

STRING DIAGRAMS AND WIRE CALCULUS

Pawel Sobocinski
CCS '09, Grenoble 26/11/09

PLAN OF TALK

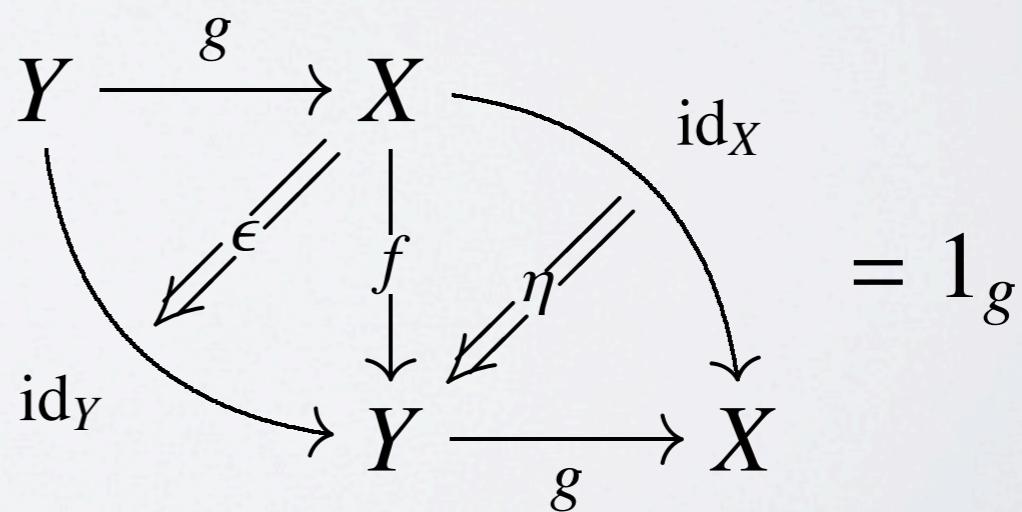
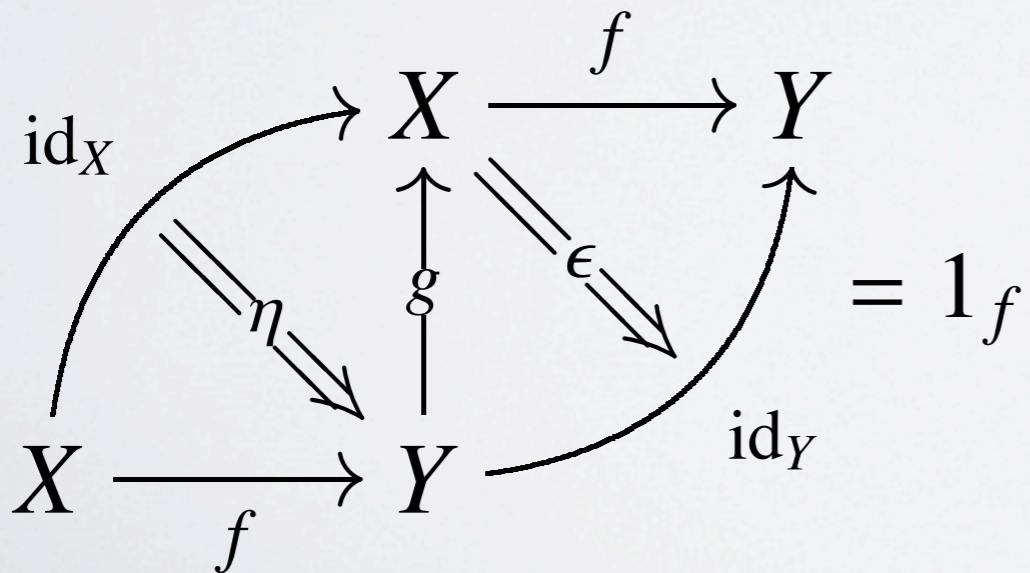
- **String diagrams at work**
- Process calculi
- Wire calculus

EQUIVALENCES IN 2-CATEGORIES

- Equivalences in 2-cats

$$\begin{array}{ll} f : X \rightarrow Y & \eta : \text{id}_X \Rightarrow gf \\ g : Y \rightarrow X & \epsilon : fg \Rightarrow \text{id}_Y \end{array}$$

- Adjoint equivalences in 2-cats, additionally



ADJOINT EQUIVALENCE LEMMA

- **Folklore:** If $f : X \rightarrow Y$ is part of an equivalence then it is part of an adjoint equivalence

- Proof:

$$f : X \rightarrow Y \quad \eta : \text{id}_X \Rightarrow gf$$

$$g : Y \rightarrow X \quad \epsilon : fg \Rightarrow \text{id}_Y$$

$$\epsilon' = fg \xrightarrow{\epsilon^{-1}fg} f g f g \xrightarrow{f\eta^{-1}g} fg \xrightarrow{\epsilon} \text{id}_Y$$

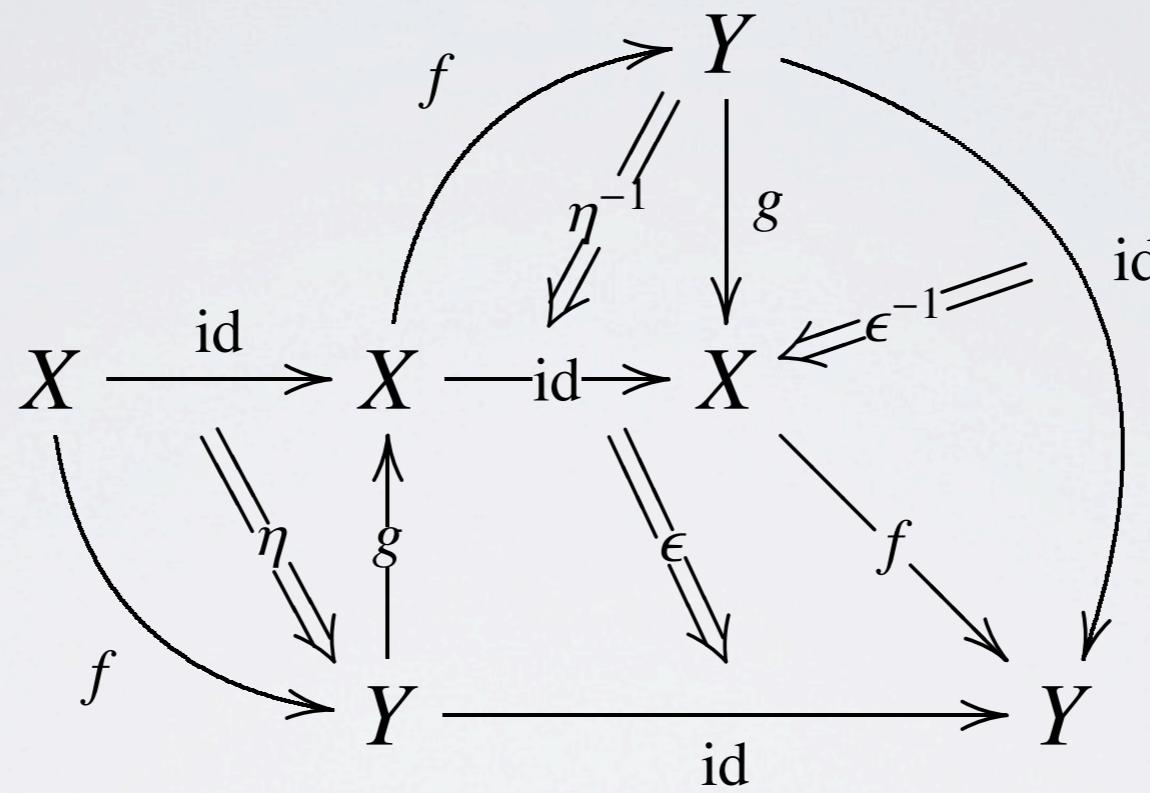
- Need to check:

$$\epsilon f \bullet f\eta^{-1}gf \bullet \epsilon^{-1}fgf \bullet f\eta = 1_f$$

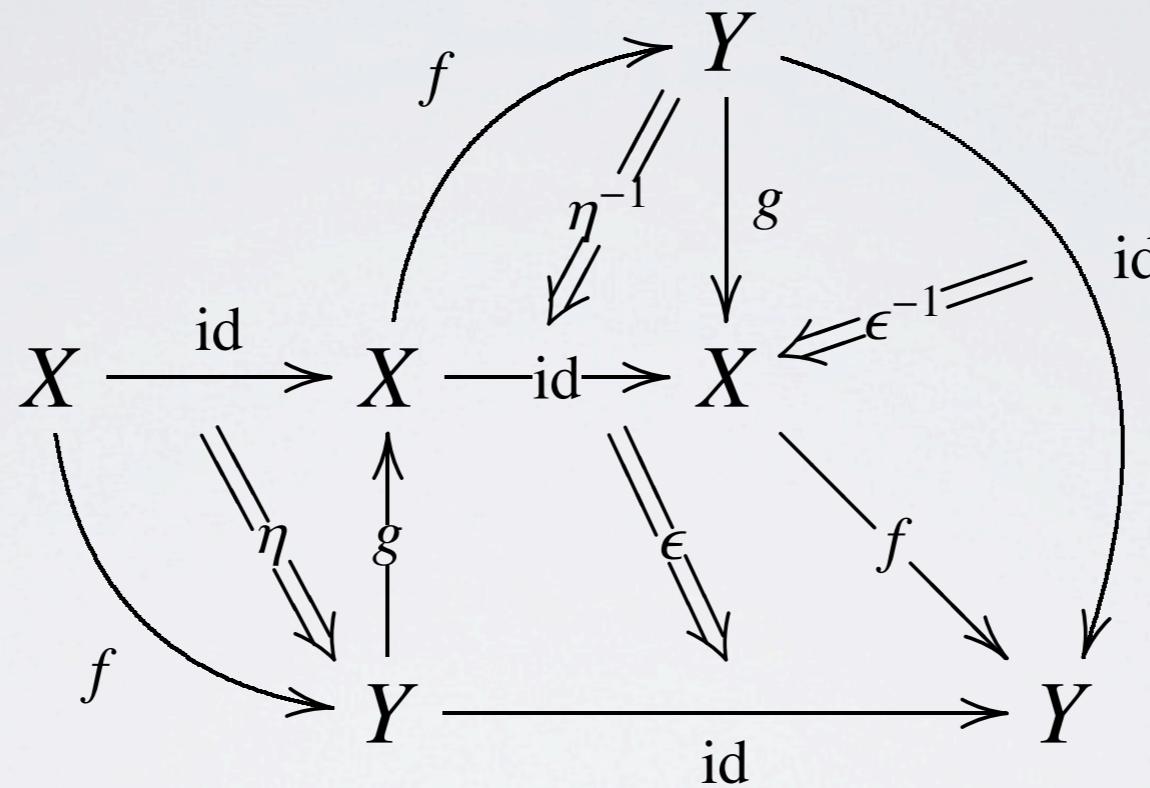
$$g\epsilon \bullet g f \eta^{-1} g \bullet g\epsilon^{-1}fg \bullet \eta g = 1_g$$

three pages of
3-dimensional diagram
chases?

