Transformations of Attributed Graphs with Cloning

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Universidade Federal do Rio Grande do Sul, Brazil

June 26., 2014 – LJK – Grenoble – Bipop-Casys seminar

- Rewriting / Transformation of:
  - terms
  - graphs
  - attributed graphs

- with algebraic methods:
  - PO (add, merge)
  - DPO (add, merge, delete)
  - SqPO (add, merge, delete, copy)

Outline

Term rewriting

Graph transformation

Attributed graph transformation: FASE 2014

Example: term rewriting

$$1 + 1 = 2\,?$$

Example: term rewriting

$$1 + 1 \rightsquigarrow 2\,?$$

Example: term rewriting

$$1 + 1 \rightsquigarrow 2\,?$$

Specification:

- N
- $0 : N$, $s : N \to N$, $+ : N, N \to N$

Rules:

$(R_0) \colon x + 0 \rightsquigarrow x \qquad (R_1) \colon x + s(y) \rightsquigarrow s(x + y)$

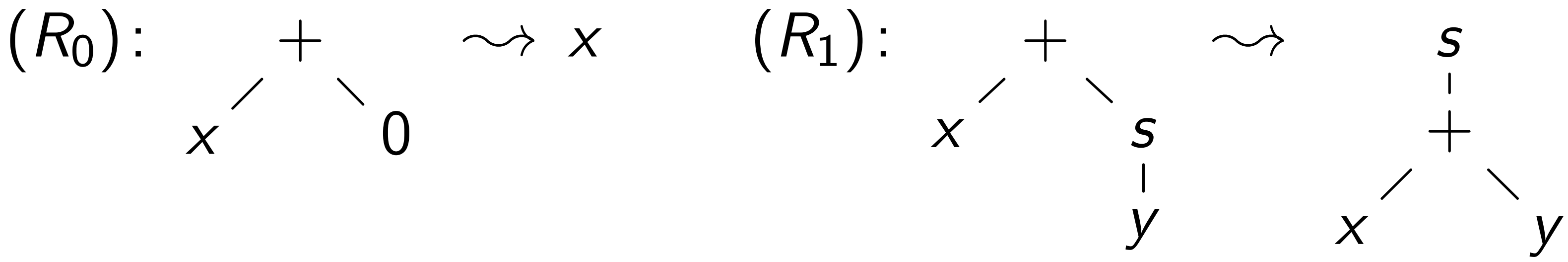
Reduction:

$$s(0) + s(0) \overset{(R_1)}{\rightsquigarrow} s(s(0) + 0) \overset{(R_0)}{\rightsquigarrow} s(s(0))$$

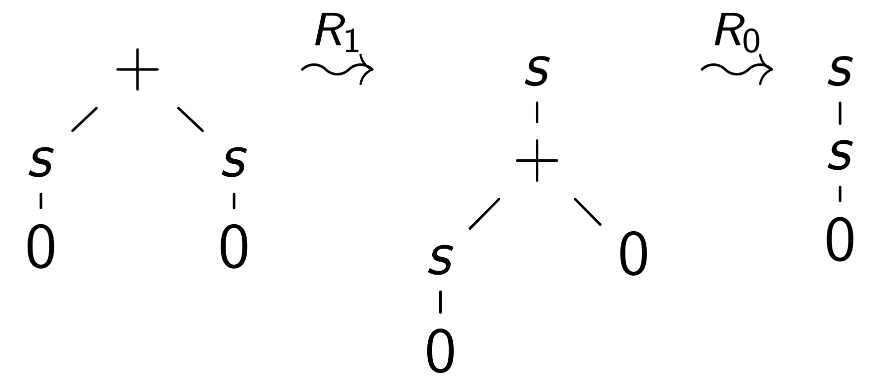
Example: term rewriting

$$1 + 1 \rightsquigarrow 2\,?$$

Rules (dim. 2):



Reduction (dim. 2):



Terms and graphs

A term "is" a tree, and a tree "is" a graph.

However:

- Trees are defined inductively ("generalized" lists):

$$T ::= r \mid TrT$$

- Graphs are defined as "presheaves" ("generalized" sets):

$$\begin{tikzcd} V & E \arrow[l, bend right, "s"'] \arrow[l, bend left, "t"] \end{tikzcd}$$

Consequence:
It is difficult to adapt term rewriting to graphs!

Outline

Term rewriting

Graph transformation

Attributed graph transformation: FASE 2014

Graph transformation

L, R, G, H are graphs.

Given a rewrite rule:

$$L \rightsquigarrow R$$

and a matching:

$$\begin{tikzcd} L \arrow[d, "\subseteq"] \\ G \end{tikzcd}$$

a rewrite step builds H by replacing
the occurrence of L in G
by some occurrence of R in H:

$$\begin{tikzcd} R \arrow[d, "\subseteq"] \\ H \end{tikzcd}$$

Graph transformation

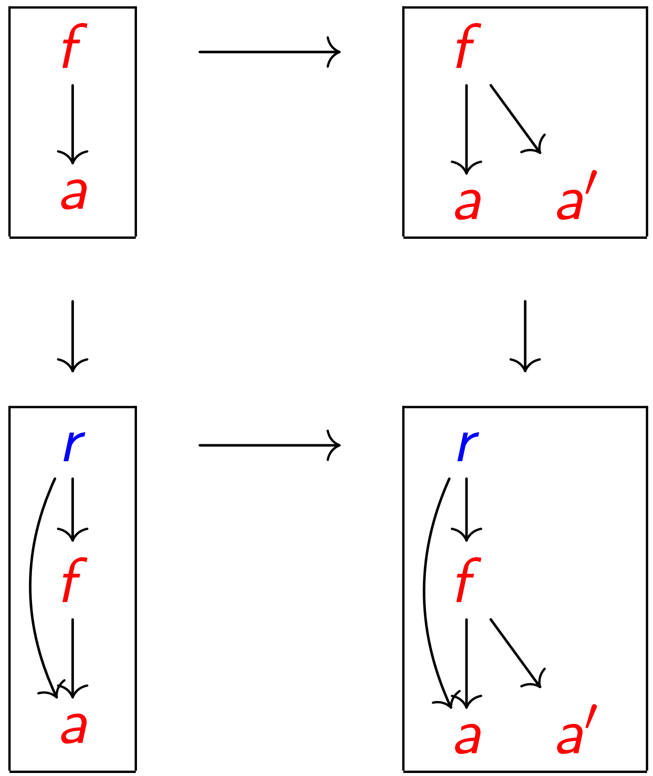
A rewrite step:

$$\begin{tikzcd} \boxed{L} \arrow[r, rightsquigarrow] \arrow[d] & \boxed{R} \arrow[d] \\ \boxed{G} \arrow[r, rightsquigarrow] & \boxed{H} \end{tikzcd}$$

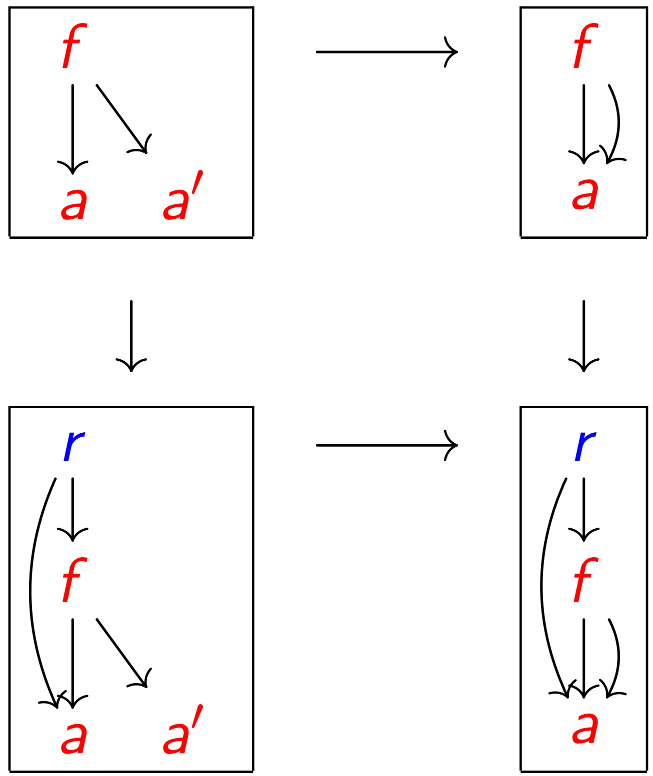
Elementary transformations:

- ADD
- MERGE
- DELETE
- COPY (= clone)

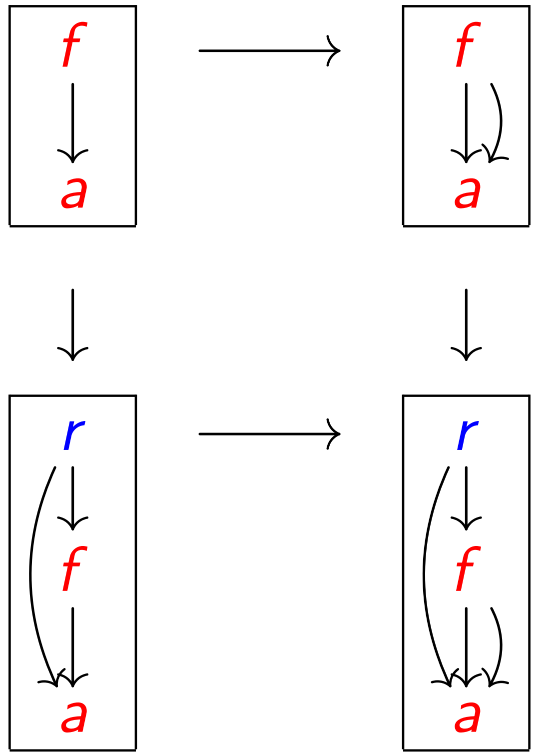
Graph transformation: ADD



Graph transformation: MERGE



Graph transformation: ADD and MERGE



Graph transformation: ADD and MERGE

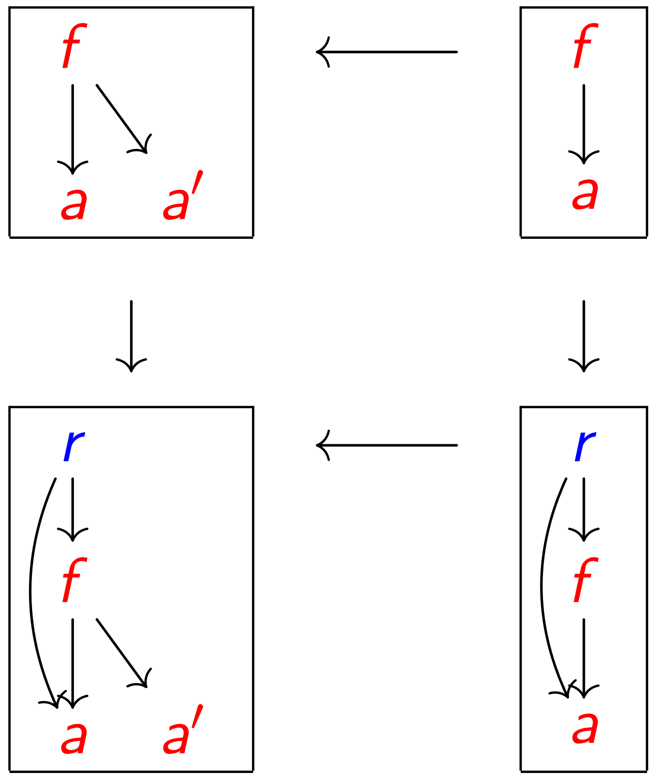
- A rule $L \rightsquigarrow R$ is a graph homomorphism $L \to R$ from L to R
- A step

$$\begin{tikzcd} \boxed{L} \arrow[r, rightsquigarrow] \arrow[d] & \boxed{R} \arrow[d] \\ \boxed{G} \arrow[r, rightsquigarrow] & \boxed{H} \end{tikzcd}$$

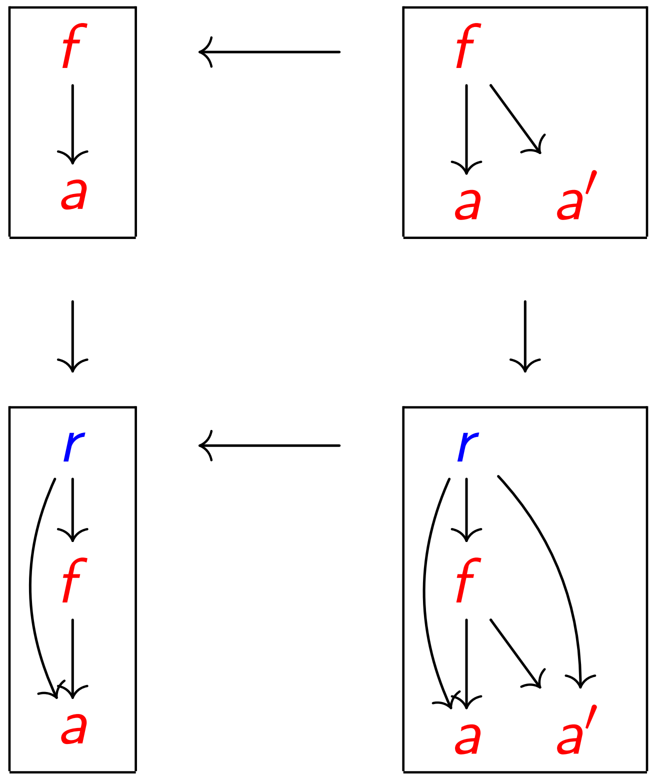
is a pushout (PO, "generalized union")

$$\begin{tikzcd} \boxed{L} \arrow[r] \arrow[d] \arrow[dr, phantom, "\mathrm{PO}"] & \boxed{R} \arrow[d] \\ \boxed{G} \arrow[r] & \boxed{H} \end{tikzcd}$$

Graph transformation: DELETE



Graph transformation: COPY



Graph transformation: DELETE and COPY

- A rule $L \rightsquigarrow R$ is a graph homomorphism $L \leftarrow R$ from R to L
- A step

$$\begin{tikzcd} \boxed{L} \arrow[r, rightsquigarrow] \arrow[d] & \boxed{R} \arrow[d] \\ \boxed{G} \arrow[r, rightsquigarrow] & \boxed{H} \end{tikzcd}$$

is a *?? kind of converse of pushout ??*

$$\begin{tikzcd} \boxed{L} \arrow[d] \arrow[dr, phantom, "??"] & \boxed{R} \arrow[l] \arrow[d] \\ \boxed{G} & \boxed{H} \arrow[l] \end{tikzcd}$$

Algebraic graph transformation

Algebraic graph rewriting is based on:

- some kind of "converse of pushout" (DELETE and COPY)
- followed by a pushout (ADD and MERGE)

$$\begin{tikzcd} L \arrow[d] \arrow[dr, phantom, "??"] & K \arrow[l, "l"'] \arrow[r, "r"] \arrow[d, dashed] \arrow[dr, phantom, "PO"] & R \arrow[d, dotted] \\ G & D \arrow[l, dashed, "l_1"] \arrow[r, dotted, "r_1"'] & H \end{tikzcd}$$

- Double-pushout: DPO: when ??=POC
  - Variant: Single-pushout: SPO
- Sesqui-pushout: SqPO: when ??=FPBC
  - Generalization of DPO and SPO

By: H. Ehrig, U. Montanari, H.J. Kreowski, M. Löwe, A. Corradini, B. König, F. Orejas, L. Ribeiro, T. Heindel, F. Hermann, U. Golas, ...

