LEVEL SET EVOLUTION WITH LOCALLY LINEAR CLASSIFICATION FOR IMAGE SEGMENTATION

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ABSTRACT

This paper presents a novel local region-based level set model for image segmentation. In each local region, we define a locally weighted least squares energy to fit a linear classification function. The local energy is then integrated over the entire image domain to form an energy functional in terms of level set function. The energy minimization is achieved by level set evolution and estimation of parameters of the locally linear function in an iterative process. By introducing the locally linear functions to separate background and foreground in local regions, our model not only ensures the accuracy of the segmentation results, but also be very robust to initialization. Experiments are reported to demonstrate the effectiveness and efficiency of our model.

Index Terms— linear classification, active contour model, level set methods

1. INTRODUCTION

In the past few years, a number of works on geometric active contours , which are implemented via level set methods, have been proposed to address several problems in computer vision, such as image segmentation, visual tracking and image denoising. Geometric active contours, which were introduced by Malladi et al. [1], are build on curve evolution theory and level set methods. The basic idea is to represent a contour as the zero level set of a higher dimensional level set function, and formulate the motion of the contour as the evolution of the level set function.

Existing active contour models can be broadly categorized into two classes: the edge-based methods [2][3] and the region-based methods [4][5]. Edge-based methods utilize image gradients to guide the evolution of the level set function. For example, the popular GAC model [3] constructs an edge stopping function to attract the active contour to the object boundaries. Unfortunately, the edge-based methods have several drawbacks, such as sensitive to image noise and weak edges. To prevent these drawbacks, region-based methods utilize the region descriptor, such as intensity, to guide contour evolution. One of the most well-known and widely used region-based active contour model [4] utilizes Mumford-Shah segmentation techniques to achieve binary phase segmentation. It assumes that the image region is statistically homogeneous. Zhang et al. [5] proposed a new region-based signed pressure force function to stop the zero level set at weak edges. Compared with the edge-based methods, the region-based models have two advantages. First, region-based models are more robust to image noise and have higher segmentation accuracy for images with weak edges. Second, they are less sensitive to the placement of initial curve. However, region-based models using global statistics may fail to segment the images with intensity inhomogeneity. In practice, intensity inhomogeneity occurs in many real-world images.

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Recently, local region-based methods [6][7][8] have been developed to handle intensity inhomogeneity. Li et al. [6] proposed a local binary fitting (LBF) energy, while Zhang et al. [7] introduced a local image fitting (LIF) energy. By extracting local image information, these methods are able to segment images with intensity inhomogeneities. Motivated by the LBF model [6], Gaussian distribution was applied to describe the local image intensities [8], then the local Gaussian distribution fitting (LGF) energy is presented to guide the evolution of the level set function. Although, local region-based methods have better performance than region-based methods and edge-based methods in segmentation accuracy, they have some drawbacks. They are very sensitive to the initial contour and easy to produce error segmentations.

Actually, a key task of local region-based models is to choose an appropriate model to separate the background and foreground in local region. Motivated by previous works [5][6][8], we propose a new locally linear classification (LLC) based model. First, we introduce an approximation *sign* function to turn the level set to the class label. In each local region, the locally linear function is fitted by a locally weighted squares energy. This energy is then integrated over the entire image domain to form a energy functional in terms of the level set function. Finally, this energy functional is minimized via a level set evolution process. Comparative experiments indicate that our algorithm has the following two main advantages:

1. Our model is a local region-based model. It assumes that local region is linearly separable. Compared with region-based methods, such as [5], the proposed method can yields higher accuracy segmentation results, especially when image is inhomogenous.
2. LBF and LGF can be regarded as local mixture models which belong to generative model, while our LLC model adopts linear classification model which belongs to discriminative model. Generally, discriminative model is more robust to model misspecification than generative model. Thus, our method is more robust to initialization than LBF and LGF.

The remainder of this paper is organized as follows. Section 2 describes our model and its level set formulation. Section 3 reports the experimental results. Conclusions are drawn in Section 4.

2. THE PROPOSED MODEL

In this section, we first introduce the weighted least squares method for locally linear classification. Then, the locally linear functions are integrated into the level set framework. Finally, we presents the implementation details of our model.