PASTING DIAGRAMS

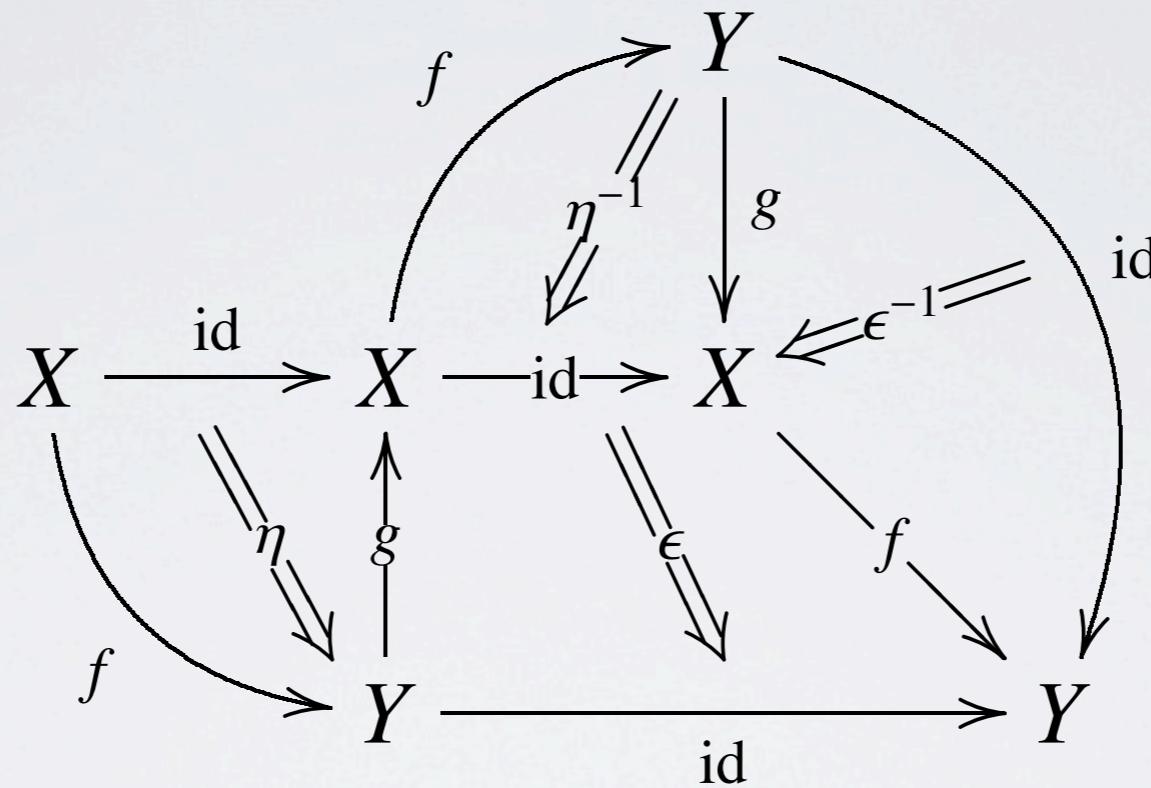


PASTING DIAGRAMS



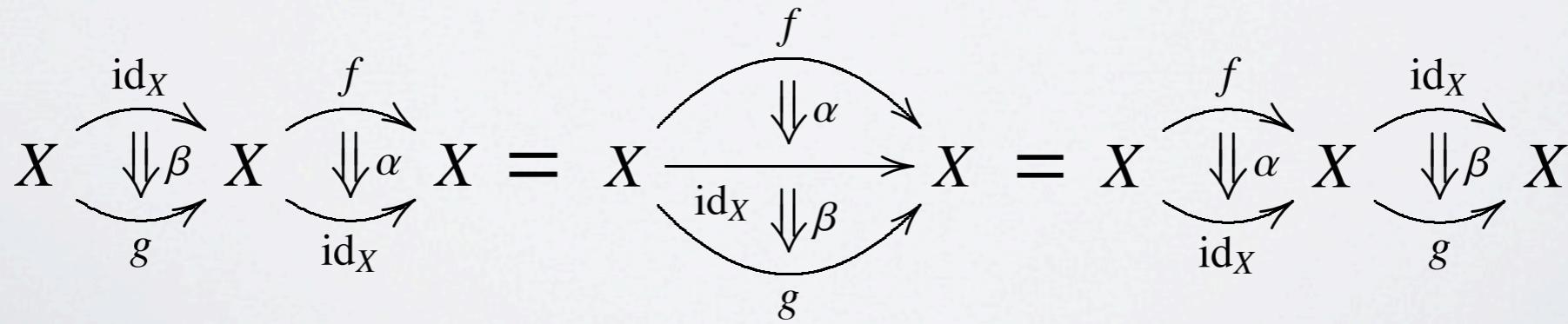
- Are not always helpful:

PASTING DIAGRAMS



- Are not always helpful:

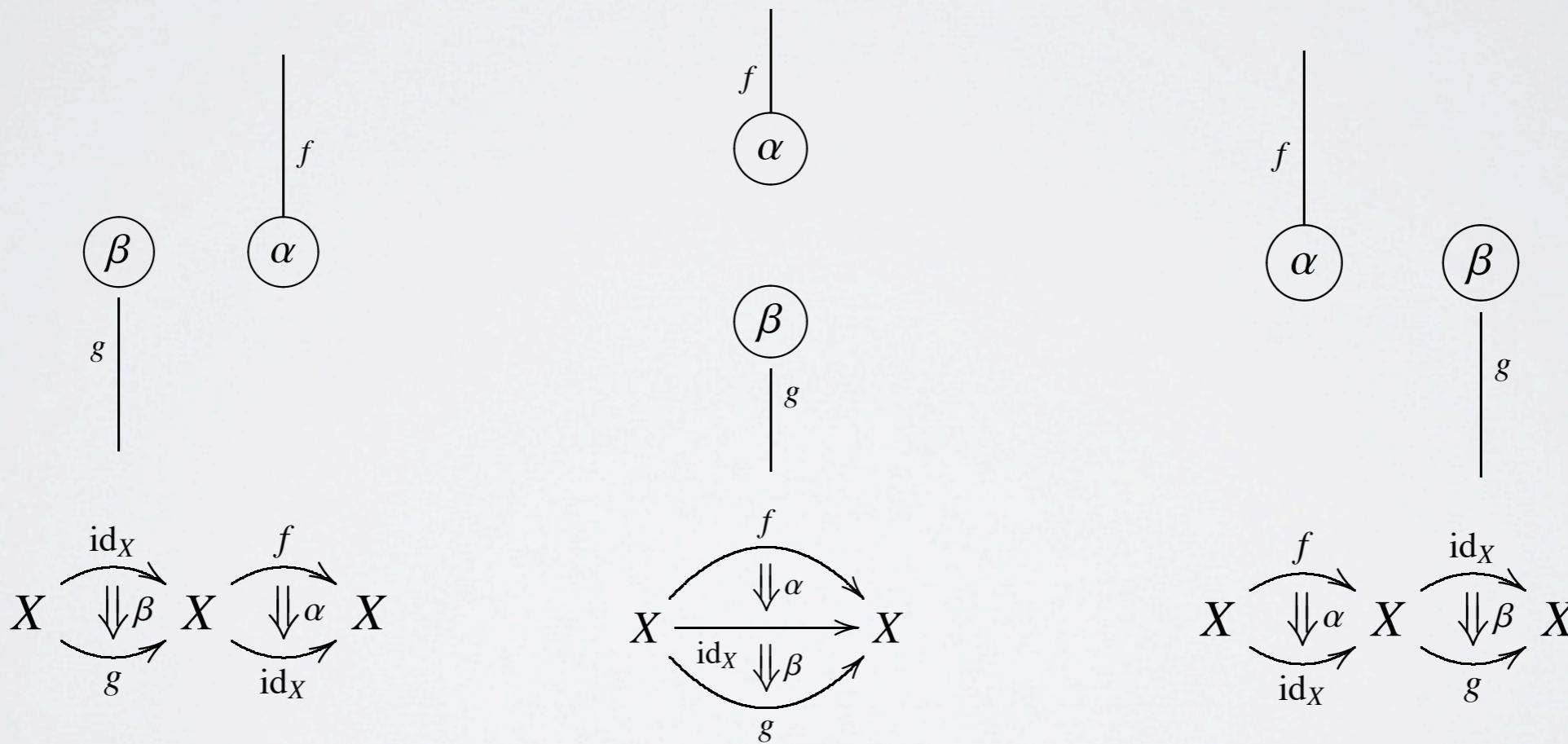
$$\alpha : f \Rightarrow \text{id}_X \quad \beta : \text{id}_X \Rightarrow g$$