FASE 2014

D. Duval, R. Echahed, F. Prost.

- TERMGRAPH 2006
- RTA 2007
- RTA 2009
- GT-VMT 2011
- ICGT 2012

D. Duval, R. Echahed, F. Prost, L. Ribeiro.

- FASE 2014

  Fundamental Approaches to Software Engineering

  Transformation of Attributed Structures with Cloning

Outline

Term rewriting

Graph transformation

Attributed graph transformation: FASE 2014

Transformations of Attributed Structures with Cloning

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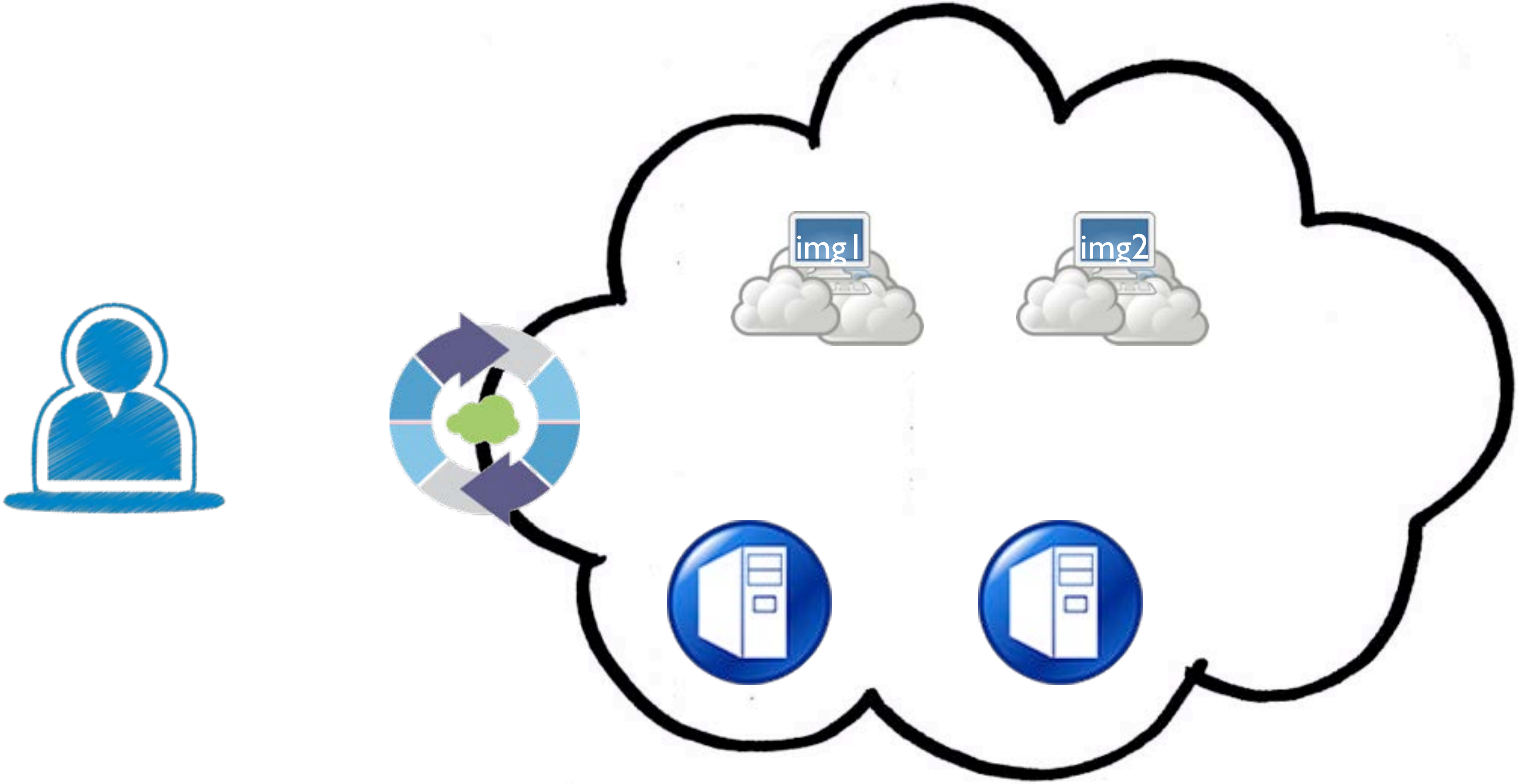
Outline

- Motivation
- Example : Cloud administration
- Attributed Structures
- Sesqui-PO Rewriting of Attributed Structures
- Conclusion and Future work