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2.1. Weighted least squares for classification

Let $\Omega \in \mathcal{R}^2$ be the image domain, and $I : \Omega \to \mathcal{R}$ be a given gray level image. Let $\mathcal{L} \in \{-1, +1\}$ be the class label, $T : \Omega \to \mathcal{L}$ be the class label in the image domain Ω. For a given point $\mathbf{x} \in \Omega$, we define the following locally linear function

$$f_{\mathbf{x}}(\mathbf{y}) = w_{\mathbf{x}} I(\mathbf{y}) + b_{\mathbf{x}}, \quad \mathbf{y} \in \mathcal{N}(\mathbf{x}), \tag{1}$$

where $w_{\mathbf{x}}$ and $b_{\mathbf{x}}$ are coefficients of the linear function (Fig. 1 (a)), and $\mathcal{N}(\mathbf{x})$ denotes the neighborhood set of point $\mathbf{x}$. Introducing the locally linear classification $f_{\mathbf{x}}$ can effectively separate the background and background of the local region $\mathcal{N}(\mathbf{x})$, which ensures the correctness of dealing images with intensity inhomogeneity and weak edges. The local error function can be written as

$$E_{\mathbf{x}}(w_{\mathbf{x}}, b_{\mathbf{x}}) = \int K_\sigma(\mathbf{y} - \mathbf{x}) \|f_{\mathbf{x}}(\mathbf{y}) - T(\mathbf{y})\|^2 d\mathbf{y}, \tag{2}$$

where K_σ is a Gaussian kernel with standard deviation σ, and $T(\mathbf{y})$ denotes the class label of point $\mathbf{y}$, specifically, $+1$ for foreground and -1 for background. Minimizing the above function (2) is to find the weighted least squares solution for classification. In order to control over-fitting problem, we introduce the regularization term. Then the objective function to be minimized takes the form

$$E_{\mathbf{x}}(w_{\mathbf{x}}, b_{\mathbf{x}}) + \lambda_1 (w_{\mathbf{x}} - w^*)^2 + \lambda_2 (b_{\mathbf{x}} - b^*)^2, \tag{3}$$

where λ_1, λ_2 are regularization coefficients, and w^*, b^* are prior value for $w_{\mathbf{x}}, b_{\mathbf{x}}$. Setting the derivative of (3) with respect to $w_{\mathbf{x}}$ and $b_{\mathbf{x}}$ to zero, we obtain $w_{\mathbf{x}}, b_{\mathbf{x}}$ by the following equations:

$$\begin{aligned} w_{\mathbf{x}} &= \frac{\lambda_1 w^* + F(\mathbf{x}) - \frac{1}{1+\lambda_2} \bar{I}(\mathbf{x}) \left(\bar{T}(\mathbf{x}) + \lambda_2 b^*\right)}{D(\mathbf{x}) - \frac{1}{1+\lambda_2} \bar{I}^2(\mathbf{x}) + \lambda_1}, \\ b_{\mathbf{x}} &= \frac{1}{1+\lambda_2} \left(\bar{T}(\mathbf{x}) - w_{\mathbf{x}} \bar{I}(\mathbf{x}) + \lambda_2 b^*\right), \end{aligned} \tag{4}$$

where $D = (I \circ I) \otimes k$, $\bar{I} = I \otimes k$, $F = (I \circ T) \otimes k$ and $\bar{T} = T \otimes k$[1]. Here k is a discrete convolution kernel of the Gaussian kernel K_σ.

2.2. Level set formulation

The local error function (2) can be written as an energy functional in terms of a level set function. Suppose $\phi : \Omega \to \mathcal{R}$ denotes the level set function, and $\phi > 0$ denotes the foreground "+1", and $\phi < 0$ denotes the background "−1". In each point $\mathbf{y}$, its class label can be obtained by $T(\mathbf{y}) = sgn(\phi(\mathbf{y}))$, where sgn is the sign function which is approximated by $\tanh$ function (Fig. 1(b))

$$T = sgn(\phi) = \tanh(\kappa\phi), \tag{5}$$

where κ is a positive constant. The derivative of $sgn(\phi)$ with respect to ϕ is

$$sgn'(\phi) = \frac{\partial sgn(\phi)}{\partial \phi} = \frac{\kappa}{\cosh^2(\kappa\phi)}. \tag{6}$$

Based on the description in Section 2.1, the total error energy function of the entire image $\mathcal{I}$ in domain Ω can be evaluated as

$$\begin{aligned} \mathcal{E}(\phi, w, b) &= \int E_{\mathbf{x}}(w_{\mathbf{x}}, b_{\mathbf{x}}) d\mathbf{x} \\ &= \int \left(\int K_\sigma(\mathbf{y} - \mathbf{x}) \|f_{\mathbf{x}}(\mathbf{y}) - T(\mathbf{y})\|^2 d\mathbf{y} \right) d\mathbf{x}. \end{aligned} \tag{7}$$

[1] $\otimes$ denotes the convolution operation and $\circ$ denotes element-by-element multiplication.

