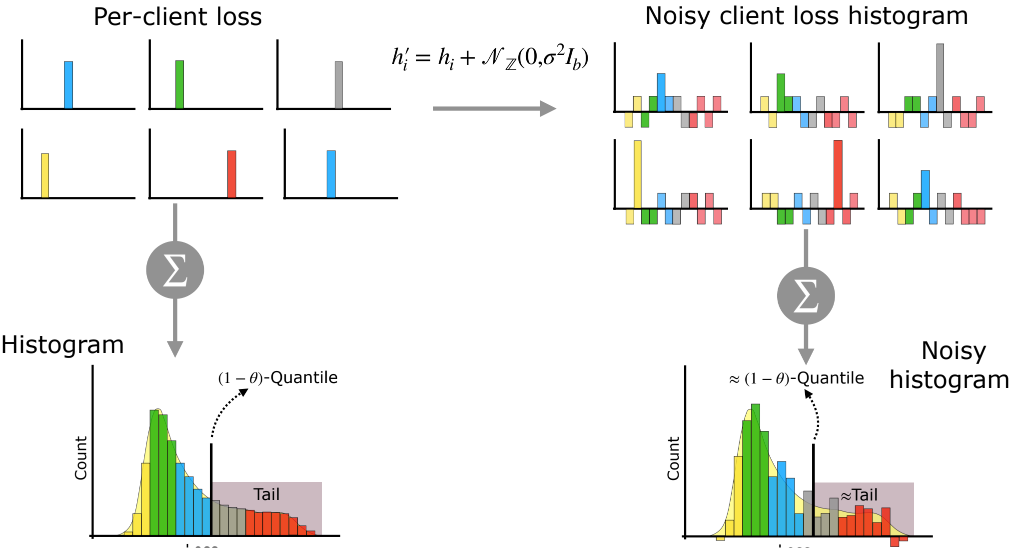
*with $t$: nb comm. rounds, $\tau$: nb local updates, and $\Delta$: initial error*

where $\Phi_\theta^\mu(x) = \inf_y \left\{\bar{R}_\theta(y) + \frac{\mu}{2}\|y-x\|^2\right\}$ (Moreau♡ enveloppe) [Davis Drus. '21]

$\bar{R}_\theta$ an approximation of $R_\theta$ with unbiased gradient [Levy *et al* '21]

+ result of linear convergence when $F_i$ are convex (add smoothing and regularization)

# Quantile by secure aggregation



# Existing statistical guarantees of WDRO

- Suppose $\xi_1, \ldots, \xi_N \sim \mathbb{P}_{\text{train}}$ (where $\xi \in \mathbb{R}^d$)
- Computations with $\widehat{\mathbb{P}}_N = \frac{1}{N}\sum_{i=1}^{N} \delta_{\xi_i}$ and guarantees with $\mathbb{P}_{\text{train}}$ ?
- We manipulate the WDRO risk : $R_\rho(x) = \max\limits_{W(\widehat{\mathbb{P}}_N, \mathbb{Q}) \leqslant \rho} \mathbb{E}_{\mathbb{Q}}[f(x, \xi)]$
- Obviously, if $\rho, N$ large enough such that $W(\mathbb{P}_{\text{train}}, \widehat{\mathbb{P}}_N) \leqslant \rho$, then

$$\underbrace{R_\rho(x)}_{\text{can compute \& optimize}} \geqslant \underbrace{\mathbb{E}_{\mathbb{P}_{\text{train}}}[f(x, \xi)]}_{\text{cannot access}}$$

- It requires $\rho \propto 1/\sqrt[d]{N}$ [Fournier and Guillin '15] (issue)
- Not optimal: $\rho \propto 1/\sqrt{N}$ suffices
  - asymptotically [Blanchet *et al* '22]
  - in particular cases [Shafieez-Adehabadeh *et al* '19]
  - or with error terms [Gao '22]

# Extended exact generalization guarantees of WDRO

Our approach: a direct "optim." approach (work to get a concentration on the dual function)

**Theorem ([Azizian, Iutzeler, M. '23], [Le, M. '24])**

*Assumptions: parametric family $f(\theta,\cdot)$ + compactness on $\theta$ + compactness on $\xi$ + non-degeneracy*

*For* $\delta \in (0,1)$, *if* $\rho \geqslant O\left(\sqrt{\frac{\log 1/\delta}{N}}\right) = \rho_n$ *then* *w.p.* $1-\delta$,

*Generalization guarantee:* $R_\rho(x) \geqslant \mathbb{E}_{\mathbb{P}_{train}}[f(x,\xi)]$

*Distribution shifts:*

$$W(\mathbb{P},\mathbb{Q})^2 \leqslant \rho(\rho-\rho_n) \quad \textit{it holds} \quad R_\rho(x) \geqslant \mathbb{E}_{\mathbb{Q}}[f(x,\xi)]$$

*Asymptotic tightness:*

$$W(\mathbb{P},\mathbb{Q})^2 \leqslant \rho(\rho+\rho_n) \quad \textit{it holds} \quad R_\rho(x) \leqslant \max_{\mathbb{Q}} \mathbb{E}_{\mathbb{Q}}[f(x,\xi)]$$

- Universal result: deep learning, kernels, family of invertible mappings (e.g. normalizing flows)
- Retrieve existing results in linear/logistic regressions [Shafieez-Adehabadeh *et al* '19]