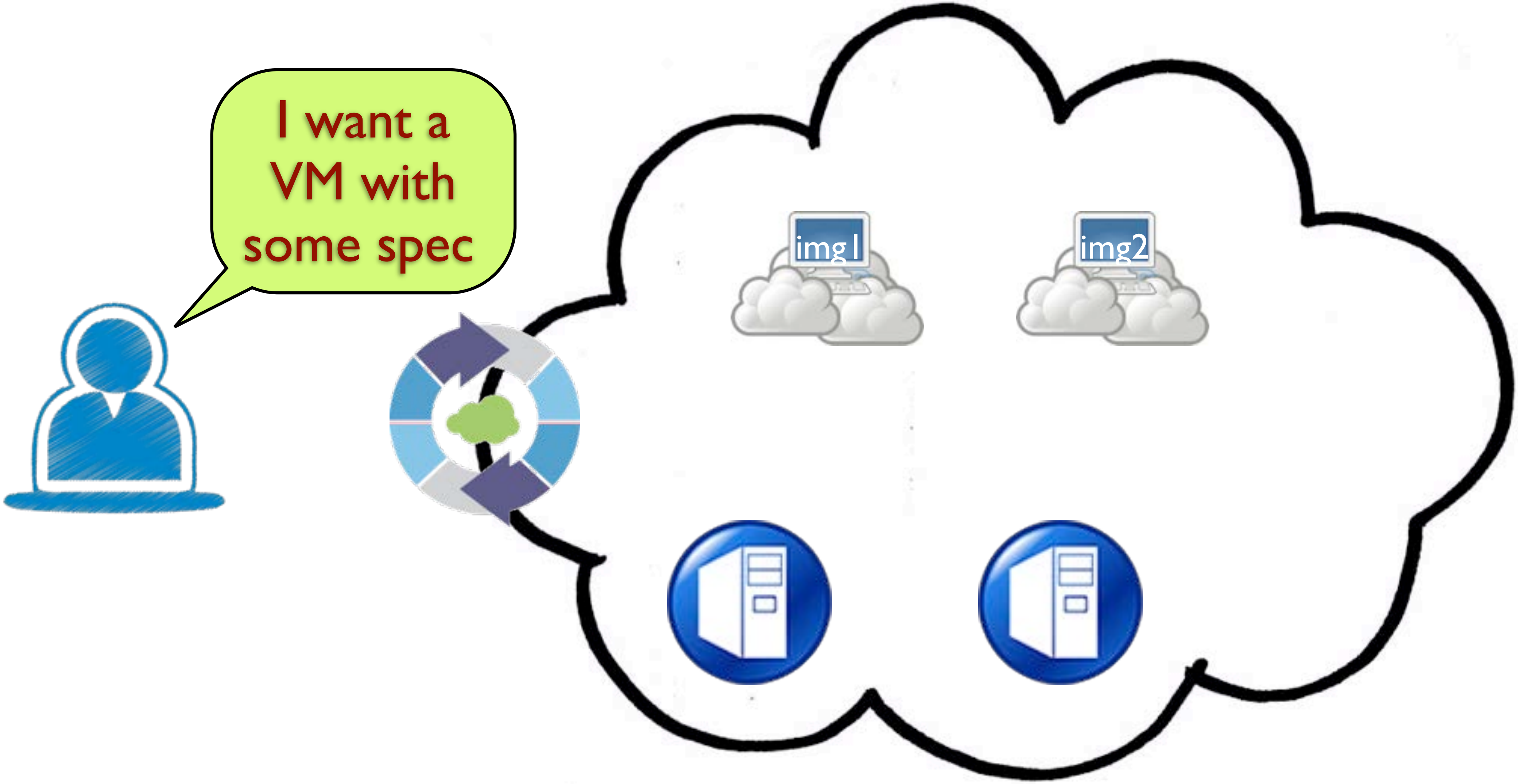
Motivation

- Simple but generic attribute notion
- Cloning possibility

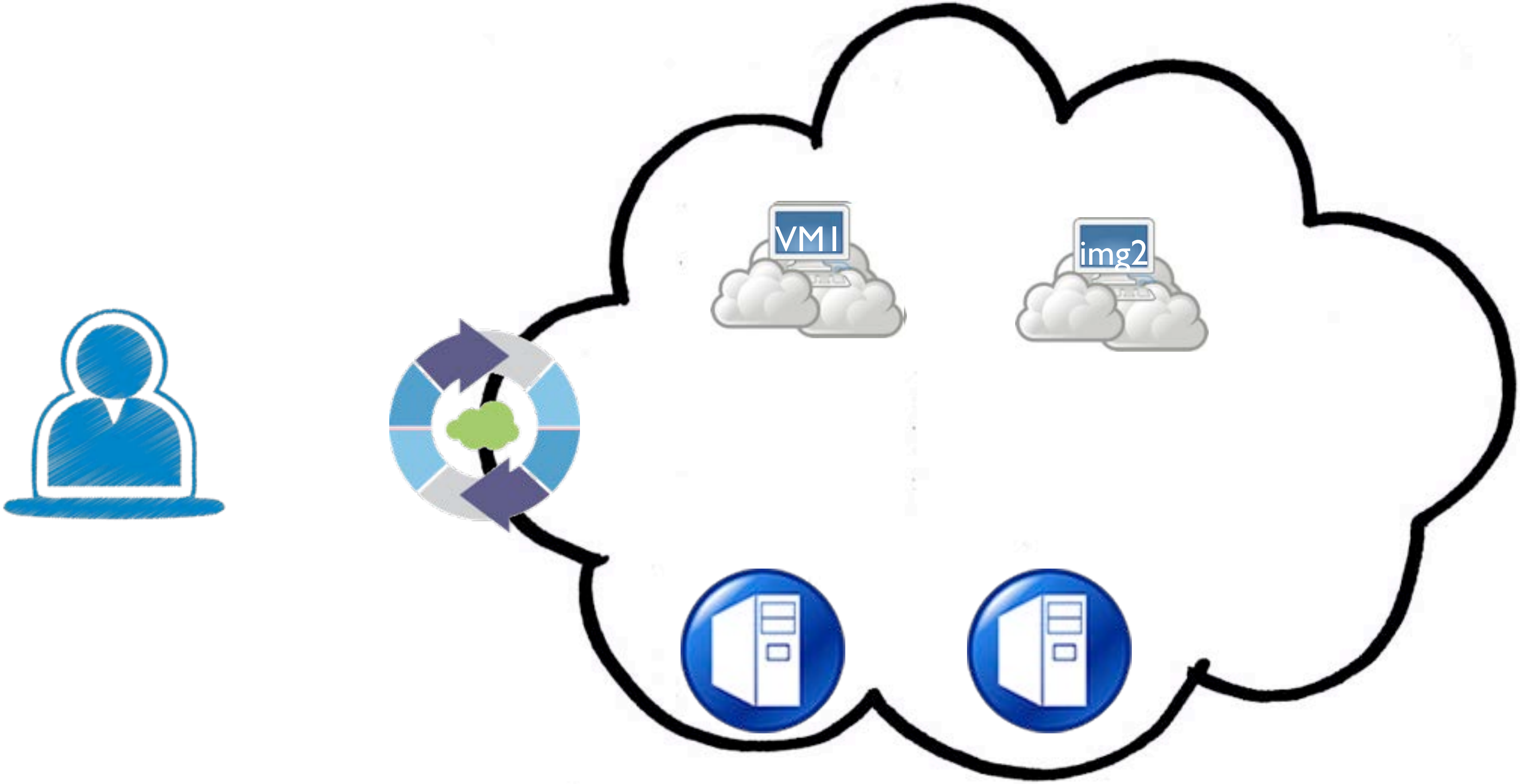
Example: Cloud Adm



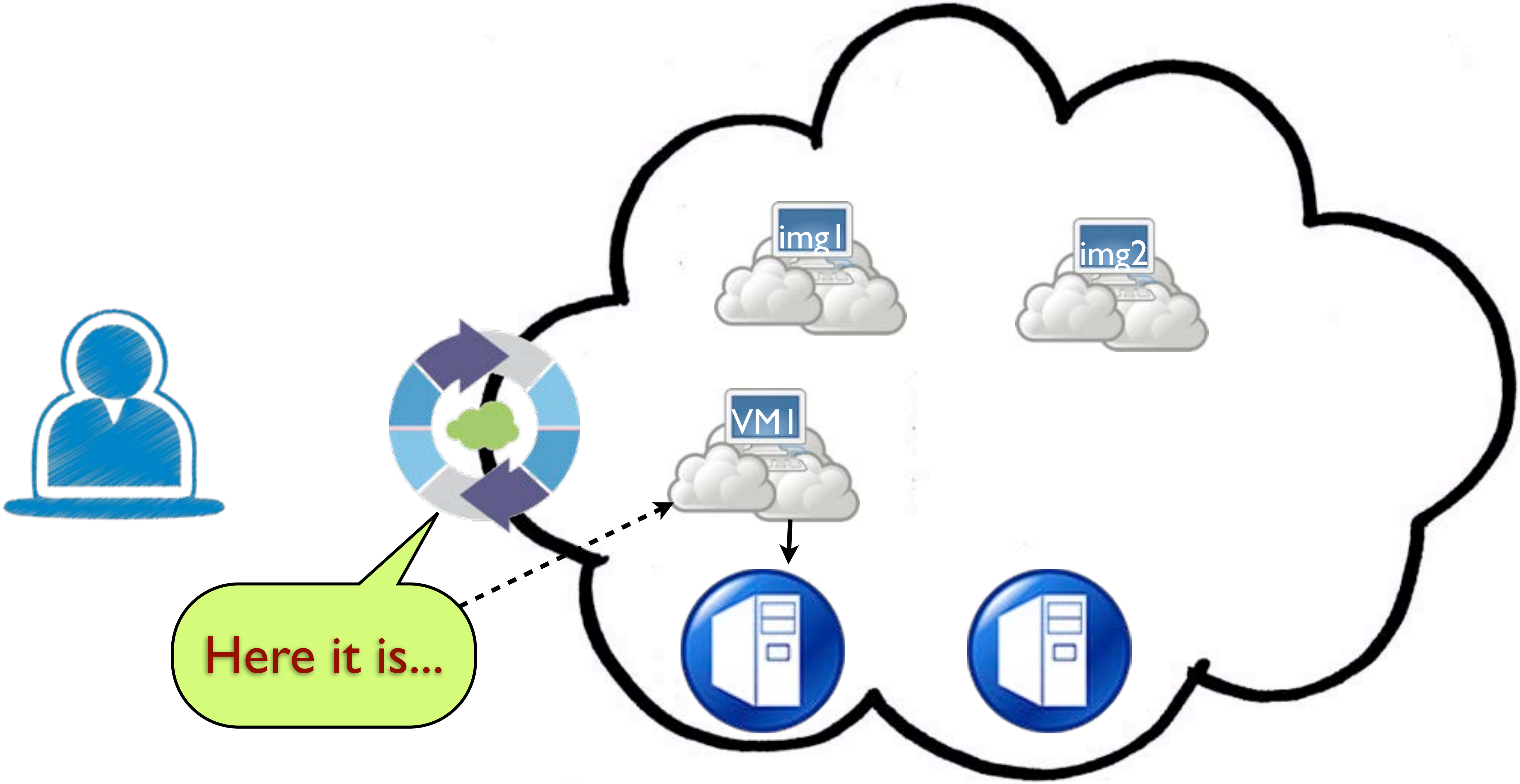
Example: Cloud Adm



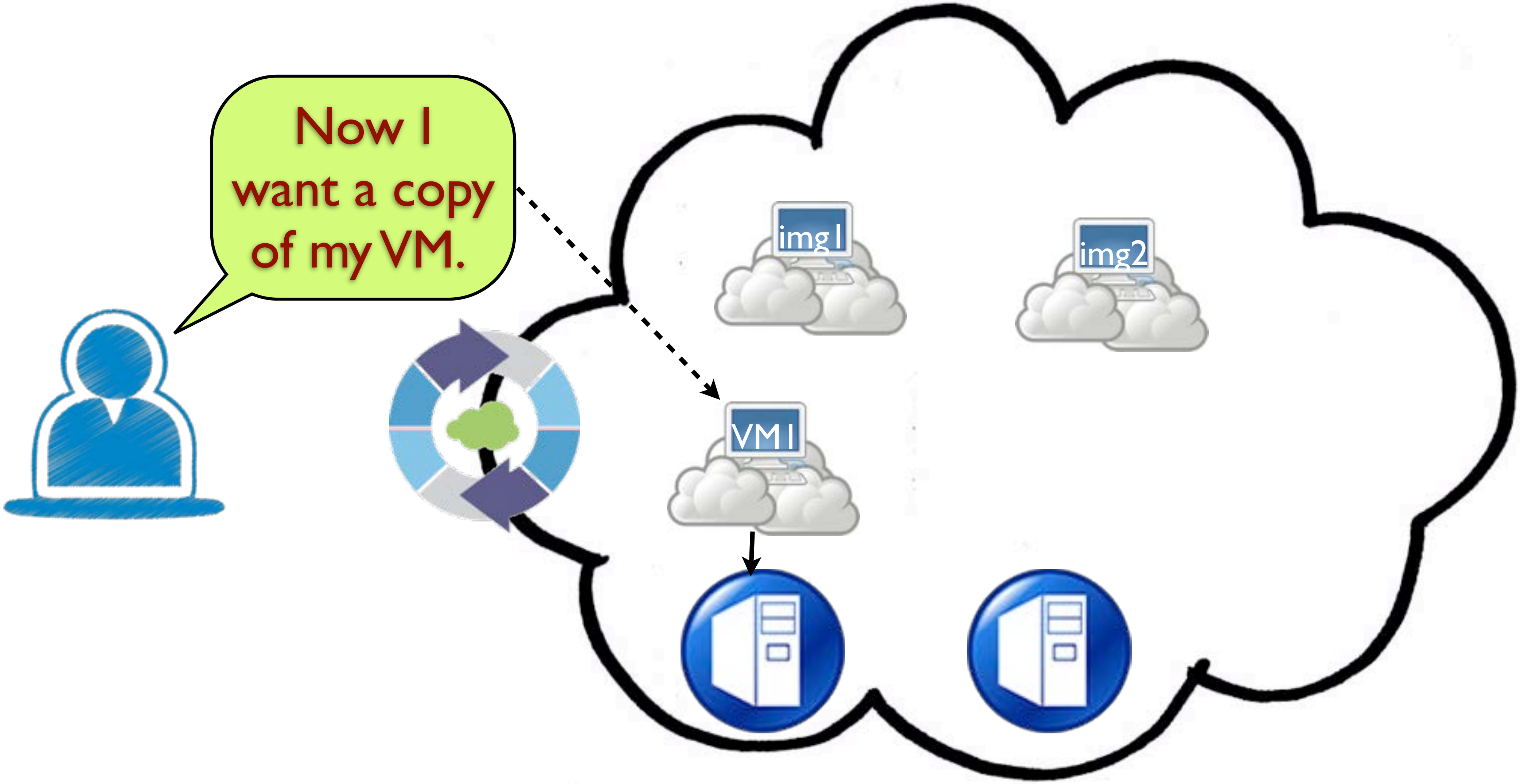
Create VM



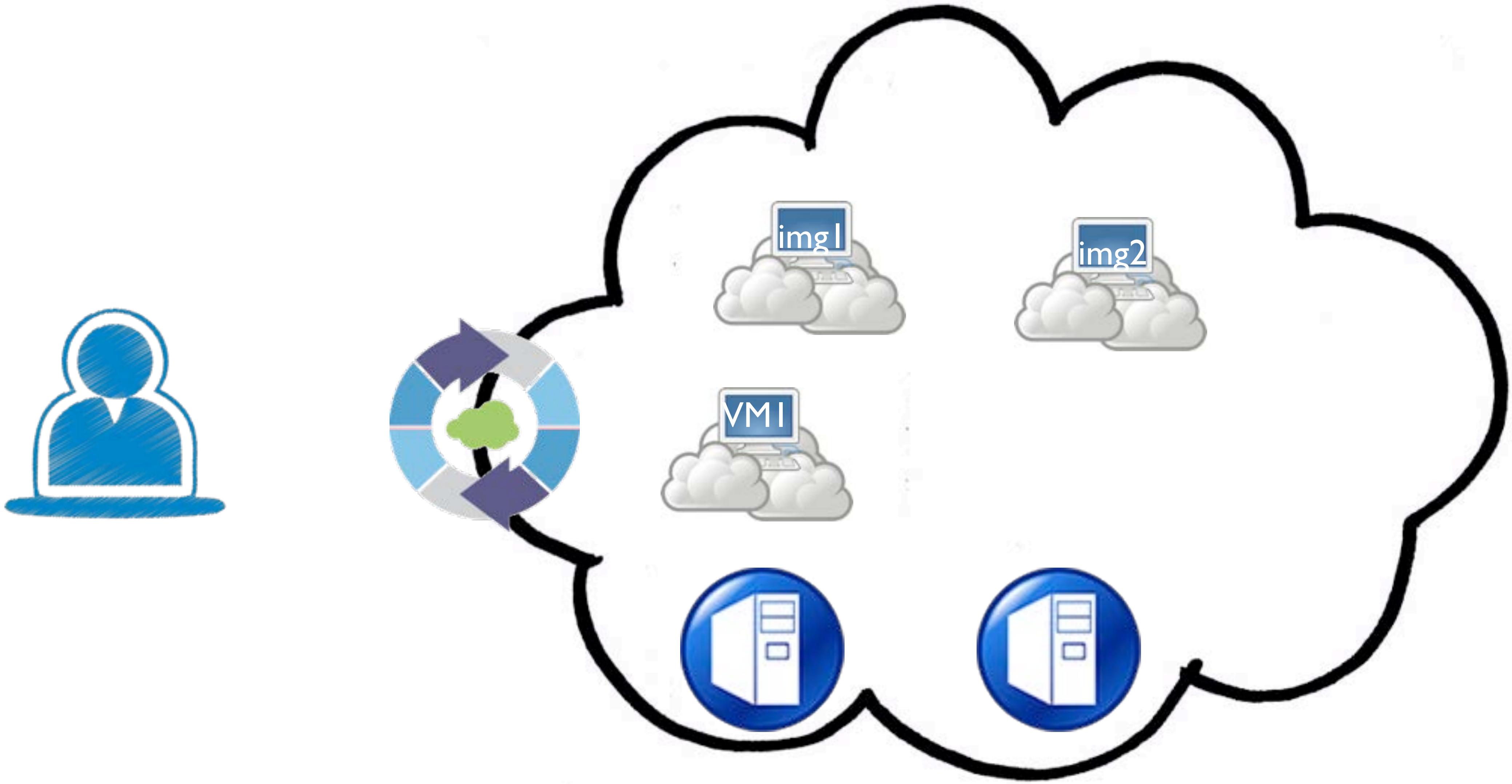
Create VM



New request...



Replicate VM



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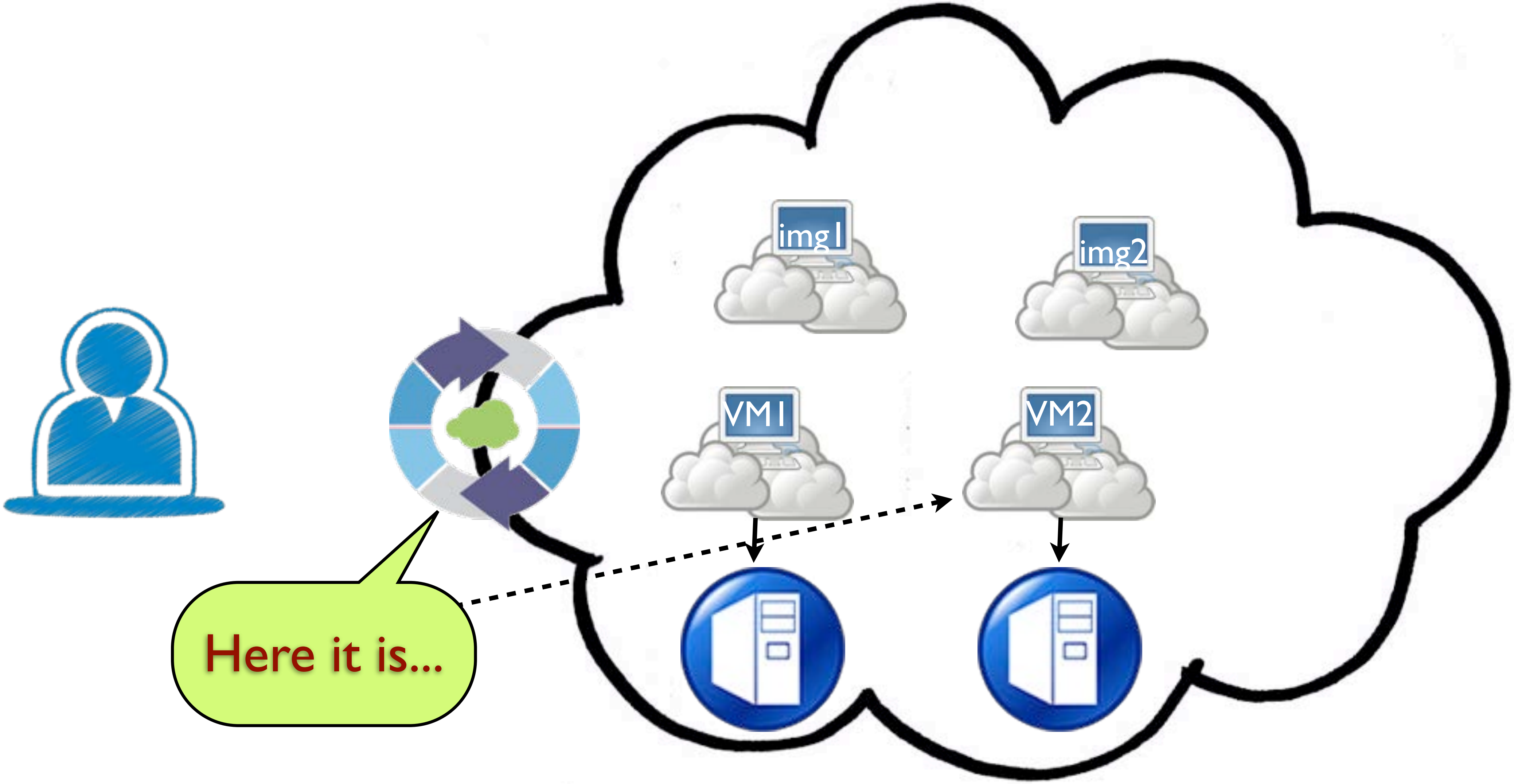

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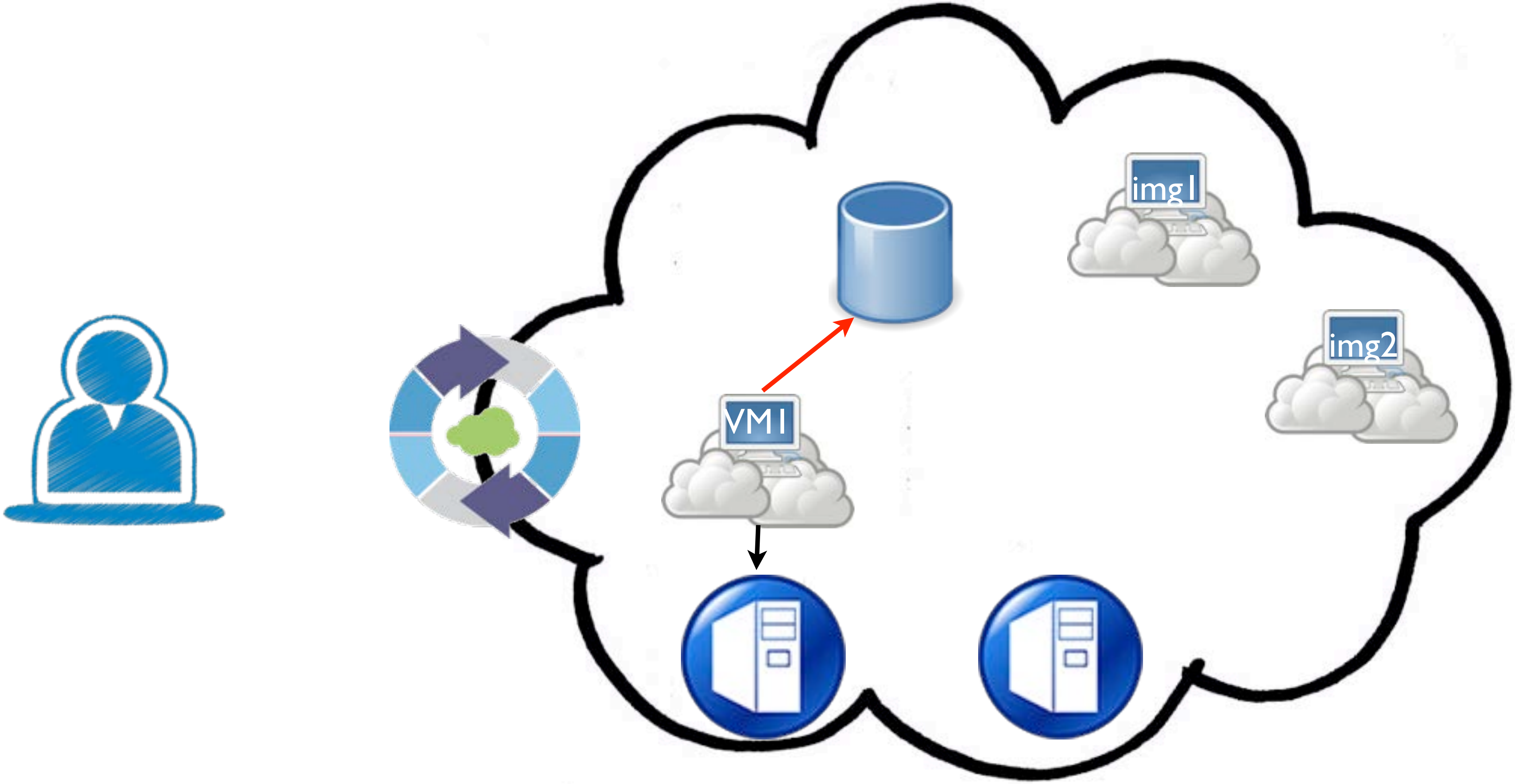

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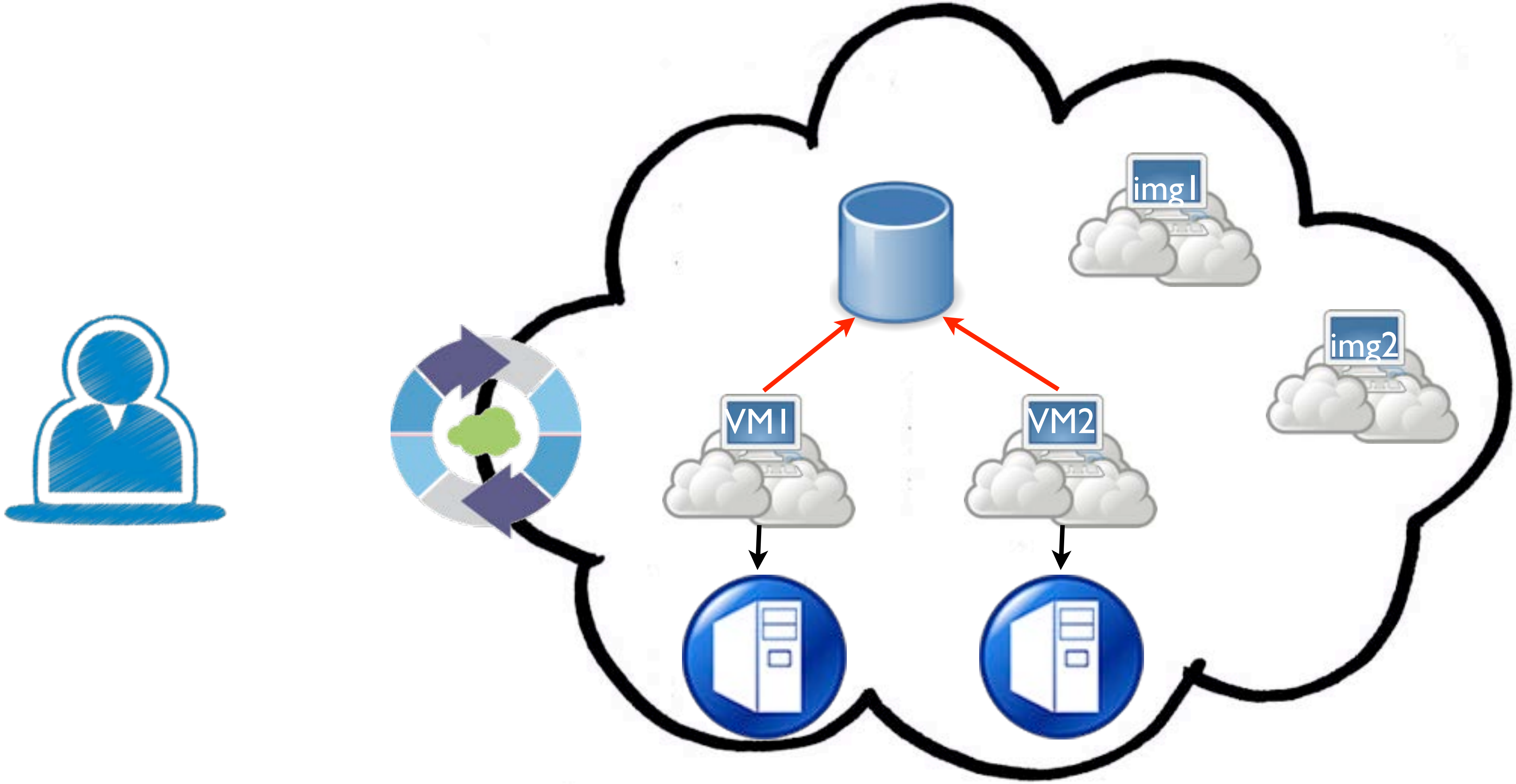