STRING DIAGRAMS

(Joyal & Street '91)

$$\alpha : f \Rightarrow \text{id}_X \quad \beta : \text{id}_X \Rightarrow g$$

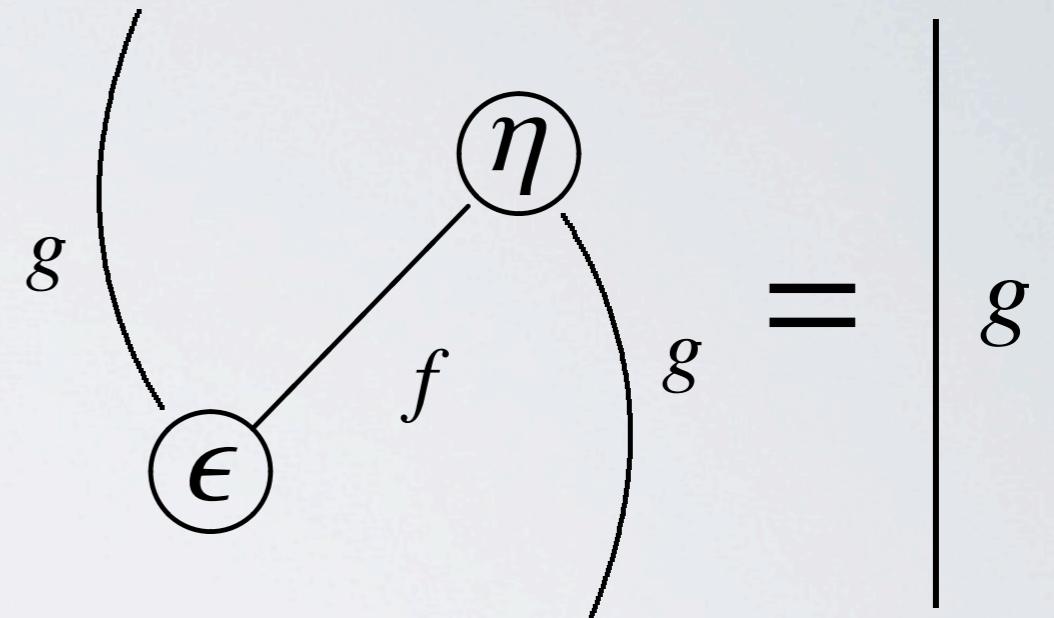
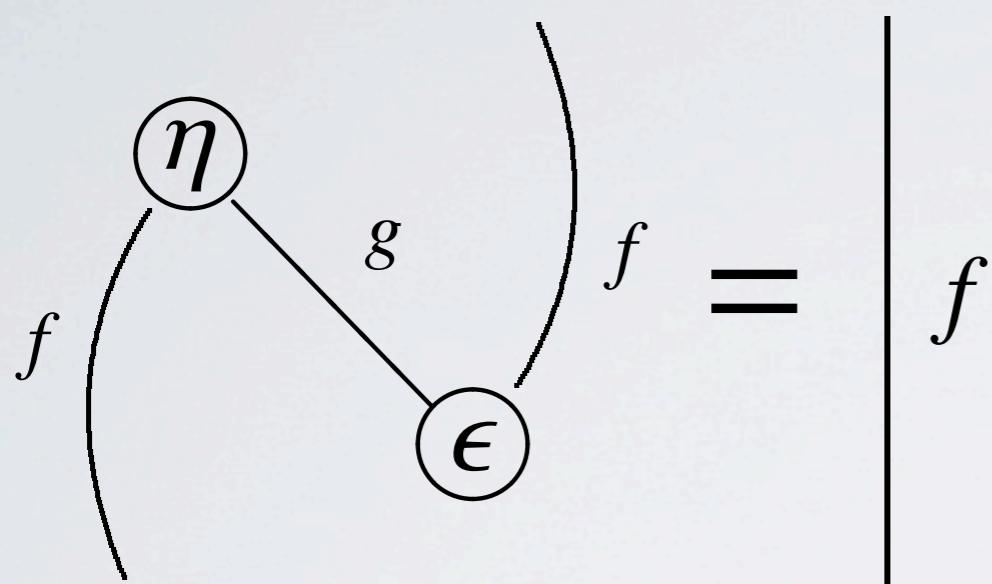


ADJOINT EQ WITH STRINGS

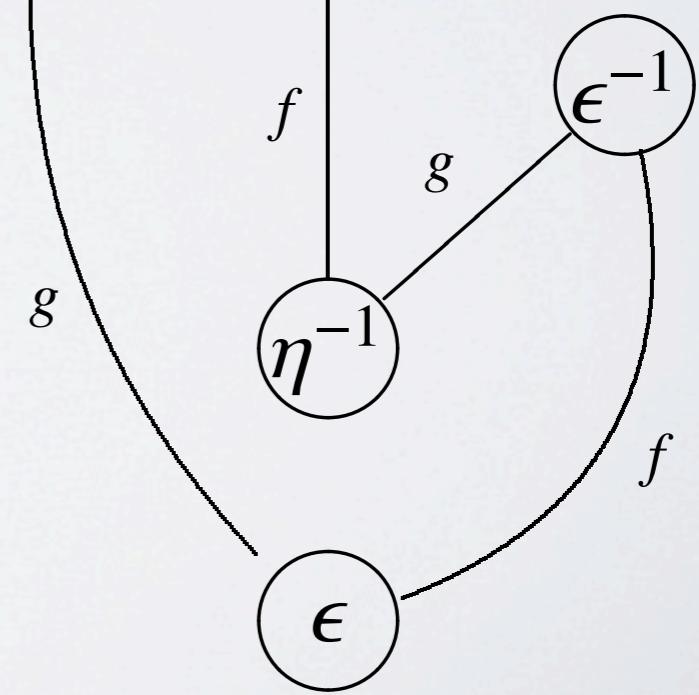
$$\begin{array}{c} \eta \\ f \swarrow \quad \searrow g \\ \epsilon \end{array} \quad f = \left| \begin{array}{c} f \end{array} \right|$$

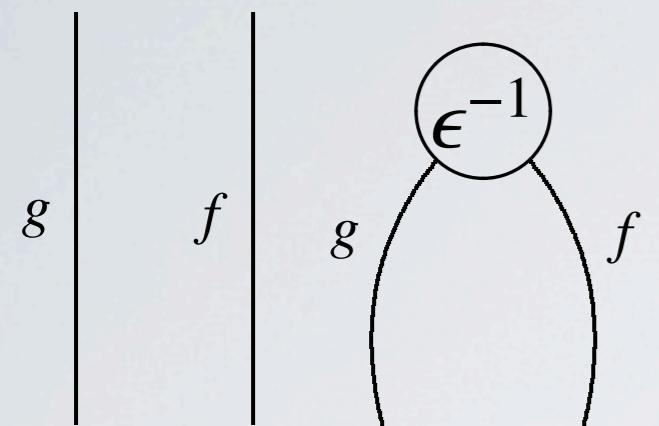
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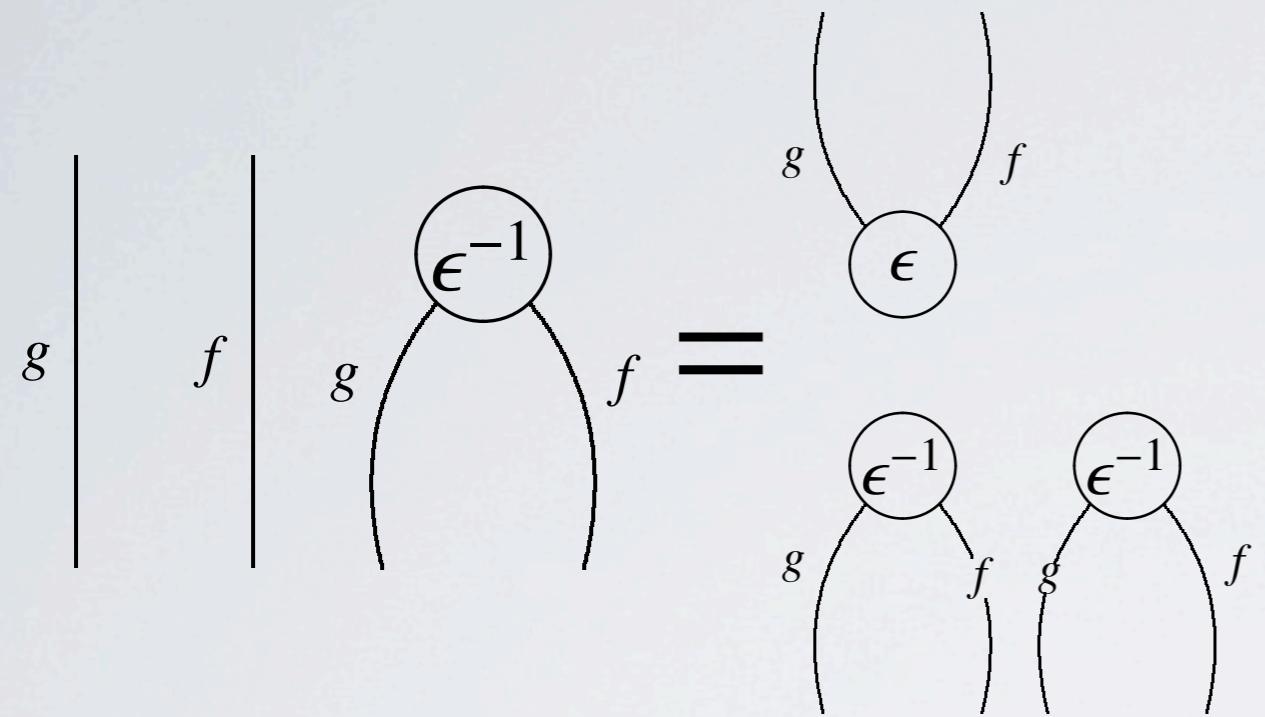
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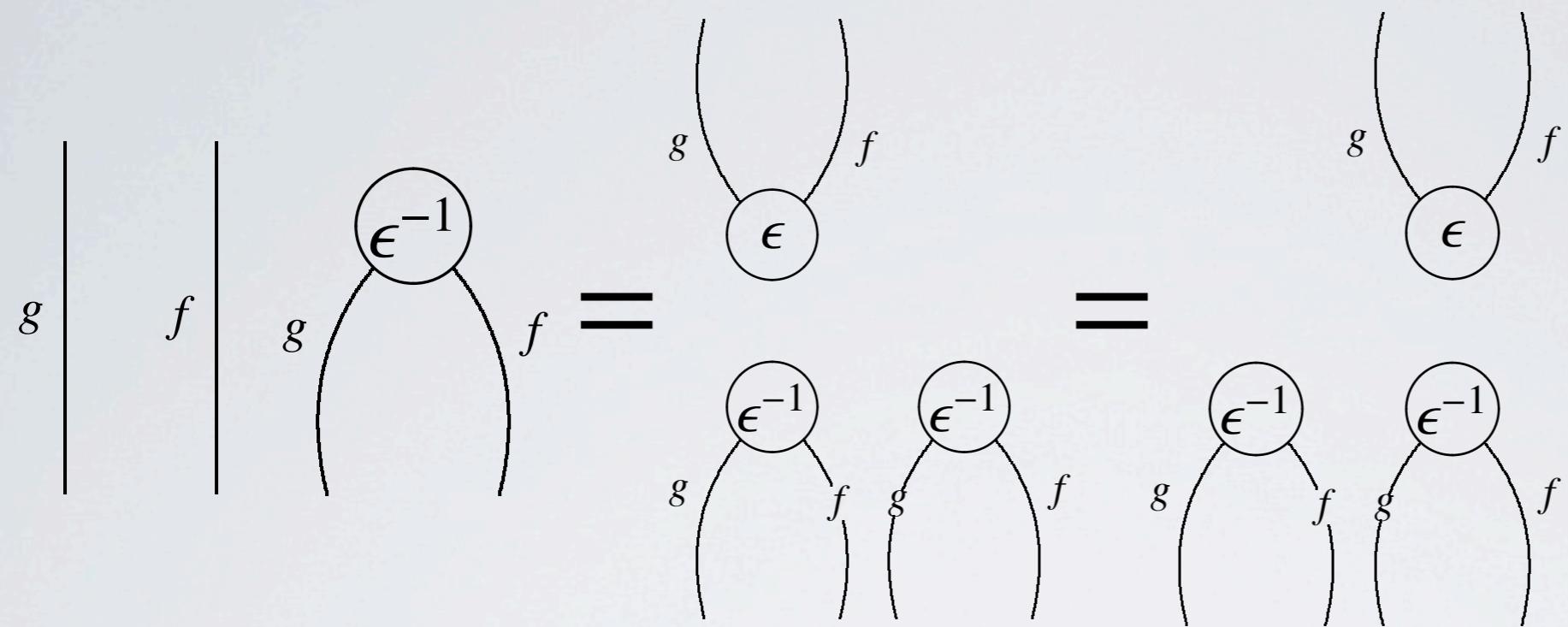