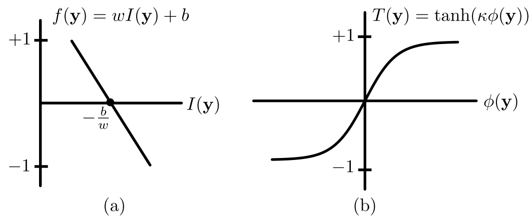

Fig. 1. (a) the linear function; (b) the approximation of the sign function

The derivative of the functional $\mathcal{E}$ with respect to ϕ is written as

$$\frac{\partial \mathcal{E}}{\partial \phi(\mathbf{y})} = sgn'(\phi(\mathbf{y})) \int (K_\sigma(\mathbf{x} - \mathbf{y})(T(\mathbf{y}) - f_{\mathbf{x}}(\mathbf{y}))\, d\mathbf{x}, \tag{8}$$

which can be formulated as discrete convolution form

$$\frac{\partial \mathcal{E}}{\partial \phi} = sgn'(\phi)\big((T - f) \otimes k\big). \tag{9}$$

To preserve the evolving level set function as an approximate signed distance function during the evolution, we introduce a level set regularization term which was proposed in [9].

$$\mathcal{P}(\phi) = \int \frac{1}{2} (\|\nabla \phi(\mathbf{y})\| - 1)^2 d\mathbf{y}. \tag{10}$$

The gradient of the functional $\mathcal{P}$ with respect to ϕ takes the form

$$\frac{\partial \mathcal{P}}{\partial \phi} = -\Delta\phi + \operatorname{div}\left(\frac{\nabla\phi}{\|\nabla\phi\|}\right). \tag{11}$$

Combining the error energy fuctional (7) and the regularization term (10), we define the entire energy functional as follows:

$$\mathcal{F}(\phi) = \mu\mathcal{E}(\phi) + \nu\mathcal{P}(\phi), \tag{12}$$

where μ and ν are positive constants. This is the energy functional which will be minimized to drive the evolution of the level set function.

2.3. Implementation and parameters settings

To minimize the energy functional (12), we adopt gradient descent method. For a fixed level set ϕ, we use the defined sign function (5) to turn the level set function to class label. Then, the locally linear classification function is solved by weighted least squares method (see Section 2.1). Specifically, for each locally linear function, its parameters $w_{\mathbf{x}}$ and $b_{\mathbf{x}}$ are given by Eqn. (4).

Keeping w and b fixed, and minimizing the energy functional $\mathcal{F}$ with respect to ϕ, we can derive the gradient descent flow:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial \mathcal{F}}{\partial \phi} = -\mu \frac{\partial \mathcal{E}}{\partial \phi} - \nu \frac{\partial \mathcal{P}}{\partial \phi}. \tag{13}$$

Thus, the minimization of energy functional $\mathcal{F}$ can be achieved by two-stage iterative optimization. First, minimizing the regularization energy (3) with respect to locally linear functions, keeping the level set function fixed. Second, minimizing the entire energy functional $\mathcal{F}$ with respect to level set function, keeping the locally linear functions fixed. This two-stage optimization is repeated until convergence. The detail steps of the proposed method are illustrated in Algorithm 1.

Input : input image $\mathcal{I}$, maximum iteration number N and initialization level set function ϕ_0

Output: level set function ϕ and segmentation results of $\mathcal{I}$

Initial the level set function $\phi^i = \phi_0$

for $i \leftarrow 1$ **to** N **do**

    $T = sgn(\phi^i)$;

    For each point $\mathbf{x}$, solve $w_{\mathbf{x}}, b_{\mathbf{x}}$ by Eqn. (4);

    Calculate the gradient descent flow $\frac{\partial \phi}{\partial t}$ by Eqn. (13);

    Evolve the level set function $\phi^{i+1} = \phi^i + \eta \frac{\partial \phi}{\partial t}$;

    if $\phi^{i+1} == \phi^i$ **then**

        break iteration;

    end

end

Output level set function $\phi = \phi^{i+1}$, obtain the segmentation according to the resultant level set function ϕ.