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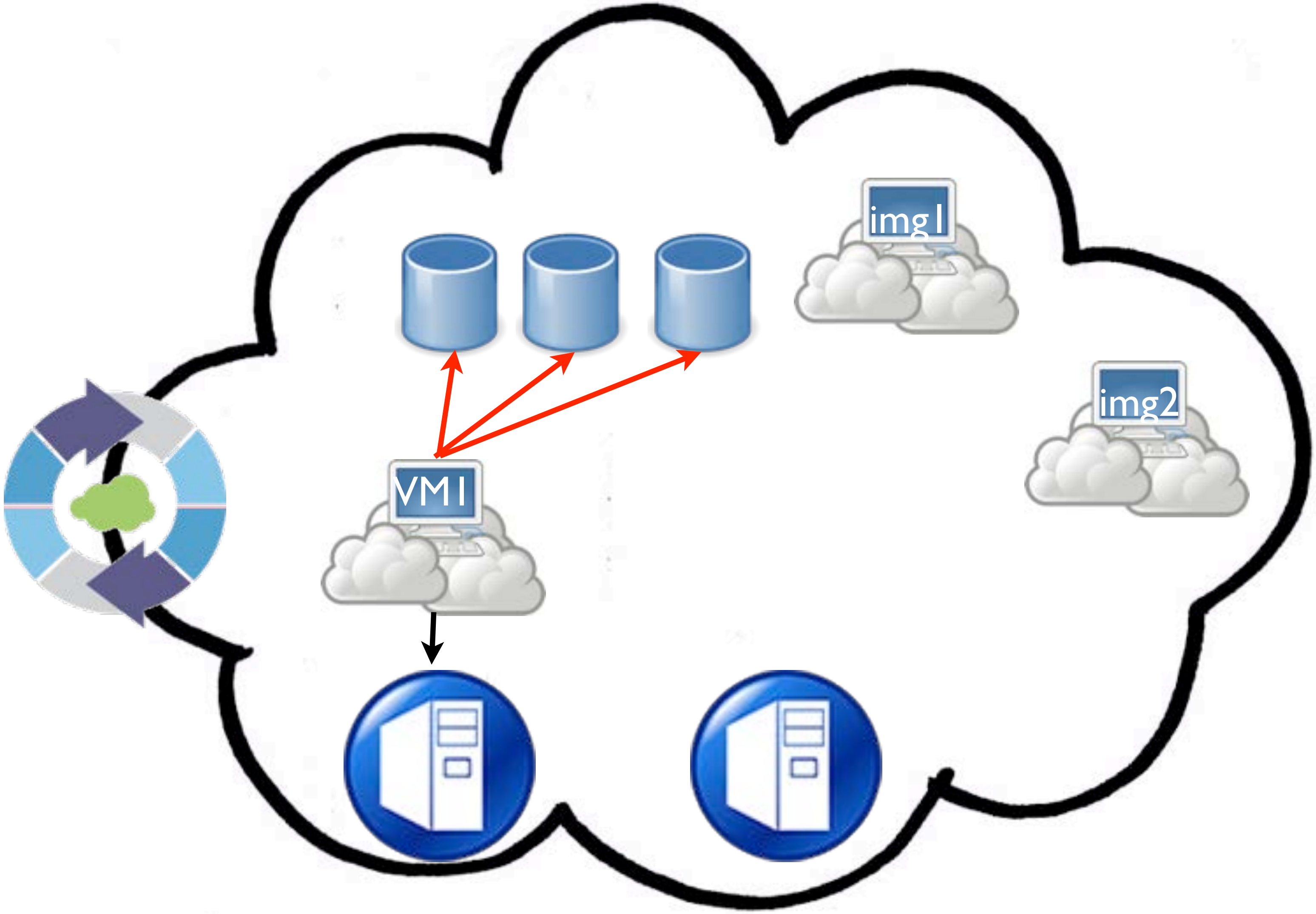
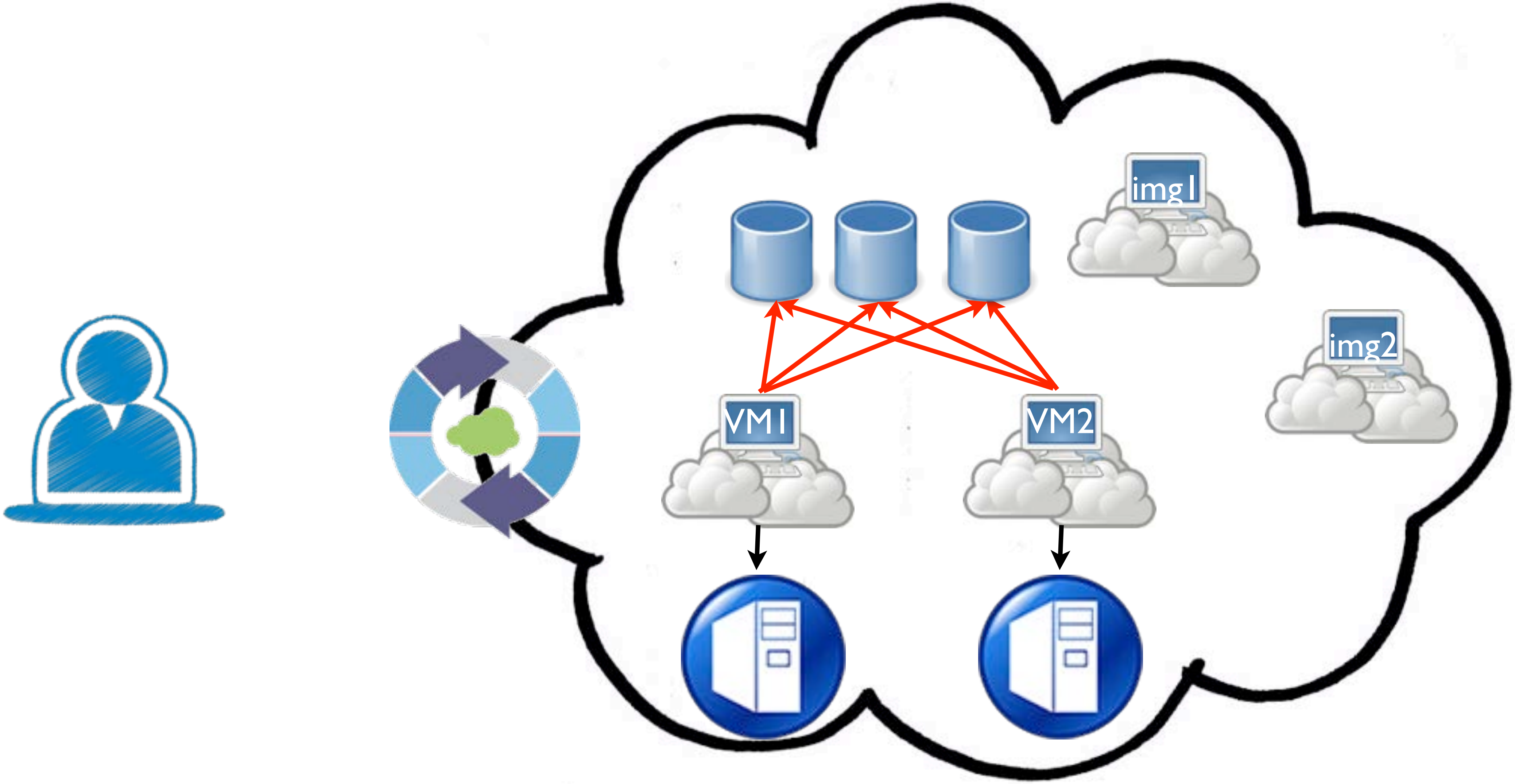
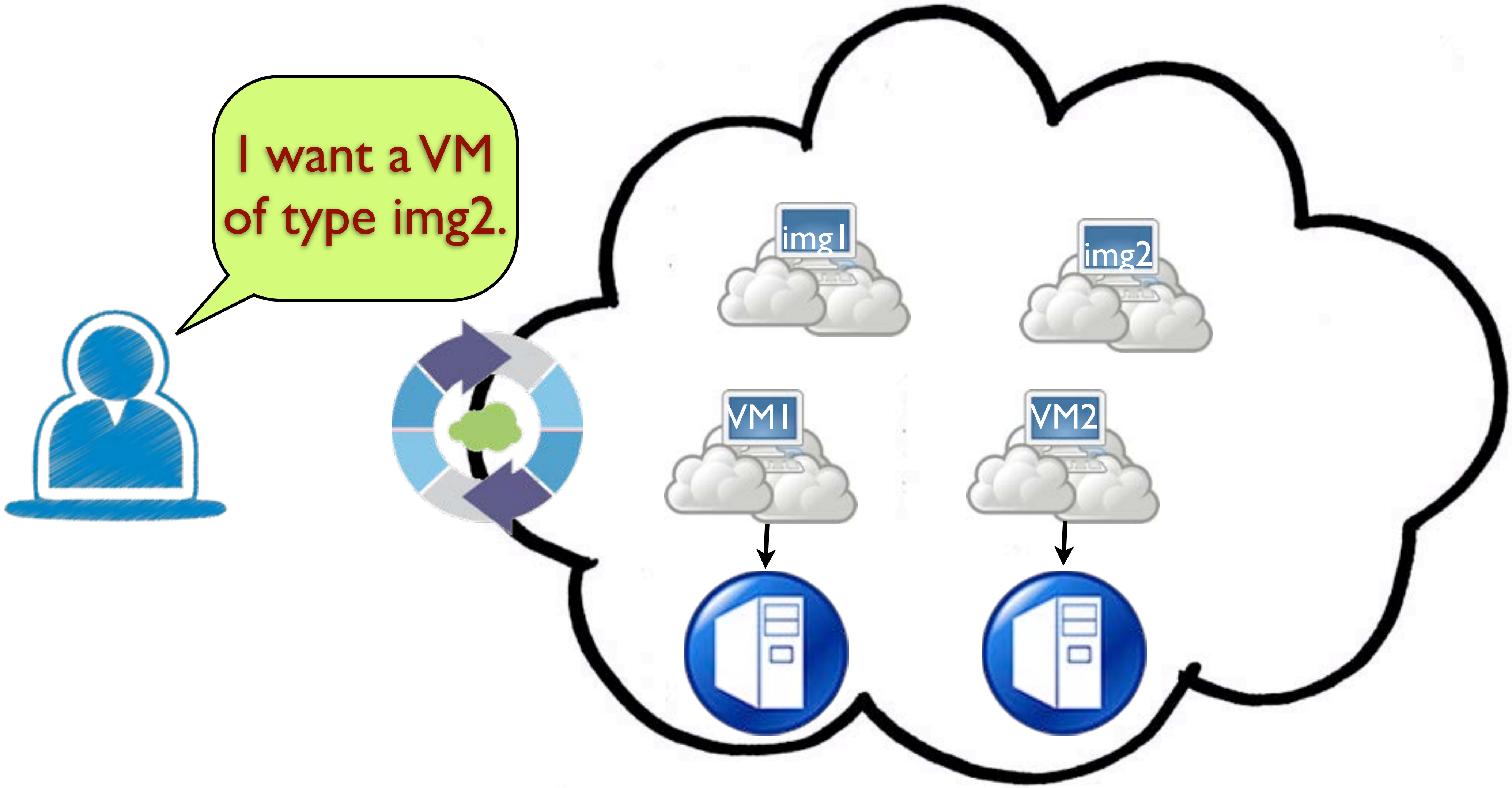


Replicate VM



Cloud needs more Mch



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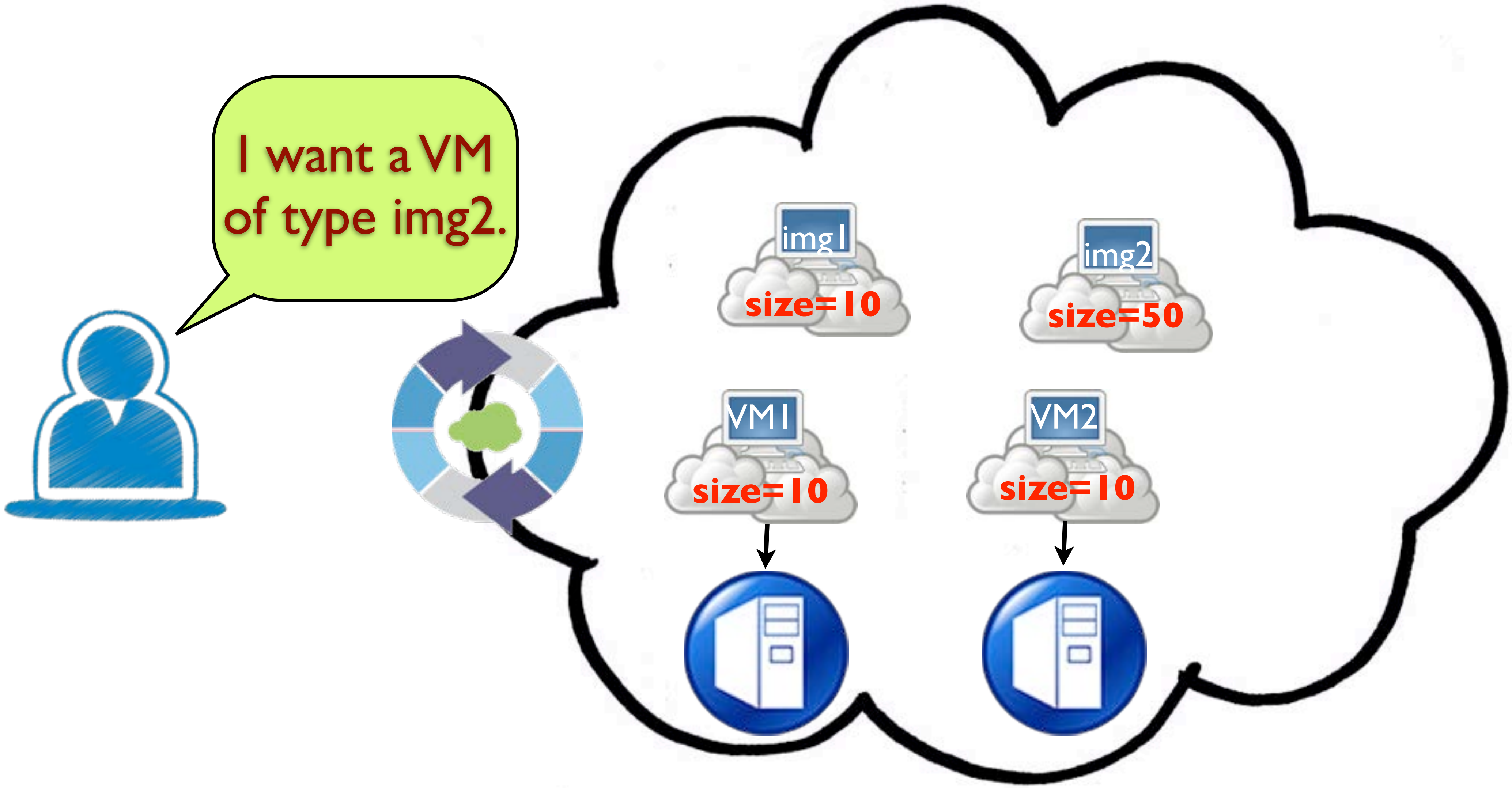
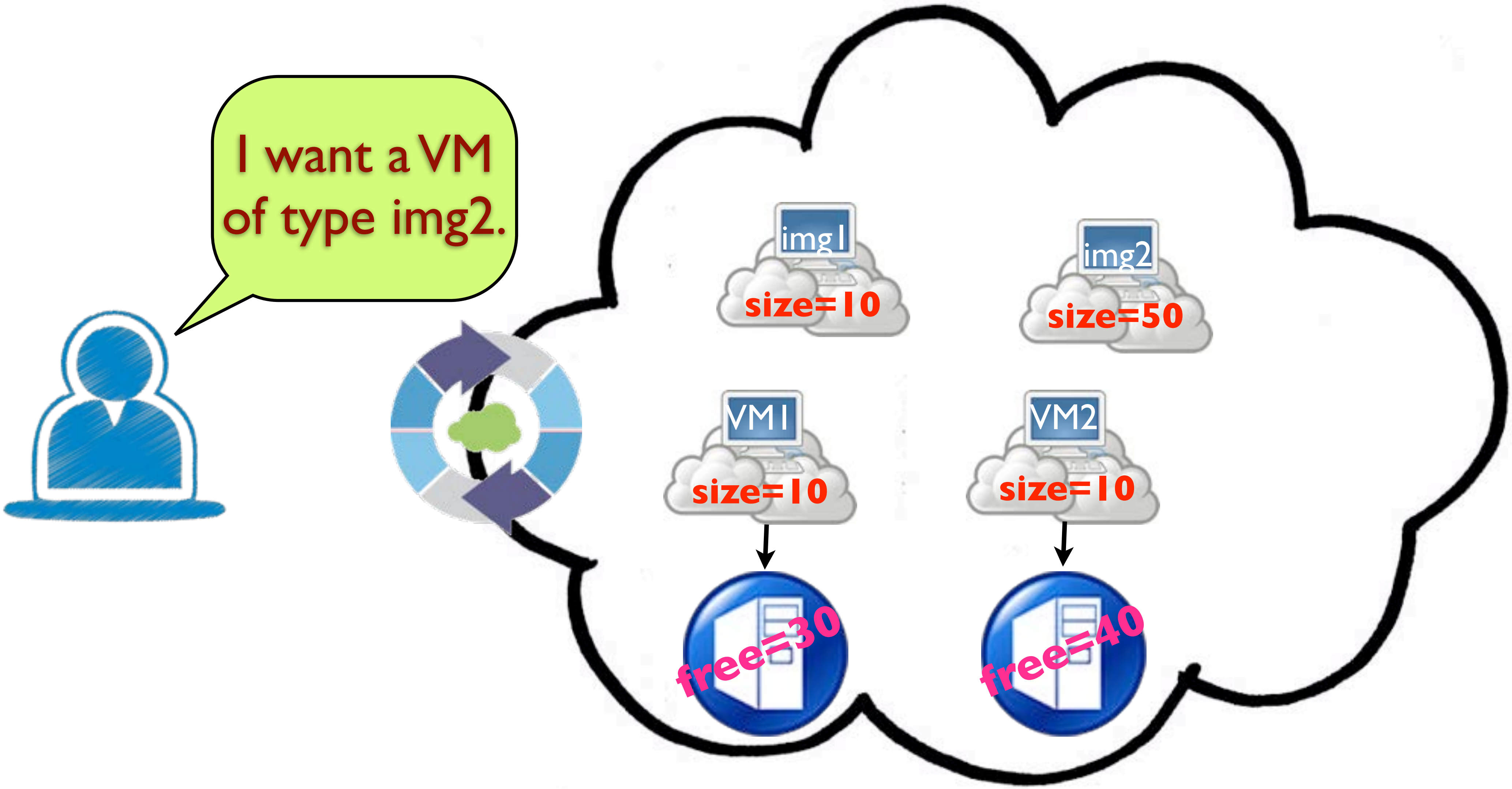
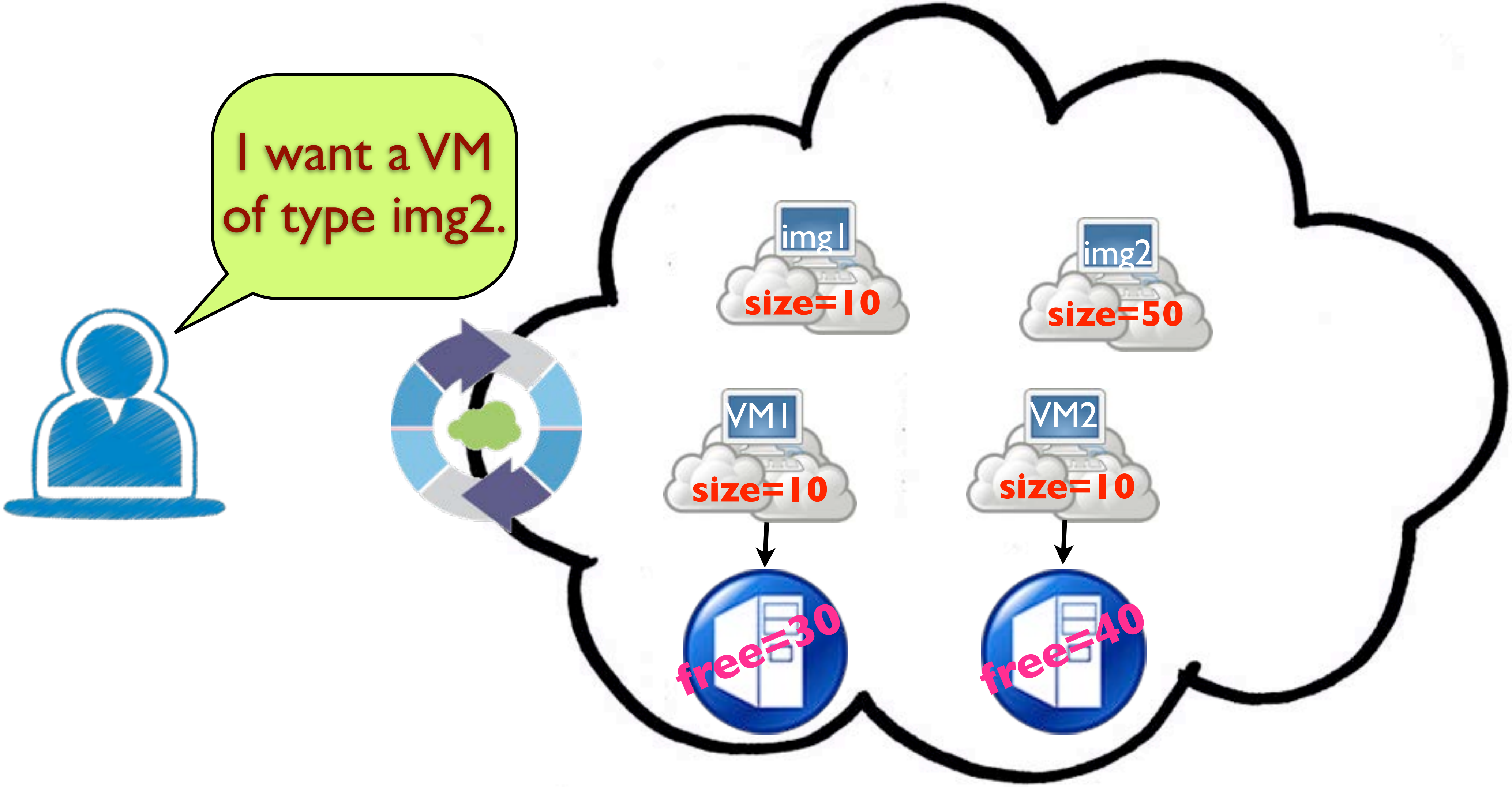