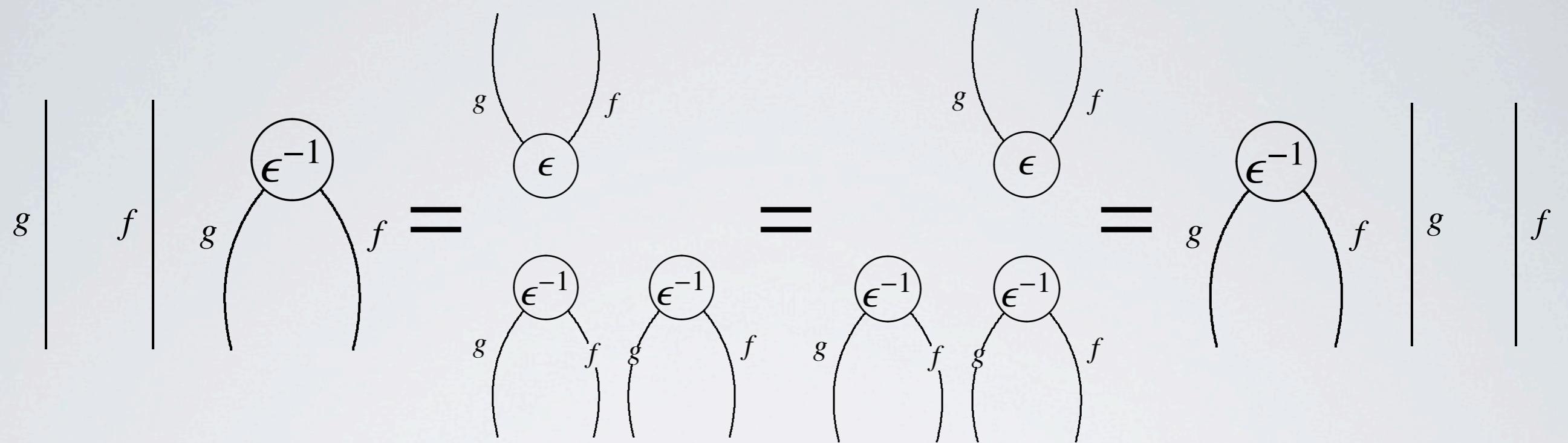
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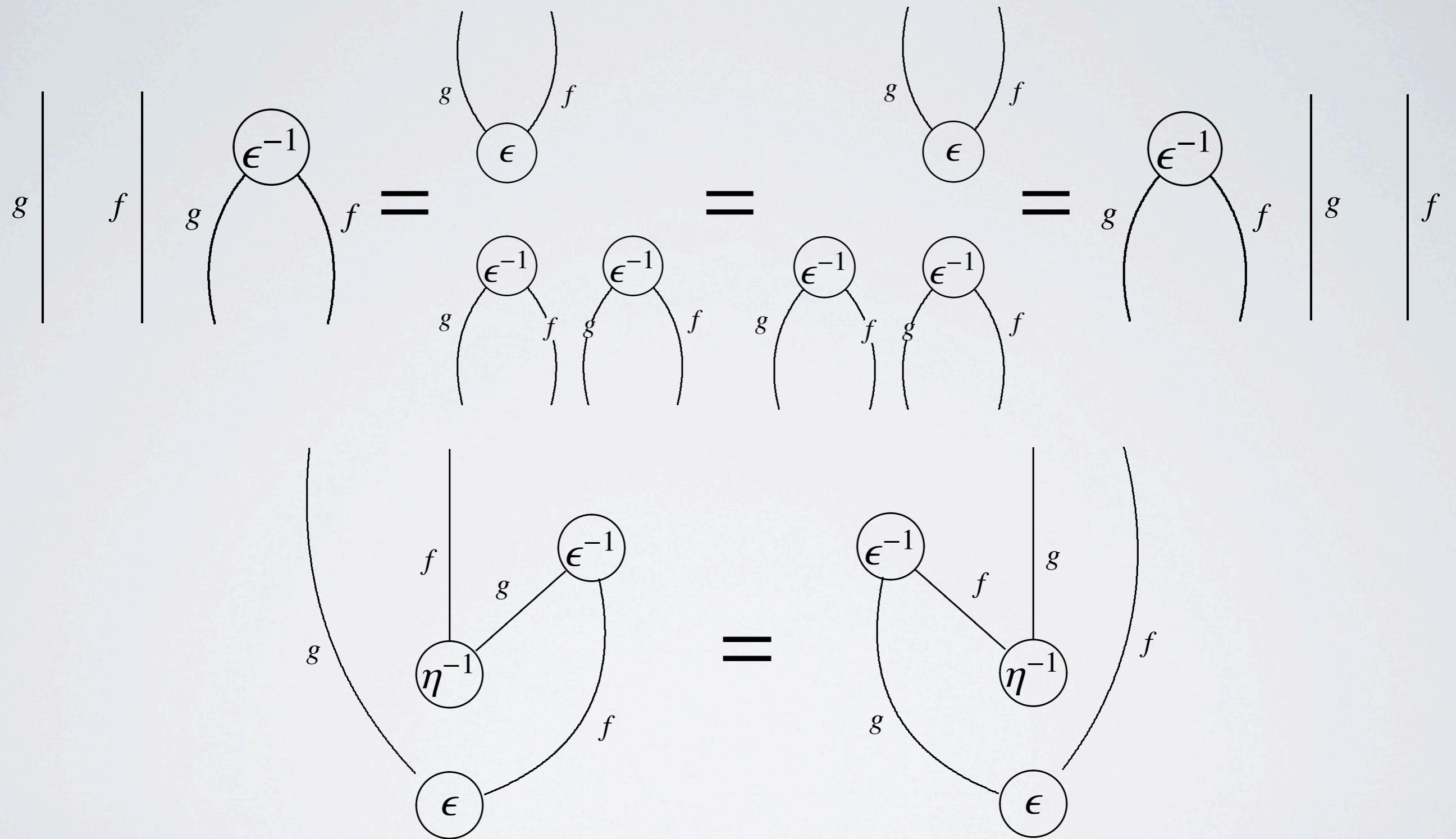