Algorithm 1: Level set evolution with locally linear classification for image segmentation

Note that the main computational cost in our method is to compute the parameters w, b in Eqn. (4) and the gradient descent flow in Eqn. (13). Both of the computations involve the convolution operations. To accelerate the computation, we can use efficient FFT to compute the convolution operations. The variables D and $\bar{I}$ in Eqn. (4) which are unchanged during the iterative process can be precomputed before the iteration.

The locally linear function defined in Eqn. (1) map the input gray-scale value to be the class label. From Fig. 1(a), we see that $-\frac{b}{w}$ is the zero-crossing point of the linear function. Actually, the zero-crossing point $-\frac{b}{w}$ plays an important role in classification. Suppose $w < 0$, if $I(\mathbf{y}) > -\frac{b}{w}$, then the label of point $\mathbf{y}$ is -1. Alternatively, $I(\mathbf{y}) < -\frac{b}{w}$ gives label $+1$. Let $\boldsymbol{m}$ be the average intensity of the image I. In this work, we set a prior value for the zero-crossing point that $-\frac{b}{w} = \boldsymbol{m}$. In addition, in our implementation, we set $w^* = -0.5$, and thus b^* is set as $b^* = -\boldsymbol{m}w^*$. The regularization coefficients λ_1, λ_2 in Eqn. (3) are selected as $\lambda_1 = \lambda_2 = 0.001$.

The function defined in Eqn. (5) is a smooth approximation of the sign function. As can be seen from Fig. 1(b), κ control the smoothness of the sign function. We choose $\kappa = 2$ in this work.

The standard deviation σ of the Gaussian kernel is set as $\sigma = 2.0$, and we truncate the Gaussian kernel as a $K \times K$ filter kernel ($K = 4\sigma + 1$).

The level set function ϕ can be simply initialized as a binary step function which takes constant -1 for background and $+1$ for foreground. For the weighing constants μ and ν, we use the following parameters: $\mu = 10, \nu = 1$. The time step η is set as $\eta = 0.1$.

3. EXPERIMENTAL RESULTS

We compare our method with Local Binary Fitting (LBF) method [6] and Selective Binary and Gaussian Filtering Regularized (SBGFR) method [5]. These two methods belong to local region-based model and region-based model, respectively. For fair comparison, all of the methods use the same initializations.

Fig. 2 and 3 show the results of X-ray images of blood vessels. As illustrated in these figures, with different initial contours, LBF may fail to segment the objects correctly. Specifically, as shown in Fig. 2, although the initial contours of the first row and the second row are very similar, the segmentation results of LBF are different from each other. It demonstrates that LBF is sensitive to initial contours. SBGFR model isn't sensitive to initial contours as shown in these figure, but it can't obtain accuracy segmentation results. By contrast, the proposed LLC model not only gives the high accuracy results but also obtains almost the same segmentation results with different initial contours. In the second row of Fig. 3, even though the initial contours didn't contrain any foreground objects, our method can still obtain precise segmentation results.

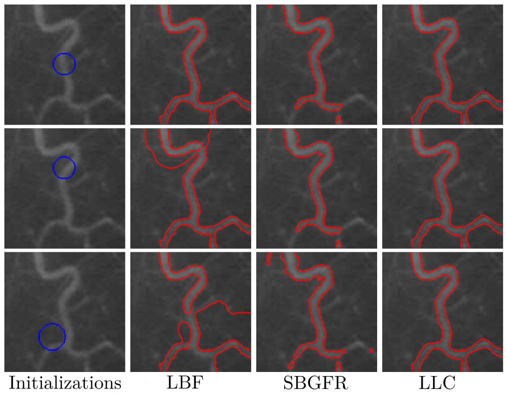

Fig. 2. Segmentation results of a blood vessel image with different initial contours.

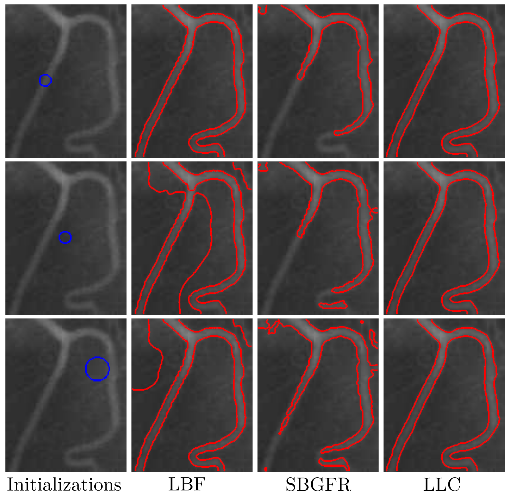

Fig. 3. Segmentation results of a blood vessel image with different initial contours.