Cloud needs more Mch



Turn on Computer



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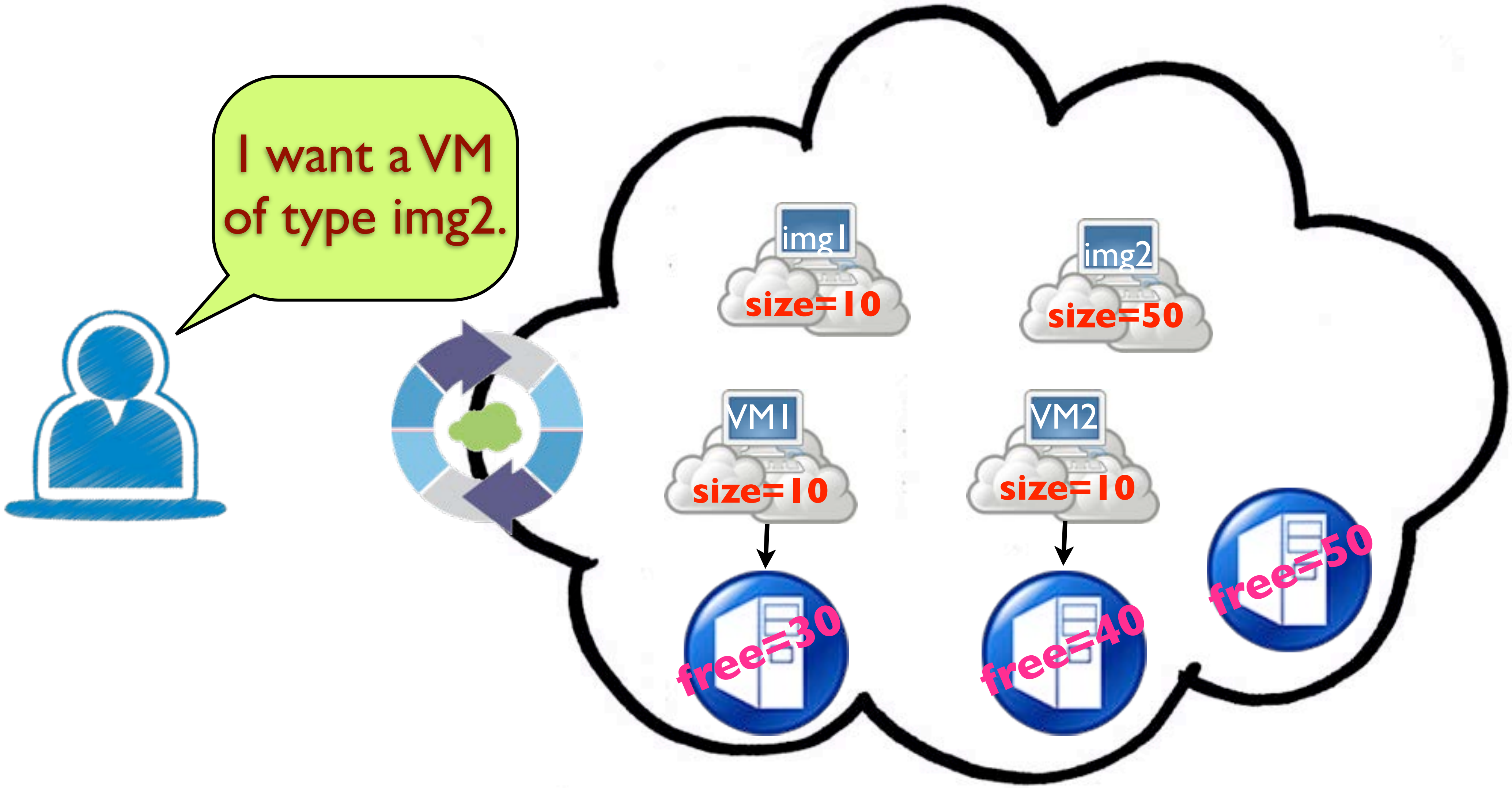
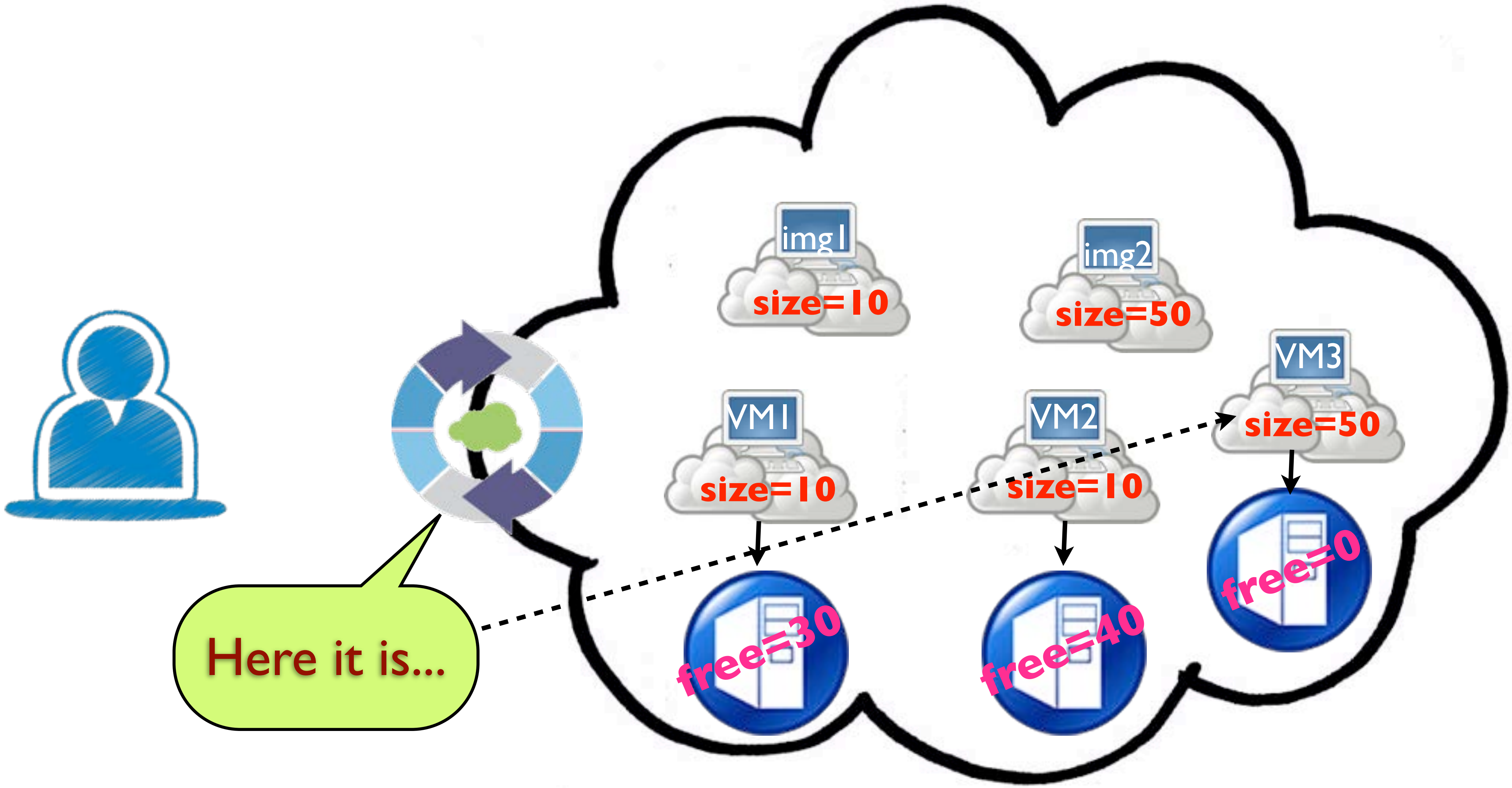


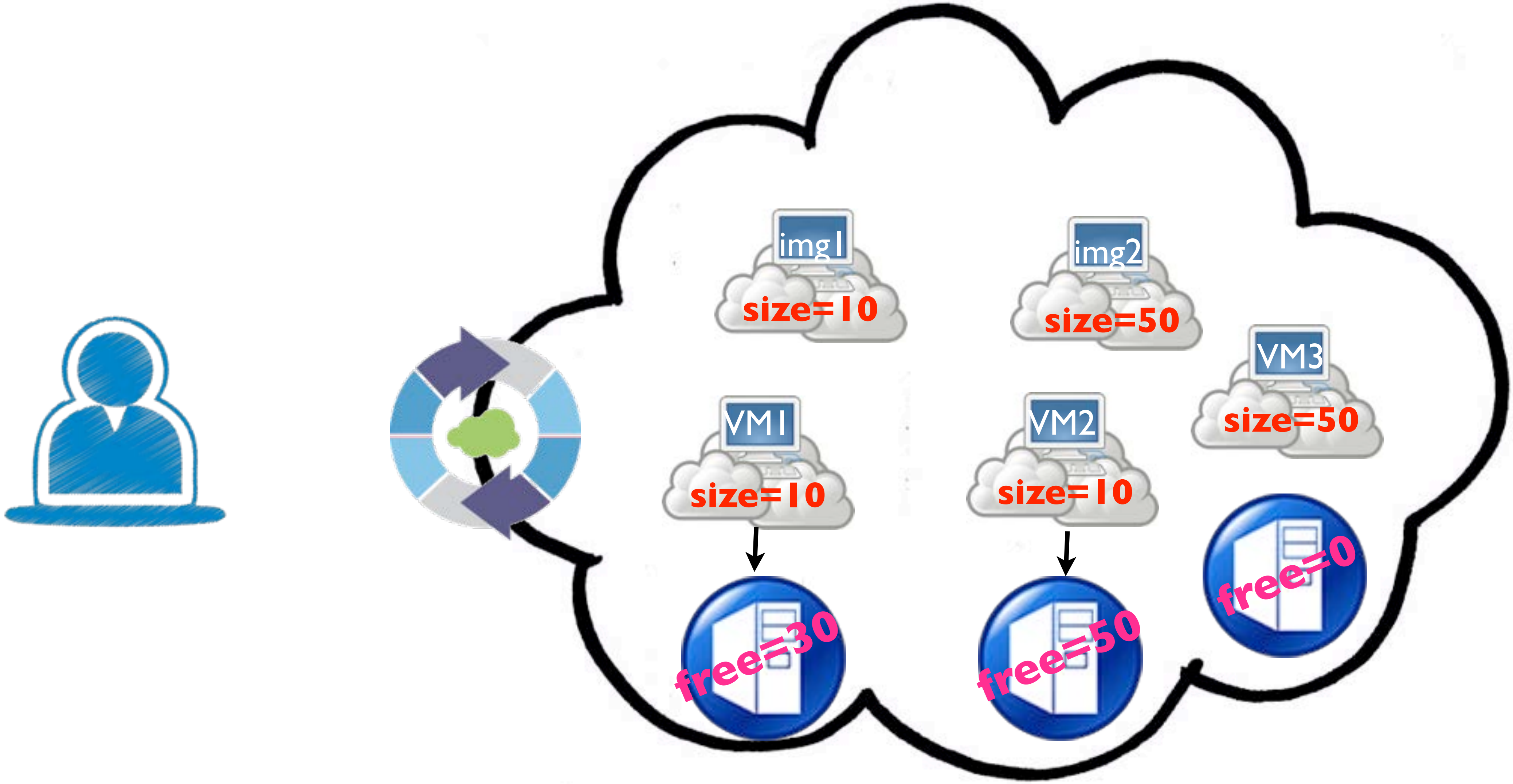
Create VM


img1
size=10
img2
size=50
VM1
size=10
VM2
size=10
free=30
free=40
free=50

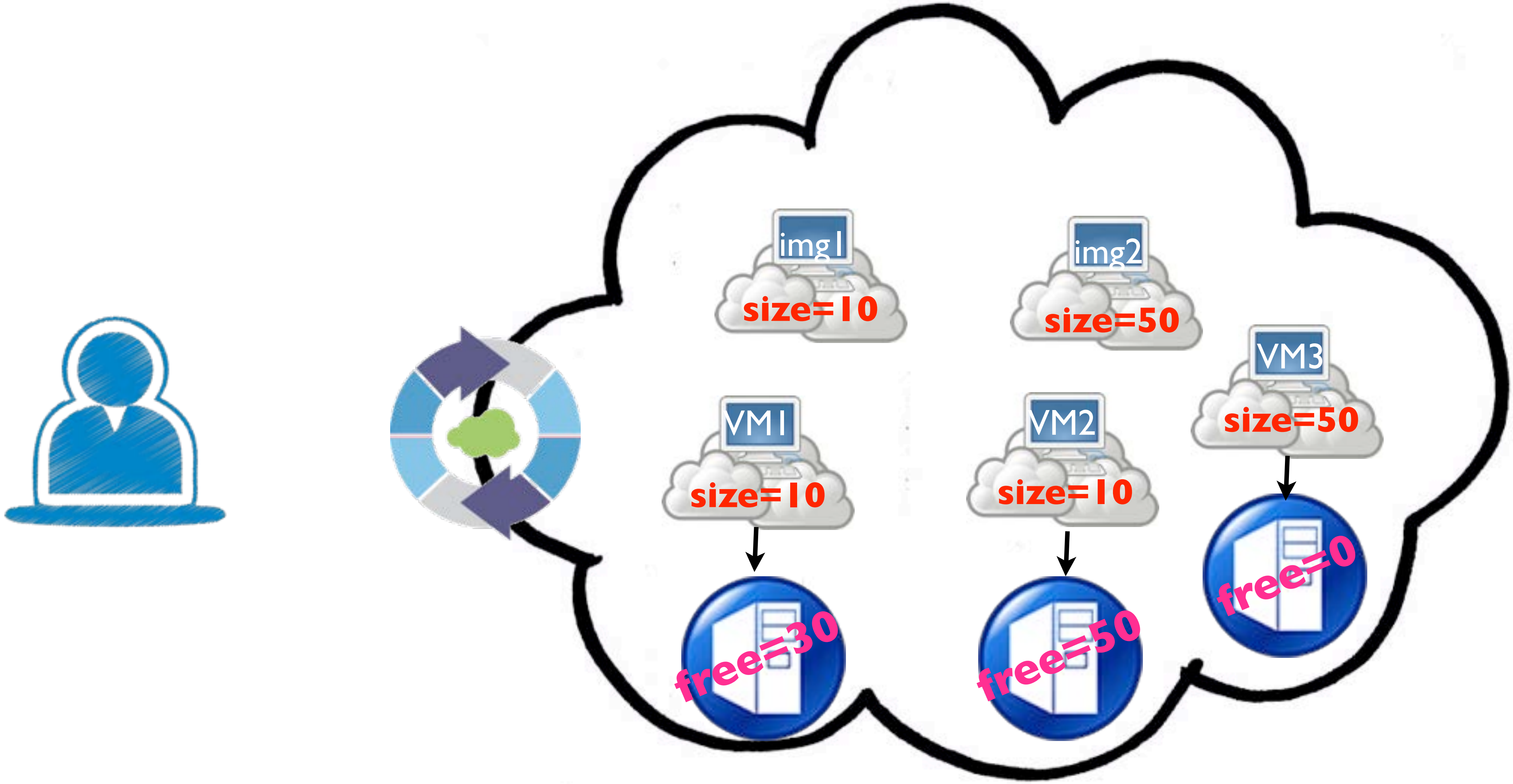
Create VM


img1
size=10
img2
size=50
VM1
size=10
VM2
size=10
VM3
size=50
free=30
free=40
free=0
Here it is...