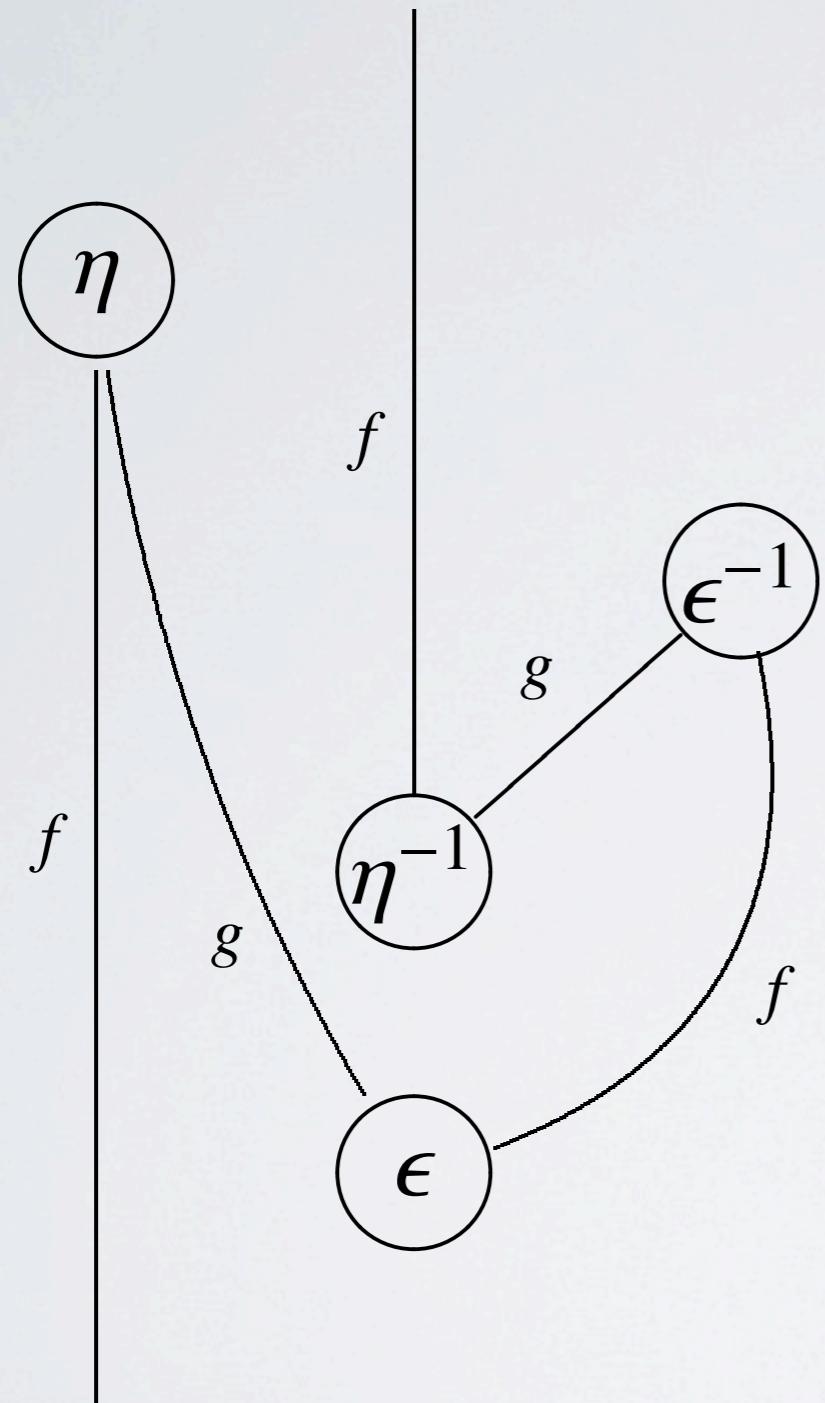


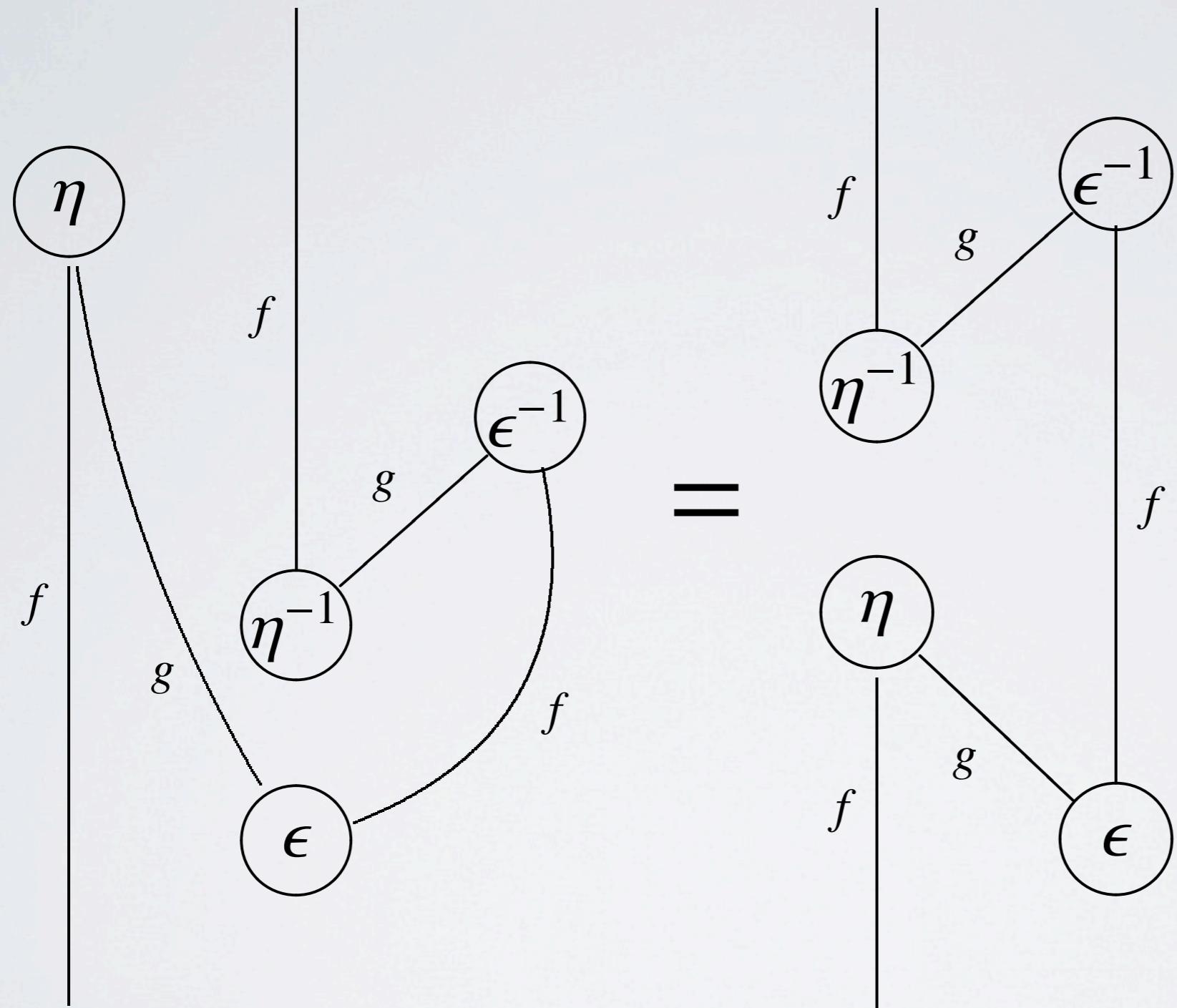


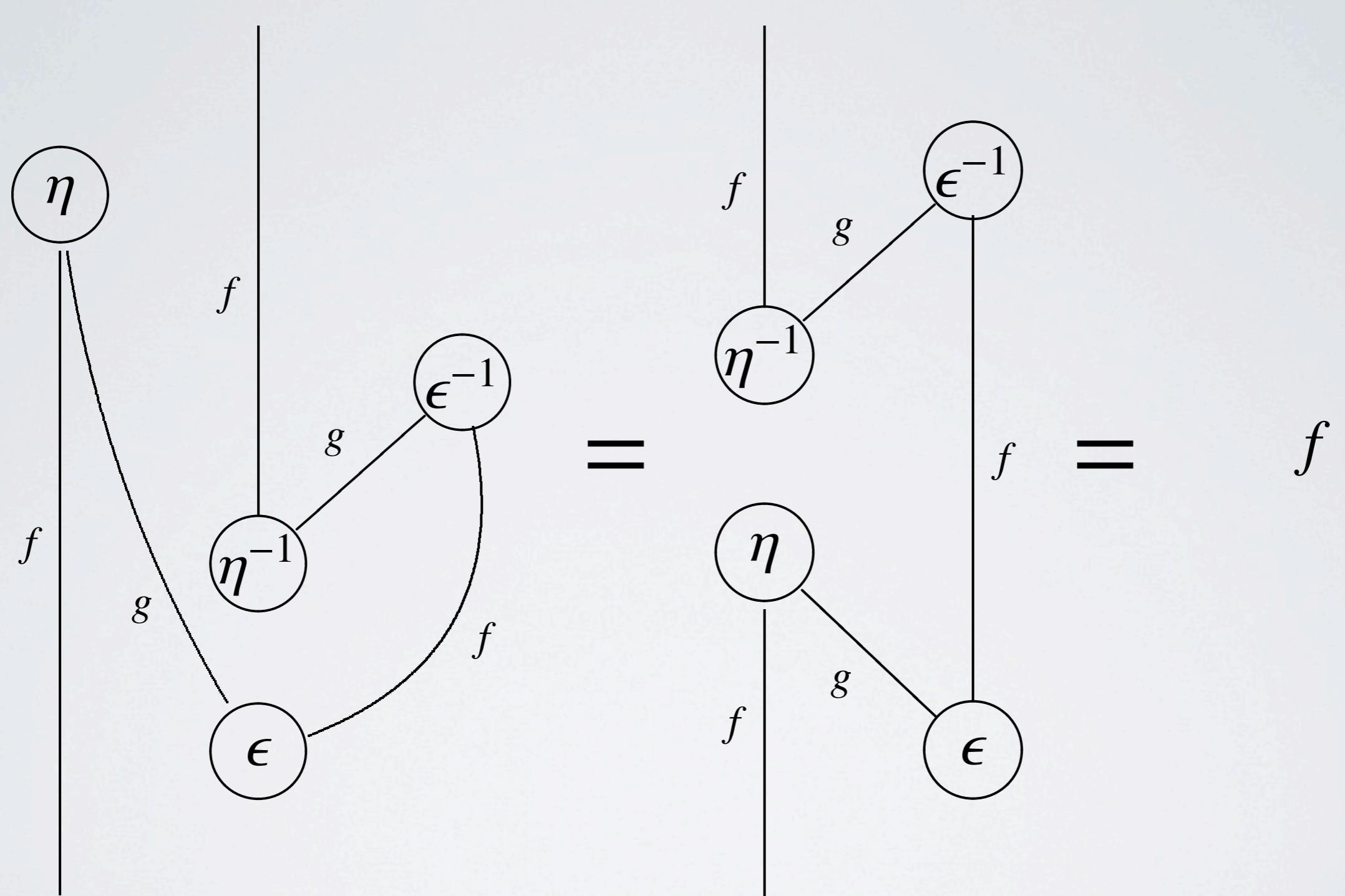


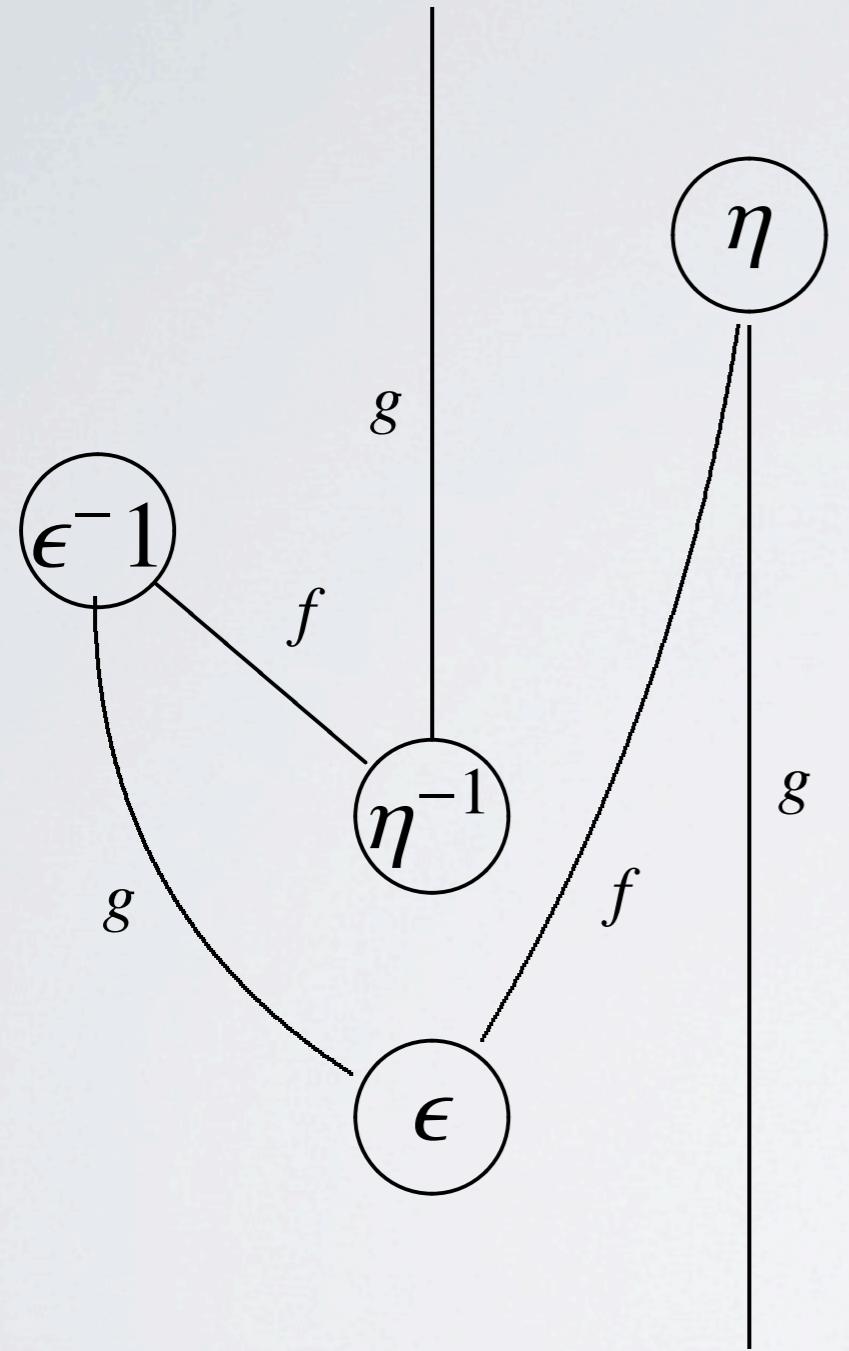


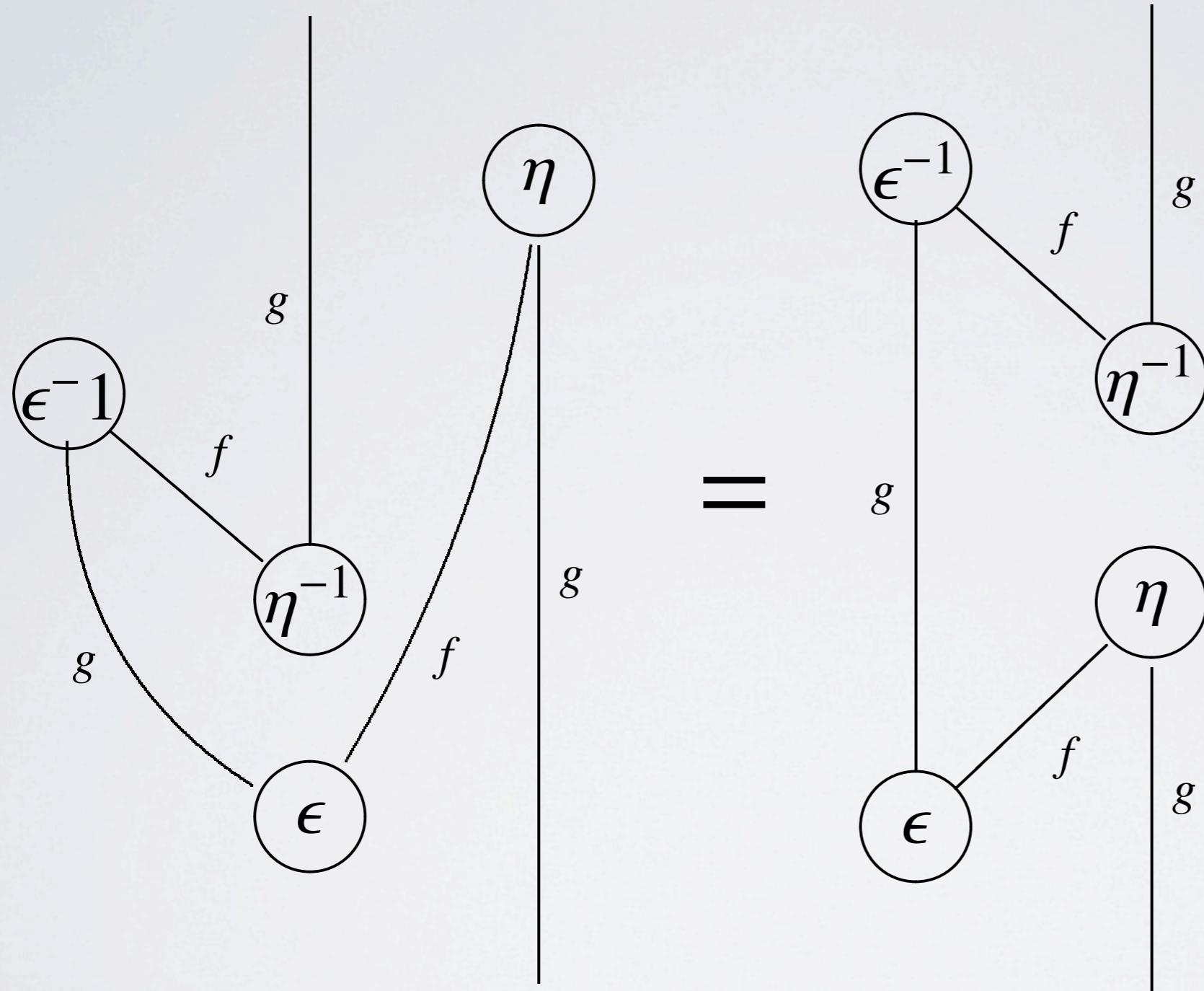


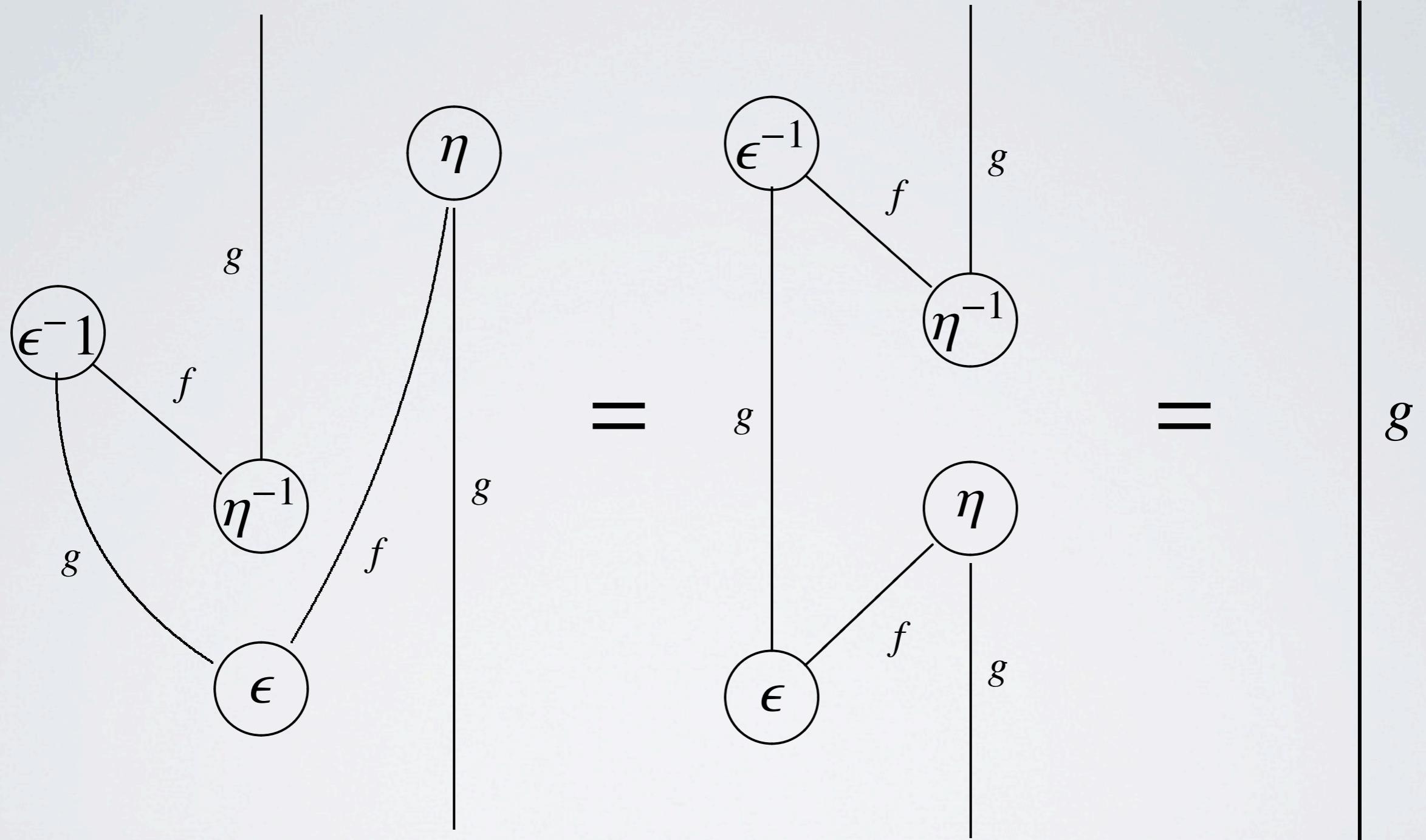












STRING DIAGRAMS

- apparently, a categorical trade secret for several years...
- used also to reason about arrows in monoidal categories
 - one object bicategory = monoidal category
 - one object 2-category = monoidal category with tensor associative “on the nose”

PLAN OF TALK

- String diagrams at work
- **Process calculi**
- Wire calculus

PROCESS CALCULI

- CCS 80s, Pi 90s, ambients, ...
- **common features:**
 - syntactic expressions represent “processes”
 - dynamics a first class entity (eg prefix, + & rec in CCS)
 - (structural) operational semantics