In order to further demonstrate the capability of our method, we compare our method with LBF model and SBGFR model for brain MR images with week edges. Fig. 6 shows the segmentation results. As can be seen from the figure, edges between white matter and gray matter are much weaker than those between white matter and background. SBGFR model segments the white matter and gray matter together as foreground objects, and can't segment the details of the white matter (see areas in blue dashed circles). LBF model is sensitive to initial contours, which easily lead to inaccurate segmentation results (see areas in green dashed circles). Compared with LBF model and SBGFR model, our LLC method achieves high accuracy segmentation results with various initial contours.

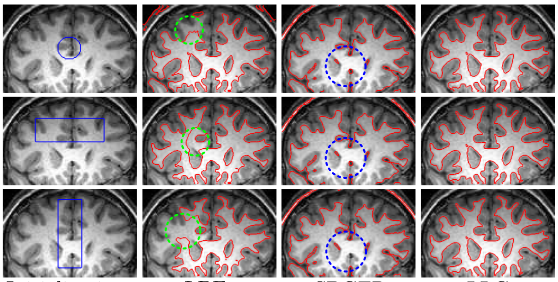

Fig. 4. Segmentation results of a brain image with intensity inhomogeneity.

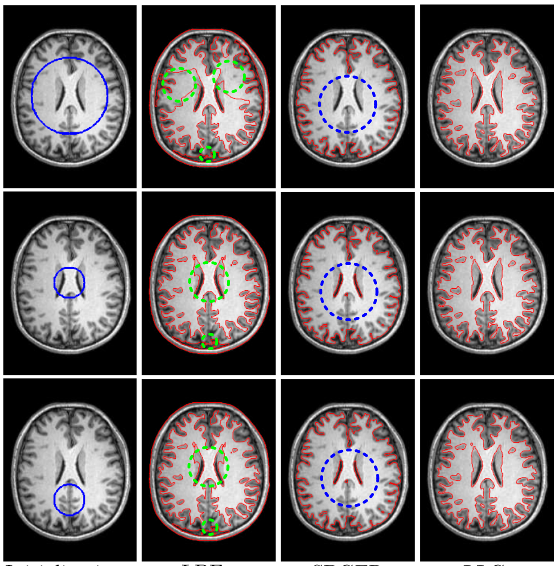

Fig. 5. Segmentation results of a brain image with intensity inhomogeneity.

We also use our method to segment images with intensity inhomogeneity. Fig. 4 and 5 are two MR brain images with intensity inhomogeneity. SBGFR can't segment the details of the white matter (see areas in blue dashed circles). The segmentation results of LBF are better than SBGFR, but there are still some error segmentations (see areas in green dashed circles). Our method successfully segments the white matter in these two images with various initial contours. In summary, all of the above experiments demonstrate that our method is superior in terms of both accuracy and robustness.

4. CONCLUSIONS

In this paper, we propose a locally linear classification based model with a level set formulation for image segmentation. The main contribution of this work lies in that we introduce the locally linear classification into level set framework. Comparative experiments on medical image segmentation show that our method can achieve accurate segmentation results with various initial contours.

In the future, we will apply our model to segment other types of images. In addition, the proposed model utilizes locally linear function to separate the local region. It has promising to adopt nonlinear

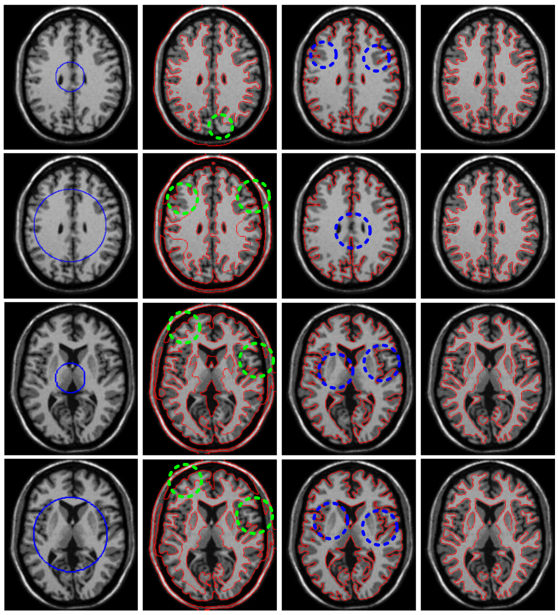

Fig. 6. Segmentation results of brain images with weak edges.

function, for example, splines [10], to further improve the performance ofour proposed approach.

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