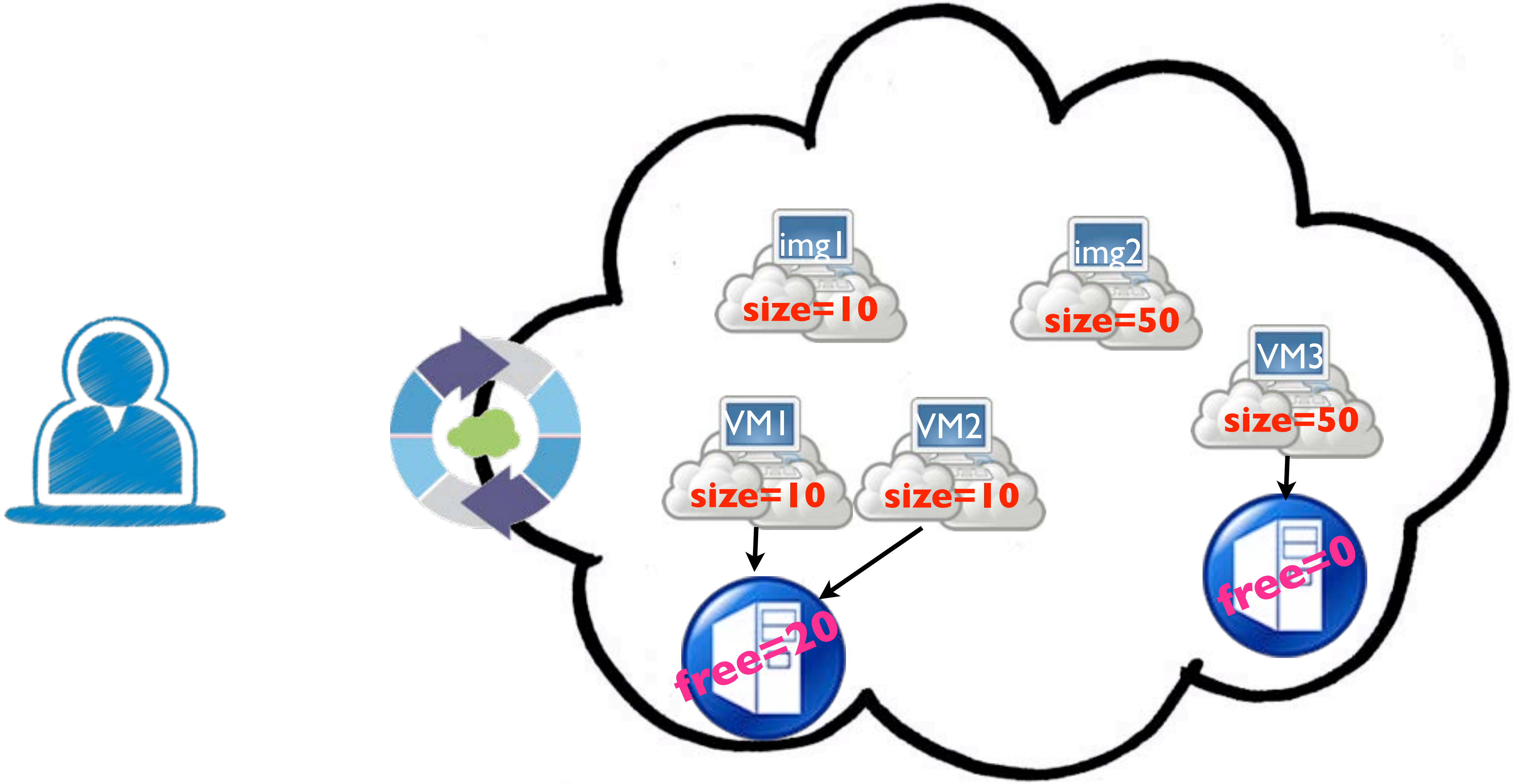
Turn Off Machine



Turn Off Machine



Turn Off Machine



Attributed Structures

Structures

S* : *G* → *Set

Attributes

T* : *A* → *Set

***AttG* = (*S* ↓ *T*)**

$$
\begin{tikzcd}
\widehat{G} \arrow[d, "\widehat{g}"'] \\
\widehat{G'}
\end{tikzcd}
=
\begin{tikzcd}
G \arrow[d, "g"'] \\
G'
\end{tikzcd}
\quad
\begin{tikzcd}
SG \arrow[r, "\alpha"] \arrow[d, "Sg"'] \arrow[dr, phantom, "="] & TA \arrow[d, "Ta"] \\
SG' \arrow[r, "\alpha'"] & TA'
\end{tikzcd}
\quad
\begin{tikzcd}
A \arrow[d, "a"] \\
A'
\end{tikzcd}
$$

Attributed Structures

Structures

$S : G \to \mathbf{Set}$

Attributes

$T : A \to \mathbf{Set}$

Graph

Alg

λ-terms

$$\textit{AttG} = (S \downarrow T)$$

$$\begin{tikzcd} \widehat{G} \arrow[d, "\widehat{g}"'] \\ \widehat{G'} \end{tikzcd} \quad = \quad \begin{tikzcd} G \arrow[d, "g"'] \\ G' \end{tikzcd} \qquad \begin{tikzcd} SG \arrow[r, "\alpha"] \arrow[d, "Sg"'] \arrow[dr, phantom, "="] & TA \arrow[d, "Ta"] \\ SG' \arrow[r, "\alpha'"] & TA' \end{tikzcd} \qquad \begin{tikzcd} A \arrow[d, "a"] \\ A' \end{tikzcd}$$

Attributed Structures

Structures

$S : G \to \mathbf{Set}$

Attributes

$T : A \to \mathbf{Set}$

Graph

Alg

λ-terms

AttG = (S ↓ T)

$$\begin{tikzcd} \widehat{G} \arrow[d, "\widehat{g}"'] \\ \widehat{G'} \end{tikzcd} = \begin{tikzcd} G \arrow[d, "g"'] \\ G' \end{tikzcd} \quad \begin{tikzcd} SG \arrow[r, "\alpha"] \arrow[d, "Sg"'] \arrow[dr, phantom, "="] & TA \arrow[d, "Ta"] \\ SG' \arrow[r, "\alpha'"] & TA' \end{tikzcd} \quad \begin{tikzcd} A \arrow[d, "a"] \\ A' \end{tikzcd}$$

Morphisms preserve attributes

Partially Attributed Structures

Structures

S* : *G* → *Part

Attributes

T* : *A* → *Part

***PAttG* = (*S* ⇣ *T*)**

$$
\begin{tikzcd}
\widehat{G} \arrow[d, "\widehat{g}"'] \\
\widehat{G'}
\end{tikzcd}
\quad = \quad
\begin{tikzcd}
G \arrow[d, "g"'] & S_pG \arrow[r, "\alpha"] \arrow[d, "S_pg"'] \arrow[dr, phantom, "\geq"] & T_pA \arrow[d, "T_pa"] & A \arrow[d, "a"] \\
G' & S_pG' \arrow[r, "\alpha'"] & T_pA' & A'
\end{tikzcd}
$$

Partially Attributed Structures

Structures

S : G → Part

Attributes

T : A → Part

PAttG = (S ⇣ T)

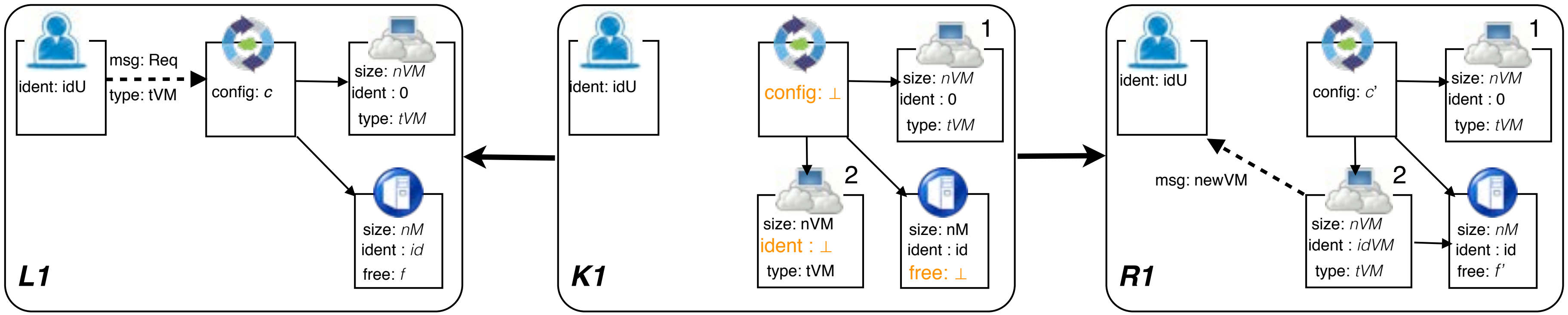
$$
\begin{tikzcd}
\widehat{G} \arrow[d, "\widehat{g}"'] \\
\widehat{G'}
\end{tikzcd}
\quad = \quad
\begin{tikzcd}
G \arrow[d, "g"'] \\
G'
\end{tikzcd}
\quad
\begin{tikzcd}
S_pG \arrow[r, "\alpha"] \arrow[d, "S_pg"'] \arrow[dr, phantom, "\geq"] & T_pA \arrow[d, "T_pa"] \\
S_pG' \arrow[r, "\alpha'"'] & T_pA'
\end{tikzcd}
\quad
\begin{tikzcd}
A \arrow[d, "a"] \\
A'
\end{tikzcd}
$$

Morphisms preserve defined attributes

Graph Transformation Rule

L	←——□K□——→	R
AttrG	**PAttrG**	**AttrG**

rule CreateVM

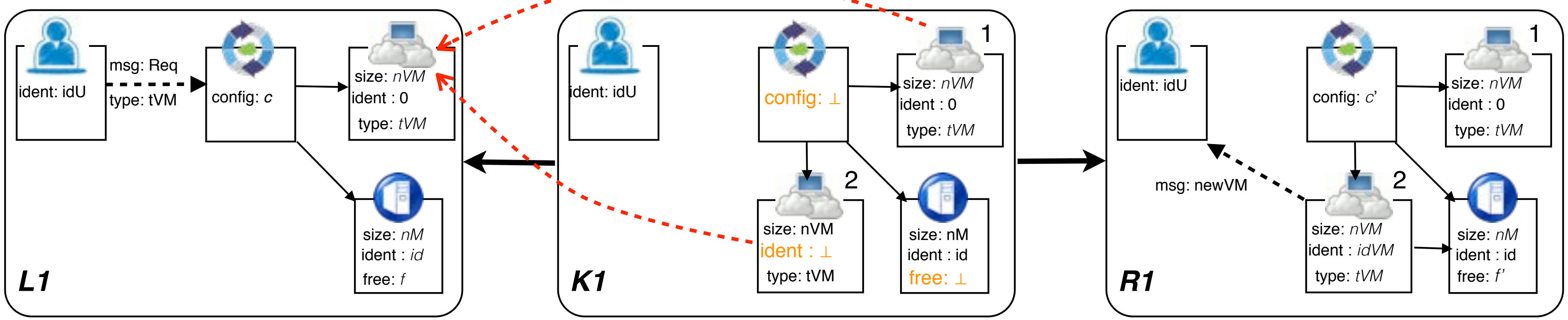


eqns: $idVM = newId(c)$;
$\leq (nVM, f) = true$;
$f' = f - nVM$;
$c' = newVM(c, idU, idVM, nVM, tVM)$

Graph Transformation Rule

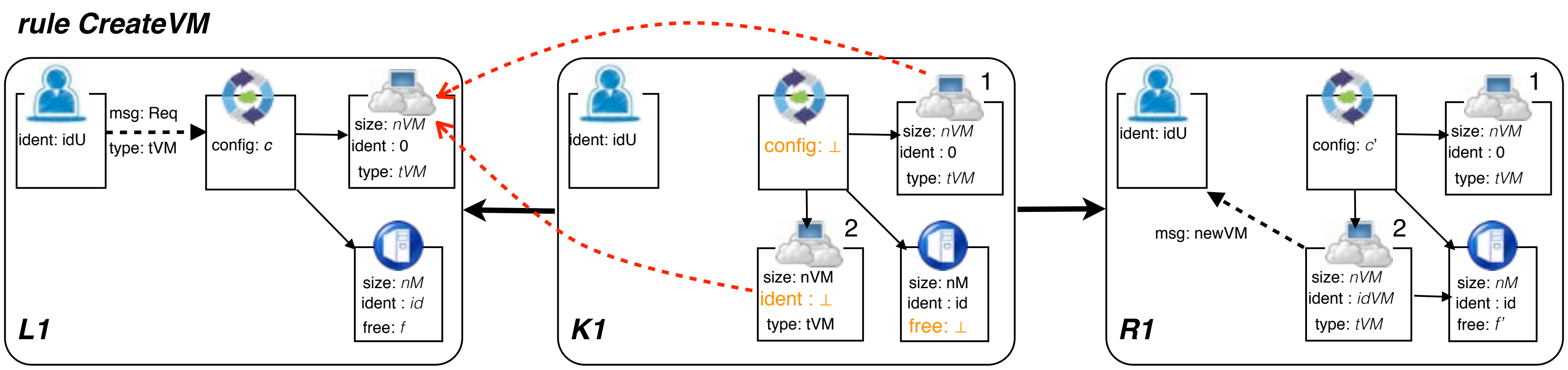
L	← K →	R
AttrG	**PAttrG**	**AttrG**

rule CreateVM

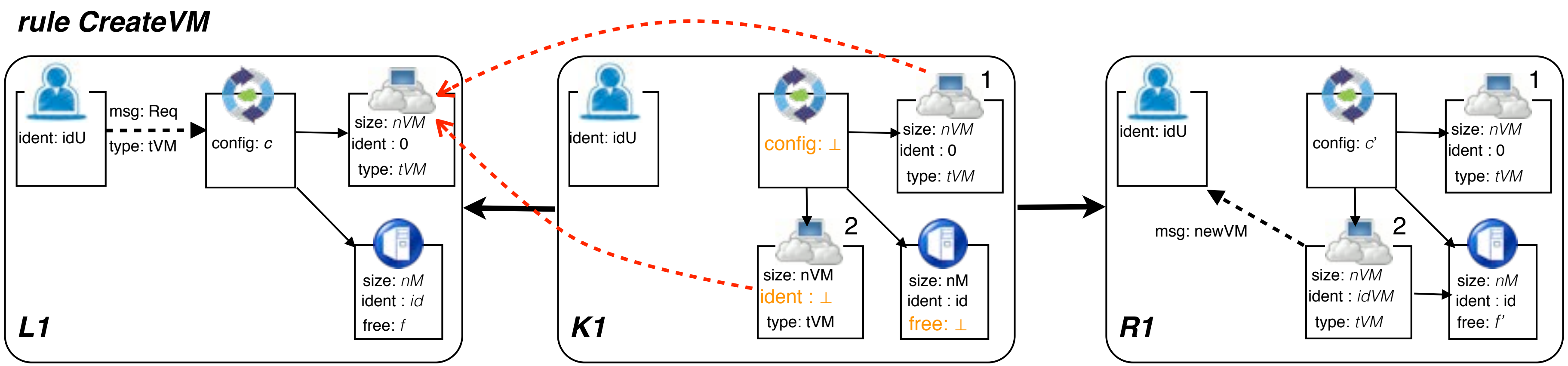


eqns: *idVM = newId(c) ;*
≤ (nVM,f) = true ;
f' = f - nVM ;
c'= newVM(c,idU,idVM,nVM,tVM)