 - observational equivalence, (weak) bisimulation
 - (weak) bisimulation induces an algebra on syntactic terms
 - Dijkstra-Hoare-Milner parallel composition ||

ALGEBRA OF PROCESSES

- Milner's SOS:

$$\frac{P \xrightarrow{a} P'}{P \| Q \xrightarrow{a} P' \| Q} \text{ (||L)} \quad \frac{Q \xrightarrow{a} Q'}{P \| Q \xrightarrow{a} P \| Q'} \text{ (||R)} \quad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\bar{a}} Q'}{P \| Q \xrightarrow{\tau} P' \| Q'} \text{ (PAR)}$$

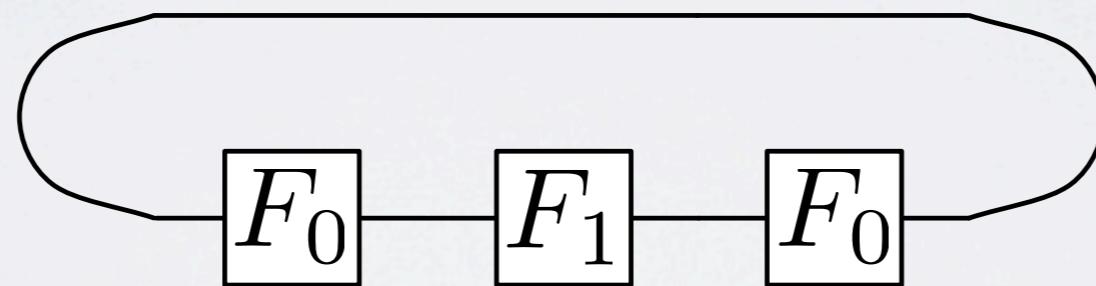
- what is the algebra induces by (weak) bisimilarity?
- the operation $\|$ is a commutative monoid
 - ie. processes live in a “chemical soup”
 - is it always the case that **concurrent universe = chemical soup?**
- interleaving gives **concurrency = nondeterminism**, always reasonable?
 - implementation issues: $\|$ is a very powerful scheduler