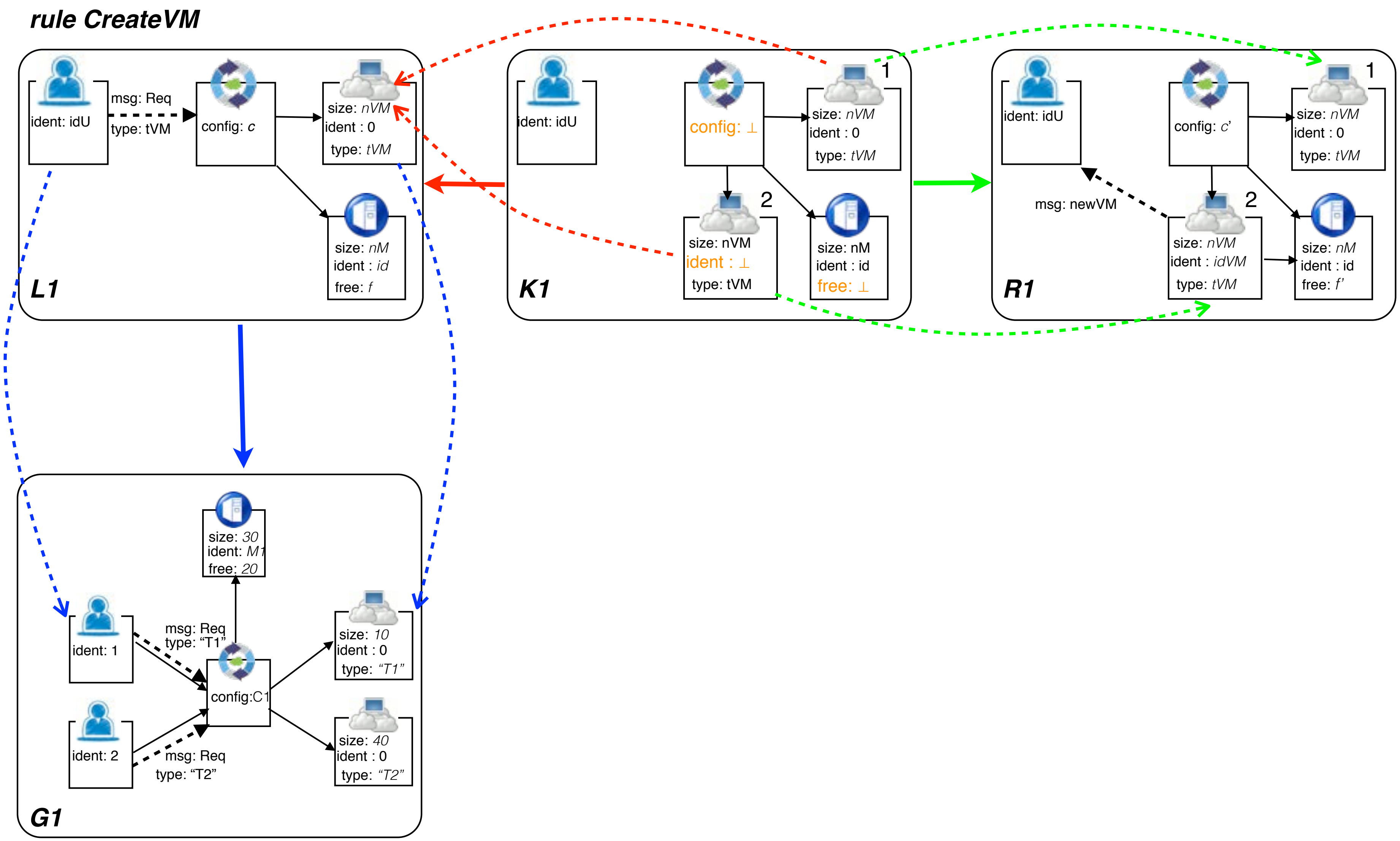
Rule Application



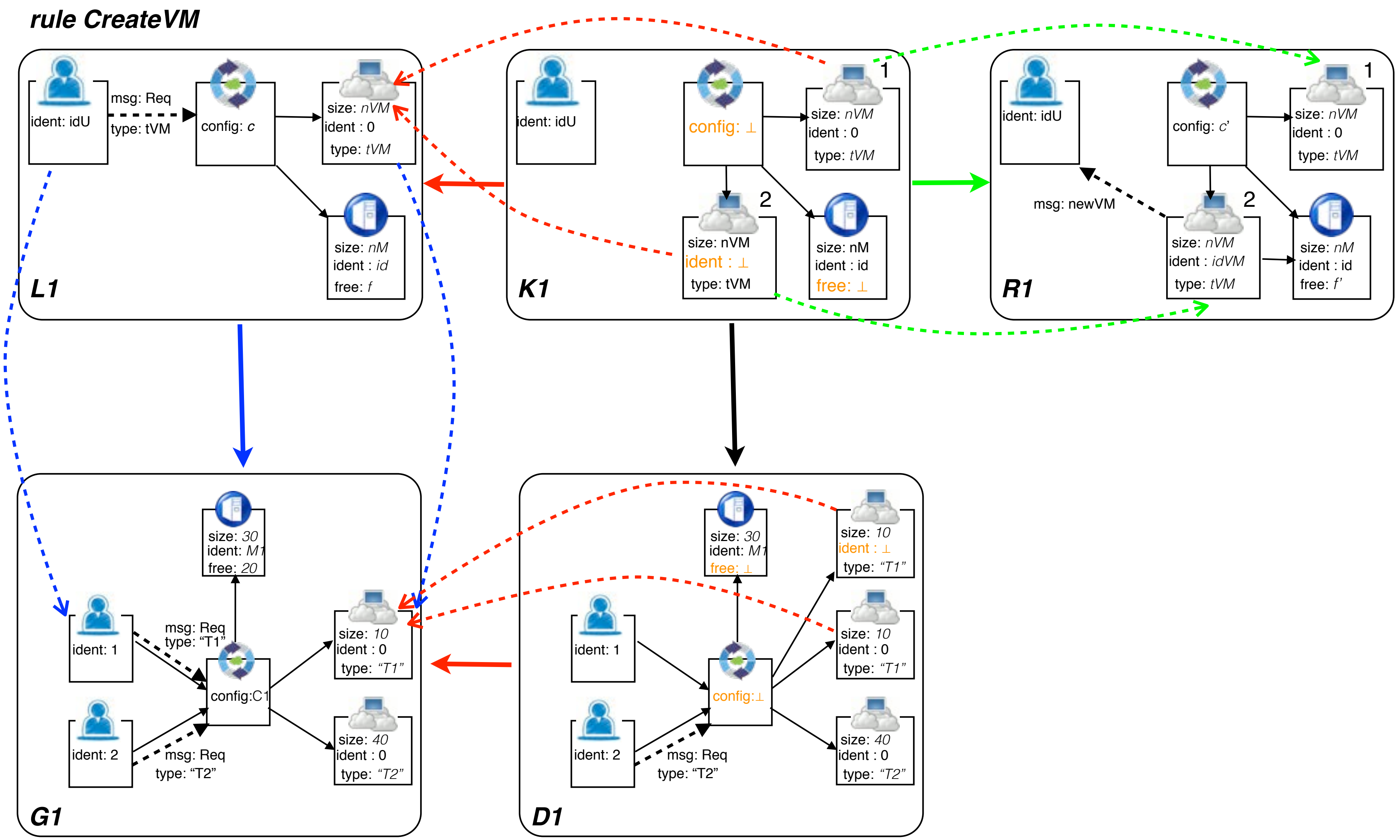
Rule Application



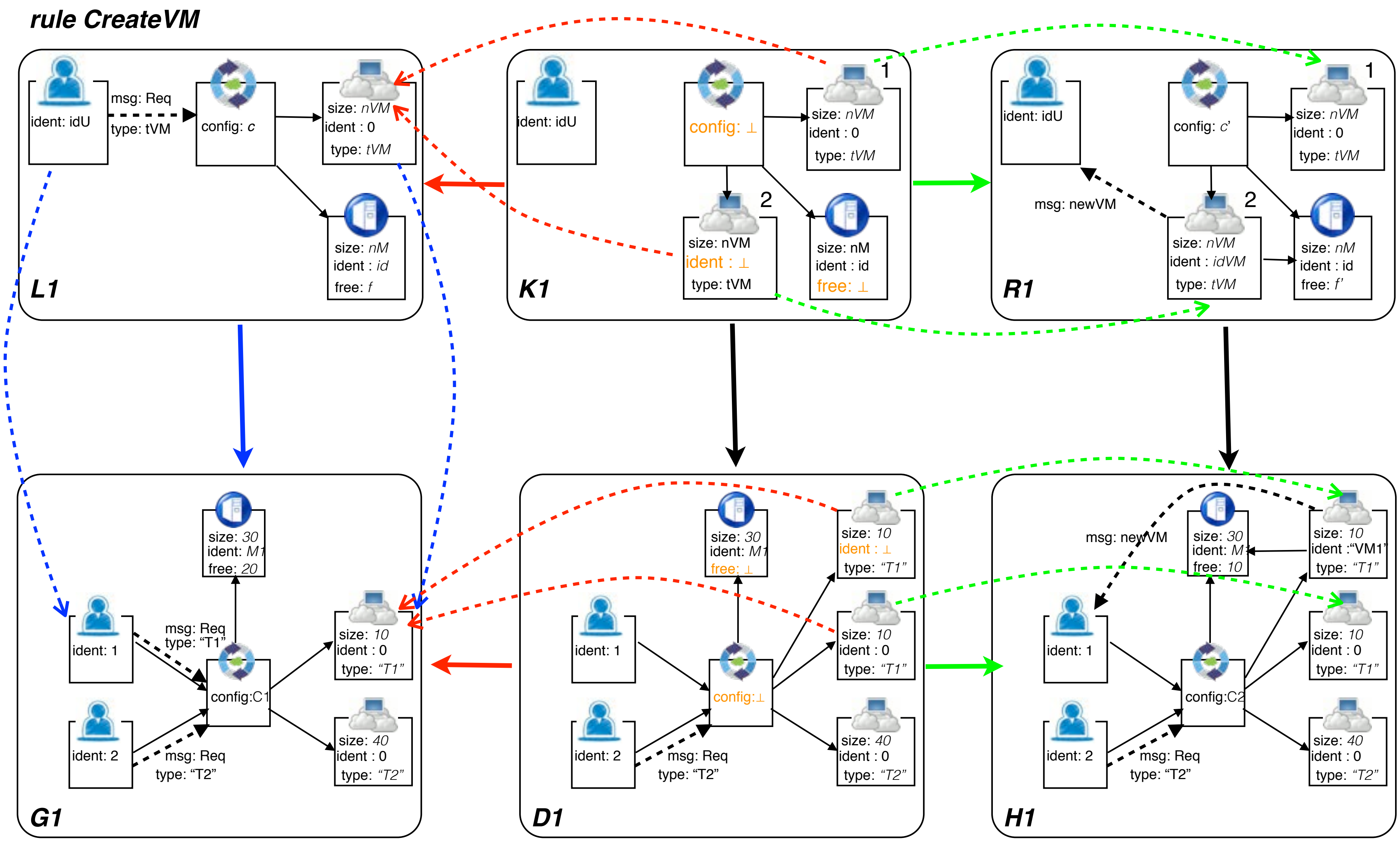
Rule Application



Rule Application



Rule Application



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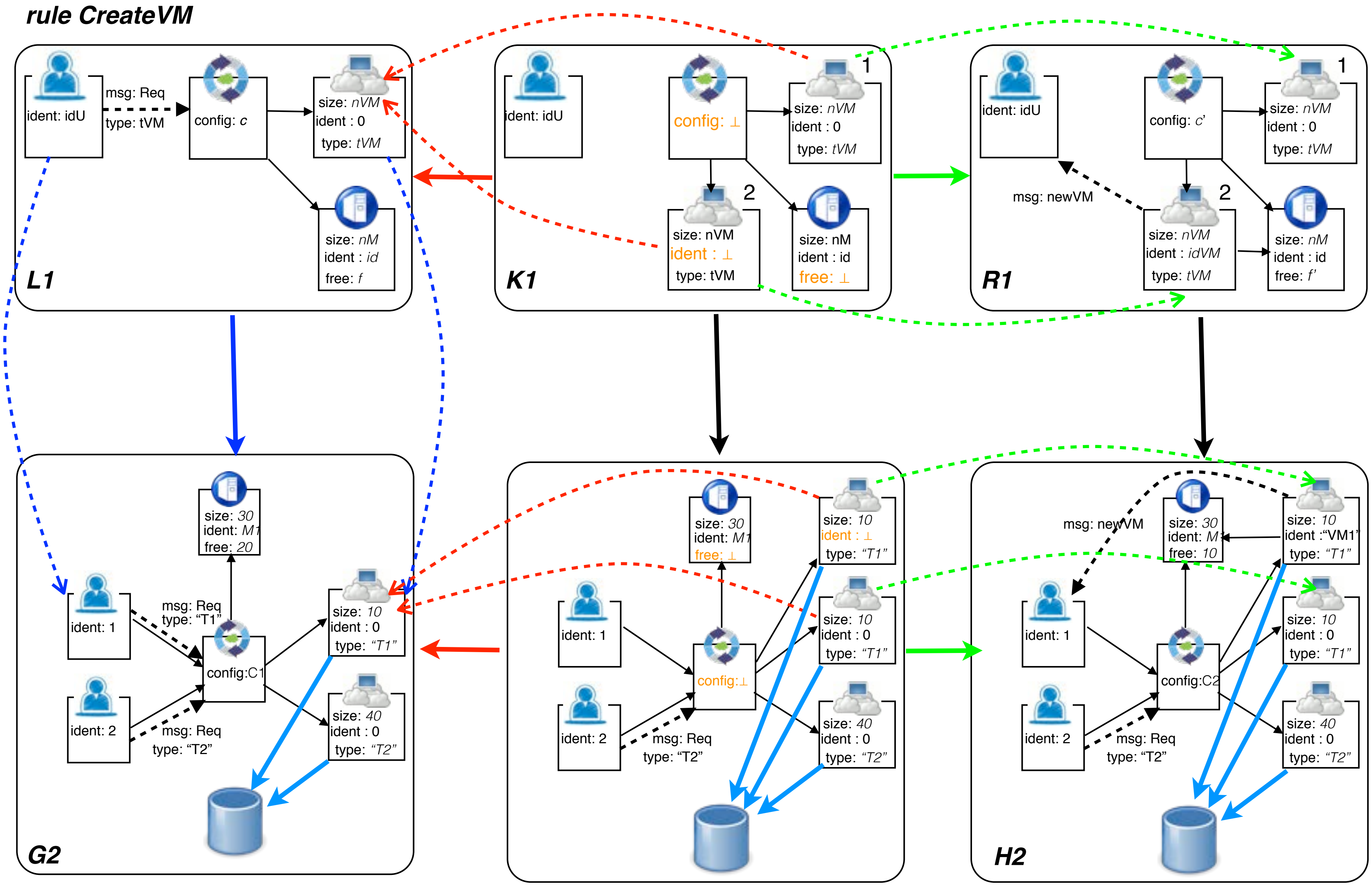
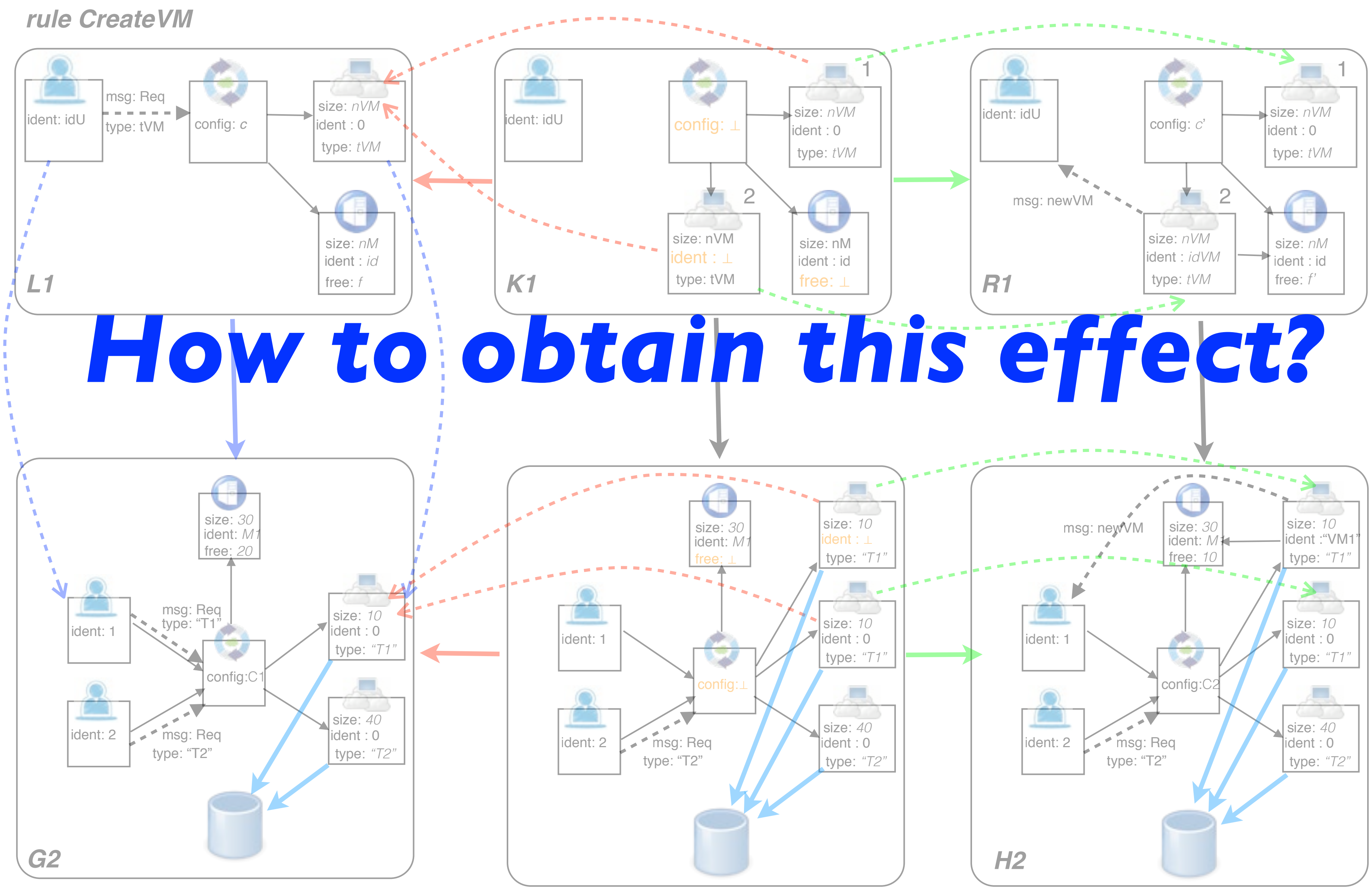


Rule Application



How to obtain this effect?

Sesqui-Pushout Approach

$$\begin{tikzcd} L \arrow[d] \arrow[dr, phantom, "\text{(FPBC)}"] & K \arrow[l] \arrow[r] \arrow[d] \arrow[dr, phantom, "\text{(PO)}"] & R \arrow[d] \\ G & D \arrow[l] \arrow[r] & H \end{tikzcd}$$

Final Pullback Complement (FPBC): Deletion and Copy

Pushout (PO): Creation and Merge

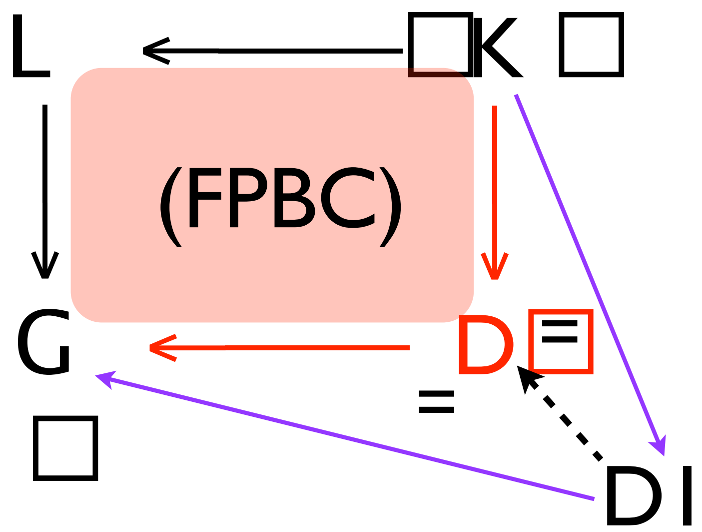
Final Pullback Complement

$$\begin{tikzcd}
L \arrow[d] & K \arrow[l] \\
G &
\end{tikzcd}$$

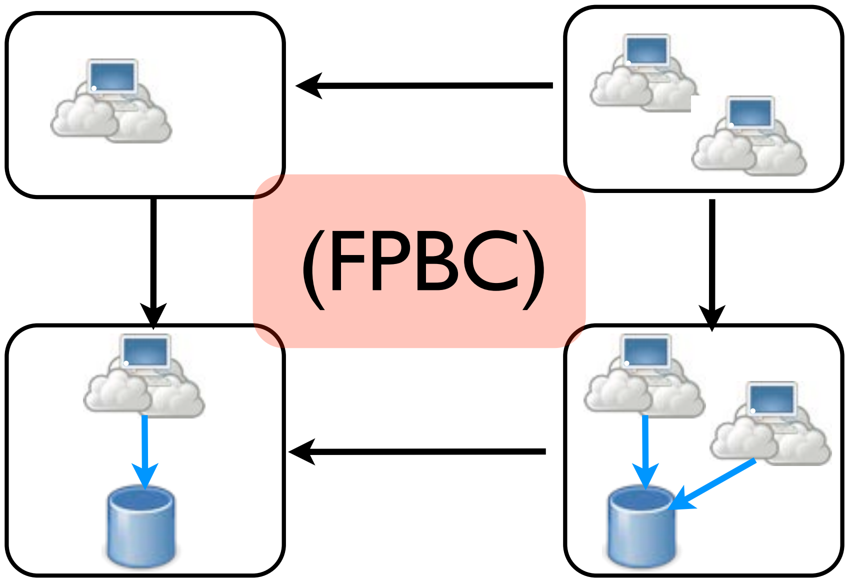
Final Pullback Complement

$$
\begin{tikzcd}
L & \square K \square \arrow[l] \arrow[d] \\
G \square & D \square \arrow[l]
\end{tikzcd}
$$