OTHER “CALCULI”

- Milner’s bigraphs
- Gadducci and Montanari’s tile systems
- RFC Walters’ Span(Graph)
- none of these is a process calculus in this traditional sense, ie
 - syntax + SOS + bisimulation congruence

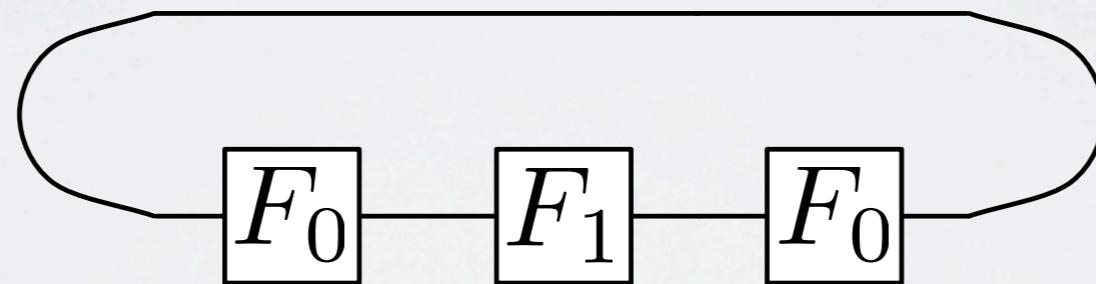
SHORTCOMINGS

How to model a network as below in a process calculus?



SHORTCOMINGS

How to model a network as below in a process calculus?



$d ; I \otimes (F_0 ; F_1 ; F_0) ; e$

PLAN OF TALK

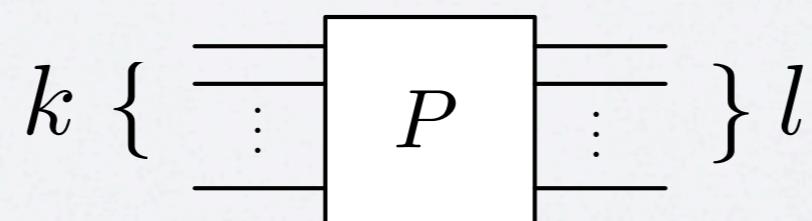
- String diagrams at work
- Process calculi
- **Wire calculus**

COMPONENTS

- Let Σ be a set of *signals* + silent action ι for int. computation
- for $k, l \in \mathbb{N}$ a (k, l) -transition is a labelled transition of the form

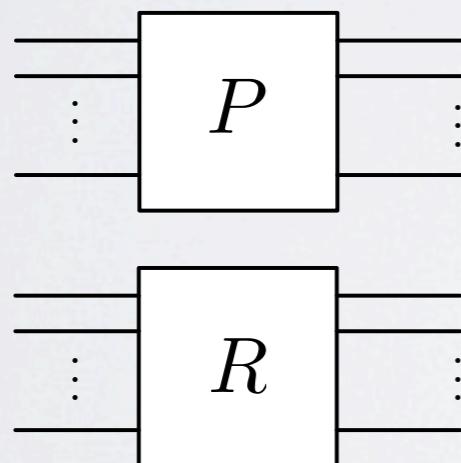
$$P \xrightarrow[\vec{b}]{} Q, \quad \#(\vec{a}) = k, \#(\vec{b}) = l$$

- any process in the wire calculus has a sort (k, l) and its semantics will be an LTS of (k, l) -transitions



ANOTHER PARALLEL COMPOSITION

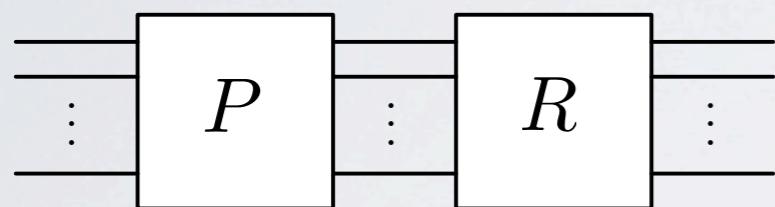
- processes are boxes with left and right boundary
- operators of the calculus allow us to specify & connect boxes
- labels have a monoidal structure (juxtaposition)



$$\frac{P \xrightarrow{\mathbf{a}} Q \quad R \xrightarrow{\mathbf{c}} S}{P \otimes R \xrightarrow{\mathbf{ac}} Q \otimes S} \text{ (TEN)}$$

- \otimes neither commutative nor interleaving

SYNCHRONISING ON BOUNDARY



$$\frac{P \xrightarrow[\mathbf{c}]{\mathbf{a}} Q \quad R \xrightarrow[\mathbf{b}]{\mathbf{c}} S}{P;R \xrightarrow[\mathbf{b}]{\mathbf{a}} Q;S} \text{ (CUT)}$$

- non commutative
- don't confuse with seq composition in imperative prog langs

DYNAMICS - CHOICE

- CSP-like (\square) external choice

$$\frac{P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q \quad (\mathbf{ab} \neq \iota)}{P+R \xrightarrow[\mathbf{b}]{\mathbf{a}} Q} (+L)$$

$$\frac{P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q \quad (\mathbf{ab} \neq \iota)}{R+P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q} (+R)$$

$$\frac{P \xrightarrow{\iota} Q \quad R \xrightarrow{\iota} S}{P+R \xrightarrow{\iota} Q+S} (+\iota)$$

DYNAMICS - RECURSION

$$\frac{P[\mu Y. P/Y] \xrightarrow{\frac{a}{b}} Q}{\mu Y. P \xrightarrow{\frac{a}{b}} Q} (\text{REC})$$

DYNAMICS - PREFIX

- signals live in some set Σ
- prefix strings: $M ::= \epsilon \mid x \mid \lambda x \mid \iota \mid \sigma \in \Sigma \mid MM$
- prefixes: $P ::= \dots \mid \frac{M}{M} P$
- $\frac{u}{v}P$ with λx in u or v binds free occurrences of x in P
- substitution: $\sigma : bd(\frac{u}{v}) \rightarrow \Sigma + \{\iota\}$

$$\frac{}{\frac{u}{v}P \xrightarrow[\frac{v}{\sigma}]{} P|_{\sigma}} \text{(PREF)}$$

Example: $\frac{\lambda x a}{\lambda y} P \xrightarrow[\beta]{\alpha a} P[\alpha/x, \beta/y]$ for all $\alpha, \beta \in \Sigma$

EXAMPLE - BASIC WIRES

Picture

Expression

$$I \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x}{\lambda x} Y$$

Behaviour

$$\frac{}{I \xrightarrow{a} I} (\text{ID})$$



$$d \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x \lambda x}{\lambda x \lambda x} Y$$

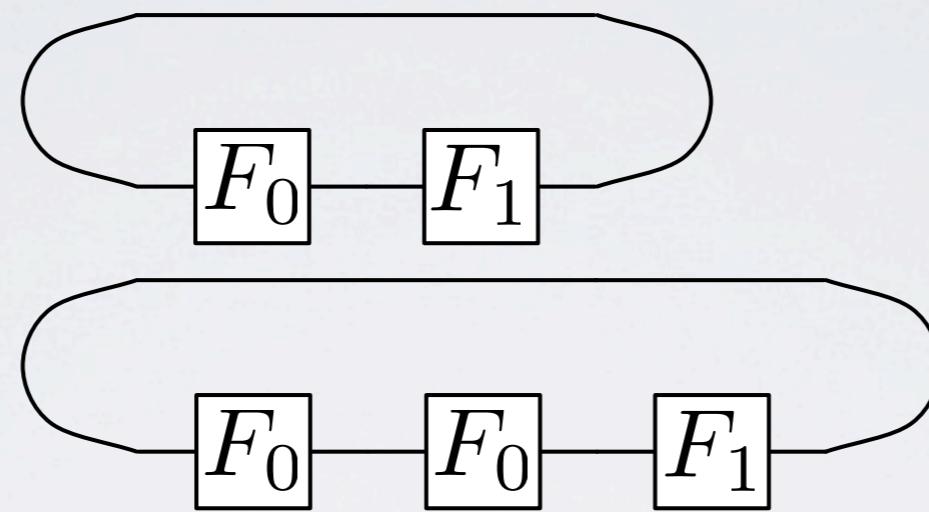
$$\frac{}{d \xrightarrow{aa} d} (\text{d})$$



$$e \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x \lambda x}{\lambda x \lambda x} Y$$

$$\frac{}{e \xrightarrow{aa} e} (\text{e})$$

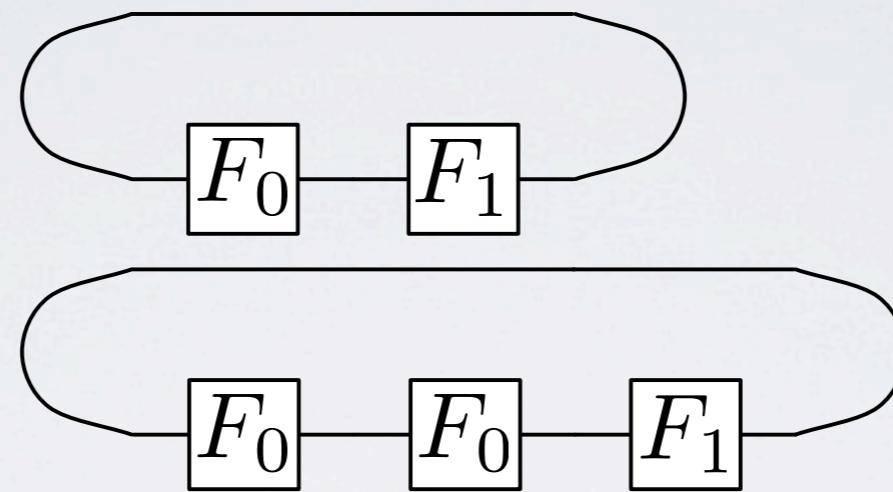
CONCURRENCY RULES I



$$\frac{}{P \xrightarrow[\iota]{\iota} P} (\text{REFL})$$

Intuition: unconnected components cannot block each other

CONCURRENCY RULES 2



$$\frac{P \xrightarrow[\iota]{\iota} R \quad R \xrightarrow[\mathbf{b}]{\mathbf{a}} Q}{P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q} (\iota L)$$

$$\frac{P \xrightarrow[\mathbf{b}]{\mathbf{a}} R \quad R \xrightarrow[\iota]{\iota} Q}{P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q} (\iota R)$$