Final Pullback Complement

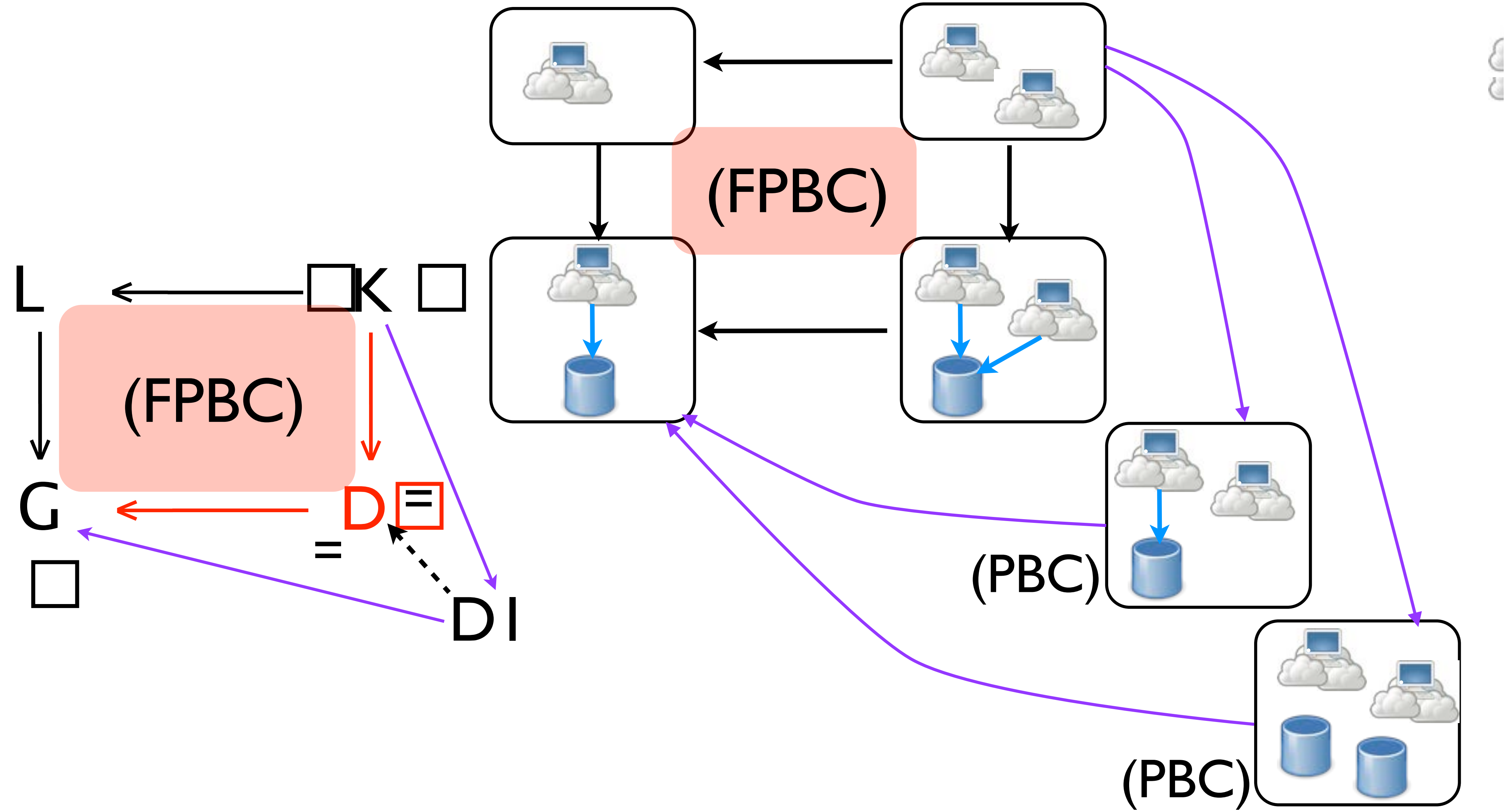


Final Pullback Complement



$$\begin{tikzcd}
L \arrow[d] & K \arrow[l] \arrow[d] \arrow[ddr, bend left=10] \arrow[dl, phantom, "\text{(FPBC)}"] & \\
G & D \arrow[l] \arrow[dl, phantom, "="] & \\
 & & D' \arrow[ull, bend left=10] \arrow[ul, dotted]
\end{tikzcd}$$

Final Pullback Complement



SqPO-Rewriting of Attributed Structures

Structures:

Δ :

$$\begin{tikzcd}
L \arrow[d, "m_L"] \arrow[dr, phantom, "(FPBC)"] & K \arrow[l, "l"'] \arrow[r, "r"] \arrow[d, "m_K"] \arrow[dr, phantom, "(PO)"] & R \arrow[d, "m_R"] \\
G & D \arrow[l, "l_1"'] \arrow[r, "r_1"] & H
\end{tikzcd}$$

SqPO-Rewriting of Attributed Structures

Structures:

Δ :

$$
\begin{tikzcd}
L \arrow[d, "m_L"] & K \arrow[l, "l"'] \arrow[r, "r"] \arrow[d, "m_K"] \arrow[dl, phantom, "(FPBC)"] \arrow[dr, phantom, "(PO)"] & R \arrow[d, "m_R"] \\
G & D \arrow[l, "l_1"'] \arrow[r, "r_1"] & H
\end{tikzcd}
$$

Attributed Structures:

$\widehat{\Delta}$:

$$
\begin{tikzcd}
\widehat{L} \arrow[d, "{(m_L,a)}"] & \widehat{K} \arrow[l, "{(l,id_A)}"'] \arrow[r, "{(r,id_A)}"] \arrow[d, "{(m_K,a)}"] \arrow[dl, phantom, "(FPBC)"] \arrow[dr, phantom, "(PO)"] & \widehat{R} \arrow[d, "{(m_R,a}"] \\
\widehat{G} & \widehat{D} \arrow[l, "{(l_1,id_{A_1})}"'] \arrow[r, "{(r_1,id_{A_1})}"] & \widehat{H}
\end{tikzcd}
$$

SqPO-Rewriting of Attributed Structures

Attributed Structures:

$\widehat{\Delta}$:

$$
\begin{tikzcd}
\widehat{L} \arrow[d, "{(m_L,a)}"] \arrow[dr, phantom, "(FPBC)"] & \widehat{K} \arrow[l, "{(l,id_A)}"'] \arrow[r, "{(r,id_A)}"] \arrow[d, "{(m_K,a)}"] \arrow[dr, phantom, "(PO)"] & \widehat{R} \arrow[d, "{(m_R,a)}"] \\
\widehat{G} & \widehat{D} \arrow[l, "{(l_1,id_{A_1})}"'] \arrow[r, "{(r_1,id_{A_1})}"] & \widehat{H}
\end{tikzcd}
$$

SqPO-Rewriting of Attributed Structures

x is context: *keep attribute*

$$l_1(x) : t_1 \longleftarrow\!\!\dashv x : t_1 \longmapsto r_1(x) : t_1$$

Attributed Structures:

$\widehat{\Delta}$:

$$\begin{tikzcd}
\widehat{L} \arrow[d, "{(m_L,a)}"'] \arrow[dr, phantom, "(FPBC)"] & \widehat{K} \arrow[l, "{(l,id_A)}"'] \arrow[r, "{(r,id_A)}"] \arrow[d, "{(m_K,a)}"] \arrow[dr, phantom, "(PO)"] & \widehat{R} \arrow[d, "{(m_R,a)}"] \\
\widehat{G} & \widehat{D} \arrow[l, "{(l_1,id_{A_1})}"'] \arrow[r, "{(r_1,id_{A_1})}"] & \widehat{H}
\end{tikzcd}$$

SqPO-Rewriting of Attributed Structures

Attributed Structures:

$\widehat{\Delta}$:

$$
\begin{tikzcd}
\widehat{L} \arrow[d, "{(m_L,a)}"] \arrow[dr, phantom, "(FPBC)"] & \widehat{K} \arrow[l, "{(l,id_A)}"'] \arrow[r, "{(r,id_A)}"] \arrow[d, "{(m_K,a)}"] \arrow[dr, phantom, "(PO)"] & \widehat{R} \arrow[d, "{(m_R,a)}"] \\
\widehat{G} & \widehat{D} \arrow[l, "{(l_1,id_{A_1})}"'] \arrow[r, "{(r_1,id_{A_1})}"] & \widehat{H}
\end{tikzcd}
$$

x is context: *keep attribute*

$$l_1(x):t_1 \longleftarrow x:t_1 \longmapsto r_1(x):t_1$$

x is preserved by the rule:

$$
\begin{tikzcd}
l(x):t \arrow[d, mapsto] & x:t \arrow[l] \arrow[r, mapsto] \arrow[d, mapsto] & r(x):t \arrow[d, mapsto] \\
l_1(x):a(t) & x:a(t) \arrow[l] \arrow[r, mapsto] & r_1(x):a(t)
\end{tikzcd}
$$

keep attribute

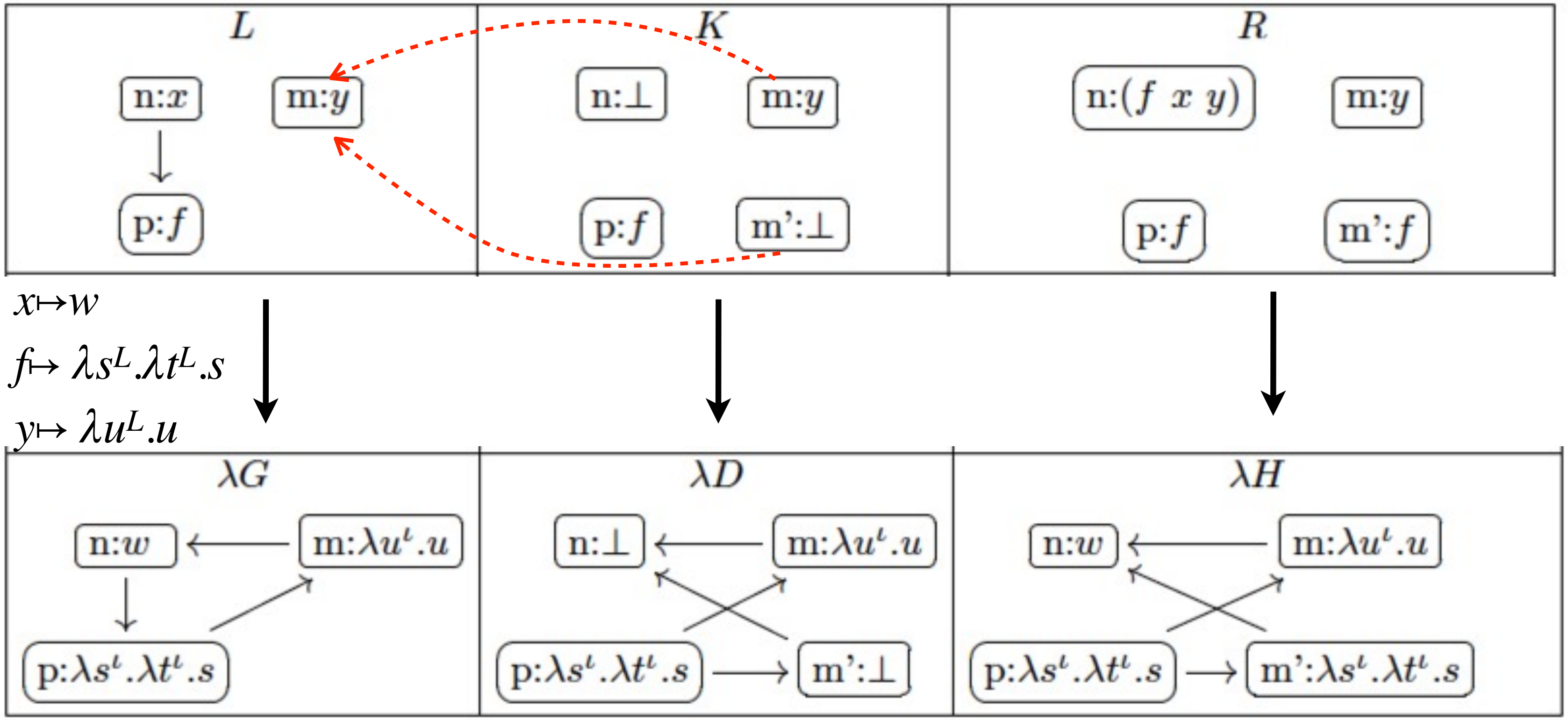
$$
\begin{tikzcd}
l()x:t \arrow[d, mapsto] & x:\bot \arrow[l] \arrow[r, mapsto] \arrow[d, mapsto] & r(x):t' \arrow[d, mapsto] \\
l_1(x):a(t) & x:\bot \arrow[l] \arrow[r, mapsto] & r_1(x):a(t')
\end{tikzcd}
$$

change attribute

SqPO-Rewriting of Attributed Structures

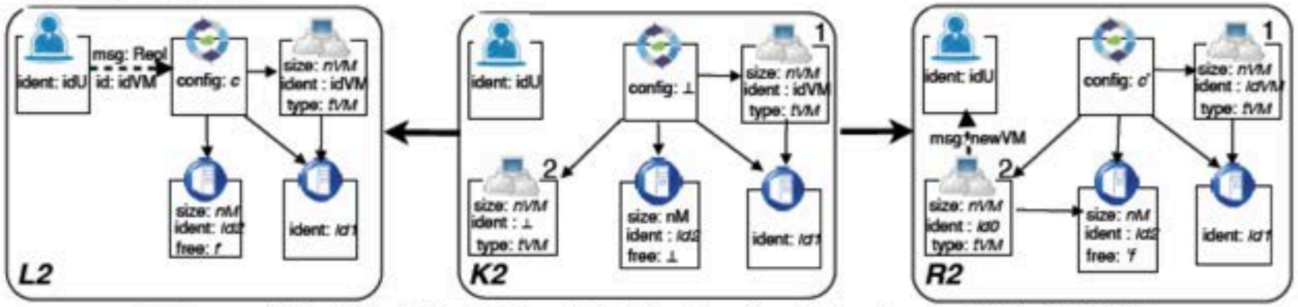
- A nice framework to define systems in the presence of cloning (and merging) operations
- Simple attribute handling:
  - ➡ allowing to use different kinds of values;
  - ➡ enabling a modular approach to prove properties (due to the independency of the structure from the attributes)

λ-Terms as Attributes



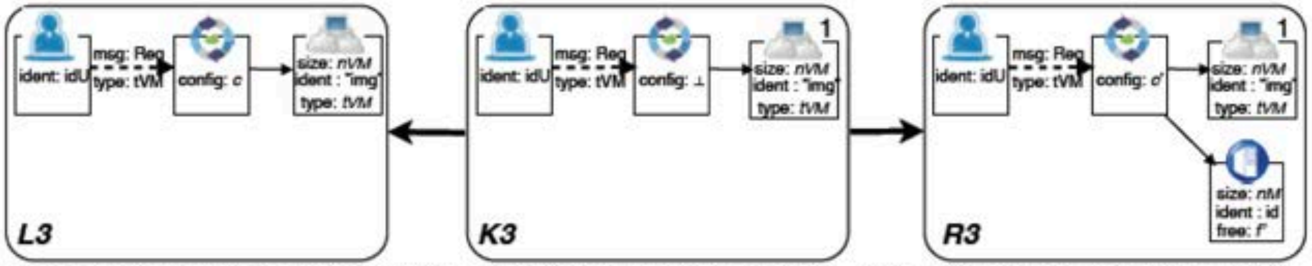
Cloud Administration

rule ReplicateVM



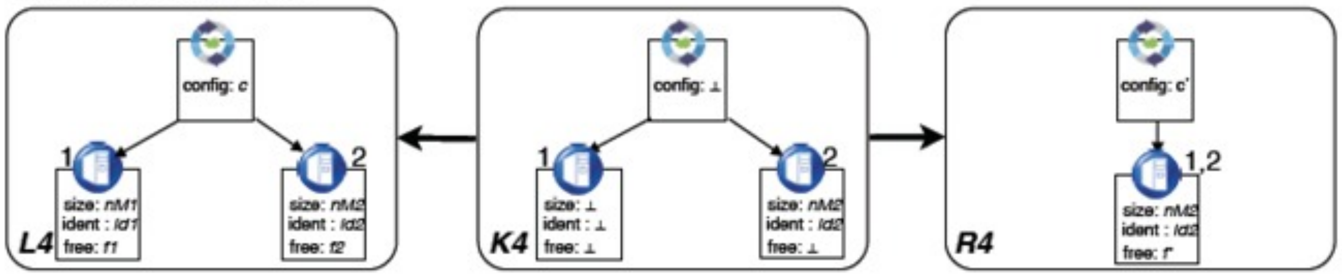
eqns: *newId(c,id0) ; id1 ≠ id2; nVM ≤ f ; f' = f - nVM ; c'= replVM(c,idU,id0)*

rule TurnOnMachine



eqns: *not(enoughSpace(c,nVM)); newId(c,id) ; nVM ≤ nM ; f'= nM-nVM ; c'=newMch(c,id,nM,f')*

rule TurnOffMachine



eqns: *nM1-f1 ≤ f2; f' = f2 - (nM1-f1) ; c'= mergeMch(c,id1,id2)*

Future Work

- Analysis of SqPO-transformation systems over attributed structures
- Case studies
- Tool support

Transformations of Attributed Structures with Cloning

Thanks for your attention!

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