Intuition: components not assumed to run at the same speed

WEAK VS STRONG ISSUES

$$\frac{}{P \xrightarrow[\iota]{\iota} P} (\text{REFL})$$

$$\frac{P \xrightarrow[\iota]{\iota} R \quad R \xrightarrow[\mathbf{b}]{\mathbf{a}} Q}{P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q} (\iota L)$$

$$\frac{P \xrightarrow[\mathbf{b}]{\mathbf{a}} R \quad R \xrightarrow[\iota]{\iota} Q}{P \xrightarrow[\mathbf{b}]{\mathbf{a}} Q} (\iota R)$$

- rules imply that ordinary (strong) bisimilarity = weak bisimilarity

SUMMARY - THE WIRE CALCULUS

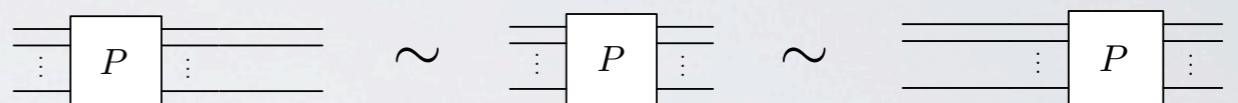
$$\begin{aligned}
 P & ::= Y \mid P ; P \mid P \otimes P \mid \frac{M}{M}P \mid P + P \mid \mu Y : \tau . P \\
 M & ::= \epsilon \mid x \mid \lambda x \mid \iota \mid \sigma \in \Sigma
 \end{aligned}$$

$$\begin{array}{c}
 \frac{}{P \xrightarrow{\iota} P} (\text{REFL}) \quad \frac{P \xrightarrow{\iota} R \quad R \xrightarrow{\mathbf{a}} Q}{P \xrightarrow{\mathbf{a}} Q} (\iota\text{L}) \quad \frac{P \xrightarrow{\mathbf{a}} R \quad R \xrightarrow{\iota} Q}{P \xrightarrow{\mathbf{a}} Q} (\iota\text{R}) \\
 \\
 \frac{P \xrightarrow{\mathbf{a}} Q \quad R \xrightarrow{\mathbf{b}} S}{P; R \xrightarrow{\mathbf{a}} Q; S} (\text{CUT}) \quad \frac{P \xrightarrow{\mathbf{a}} Q \quad R \xrightarrow{\mathbf{d}} S}{P \otimes R \xrightarrow{\mathbf{a}\mathbf{c}} Q \otimes S} (\text{TEN}) \\
 \\
 \frac{\frac{u}{v} P \xrightarrow{u|\sigma} P|\sigma}{P \xrightarrow{v|\sigma} P|\sigma} (\text{PREF}) \quad \frac{P \xrightarrow{\mathbf{a}} Q \quad (\mathbf{ab} \neq \iota)}{P + R \xrightarrow{\mathbf{a}} Q} (+\text{L}) \quad \frac{P \xrightarrow{\mathbf{a}} Q \quad (\mathbf{ab} \neq \iota)}{R + P \xrightarrow{\mathbf{a}} Q} (+\text{R}) \\
 \\
 \frac{P \xrightarrow{\iota} Q \quad R \xrightarrow{\iota} S}{P + R \xrightarrow{\iota} Q + S} (+\iota) \quad \frac{P[\mu Y. P/Y] \xrightarrow{\mathbf{a}} Q}{\mu Y. P \xrightarrow{\mathbf{a}} Q} (\text{REC})
 \end{array}$$

SIMPLE BISIMULATIONS

$$(P ; Q) ; R \sim P ; (Q ; R)$$

$$\forall P : (m, n), P ; (\otimes_n I) \sim P \sim (\otimes_m I) ; P$$



So terms up to bisim are the arrows of a category

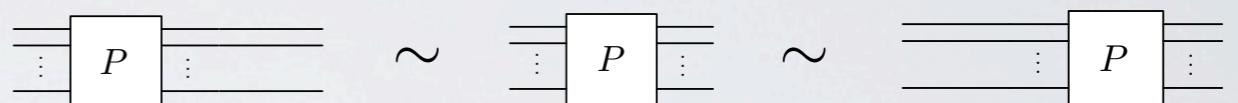
objects: natural numbers

arrows: terms up to bisimulation

SIMPLE BISIMULATIONS

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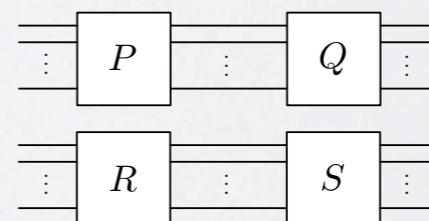
So terms up to bisim are the arrows of a category

objects: natural numbers

arrows: terms up to bisimulation

$$(P \otimes R) ; (Q \otimes S) \sim (P ; Q) \otimes (R ; S)$$

$$(P \otimes Q) \otimes R \sim P \otimes (Q \otimes R)$$

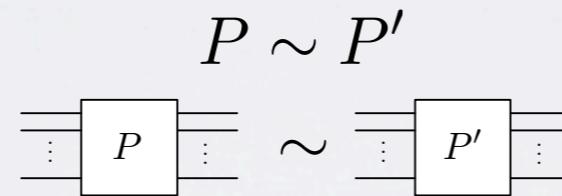


... and the category is monoidal

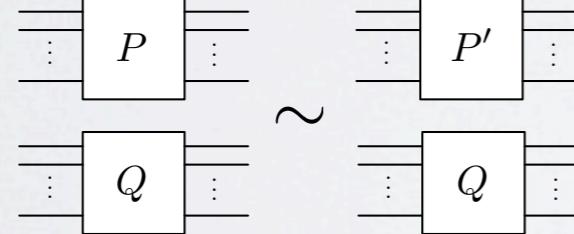
BISIMILARITY

- is a congruence wrt all operations in the syntax

$$P, P' : (k, l) \quad Q : (m, n) \quad R : (l, l') \quad S : (k', k)$$

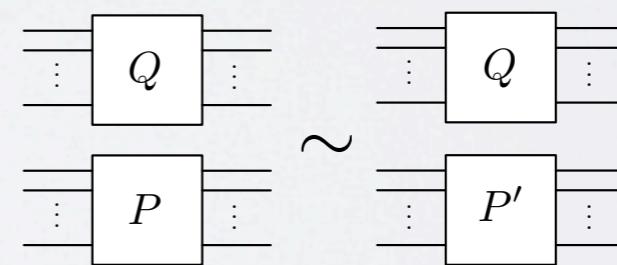


$$P \otimes Q \sim P' \otimes Q$$

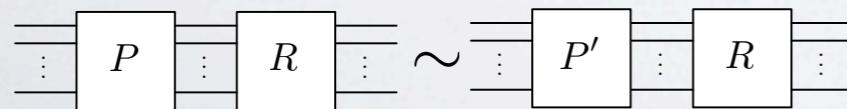


then

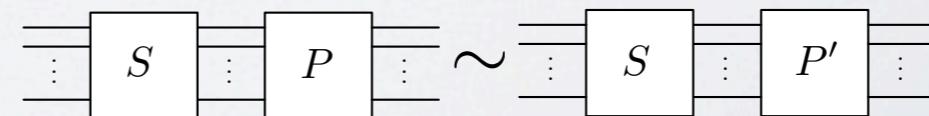
$$Q \otimes P \sim Q \otimes P'$$



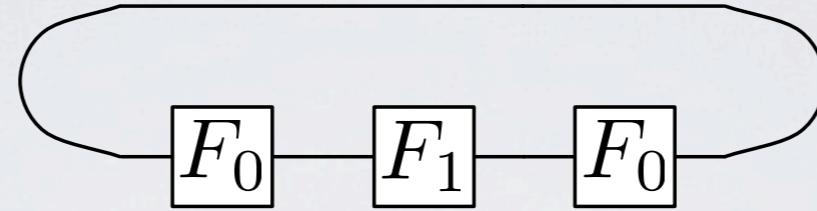
$$P ; R \sim P' ; R$$



$$S ; P \sim S ; P'$$



EXAMPLE



$$\frac{}{F_0 \xrightarrow[0]{0} F_0} \text{(0SET0)} \quad \frac{}{F_0 \xrightarrow[0]{1} F_1} \text{(0SET1)} \quad \frac{}{F_0 \xrightarrow[\iota]{\iota} F_0} \text{(0REFL)}$$

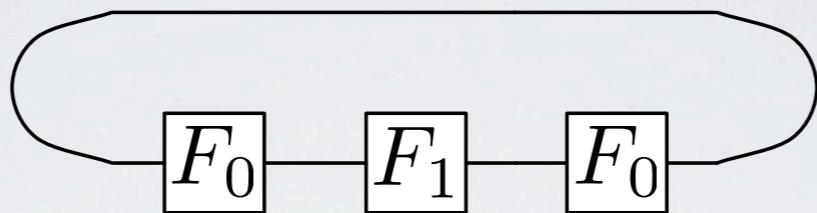
$$\frac{}{F_1 \xrightarrow[1]{1} F_1} \text{(1SET1)} \quad \frac{}{F_1 \xrightarrow[1]{0} F_0} \text{(1SET0)} \quad \frac{}{F_1 \xrightarrow[\iota]{\iota} F_1} \text{(1REFL)}$$

Expressions

$$F_0 \stackrel{\text{def}}{=} \mu Y. \frac{0}{0} Y + \frac{1}{0} \mu Z. (\frac{1}{1} Z + \frac{0}{1} Y) \quad F_1 \stackrel{\text{def}}{=} \mu Z. \frac{1}{1} Z + \frac{0}{1} \mu Y. (\frac{0}{0} Y + \frac{1}{0} Z)$$

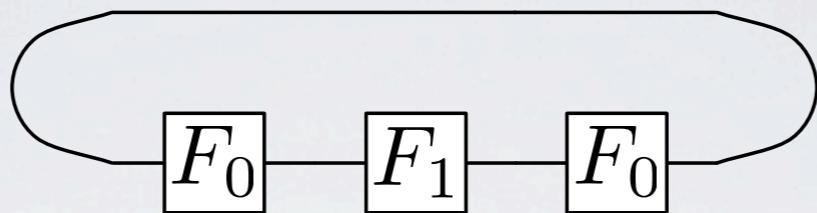
$$d ; I \otimes (F_0 ; F_1 ; F_0) ; e$$

EXAMPLE CTD



$$\frac{F_0 \xrightarrow[z]{x} X \quad F_1 \xrightarrow[y]{z} Y \qquad y = 1}{F_0; F_1 \xrightarrow[y]{x} X; Y} \text{ (CUT)} \quad z = 0 \qquad Y = F_0$$

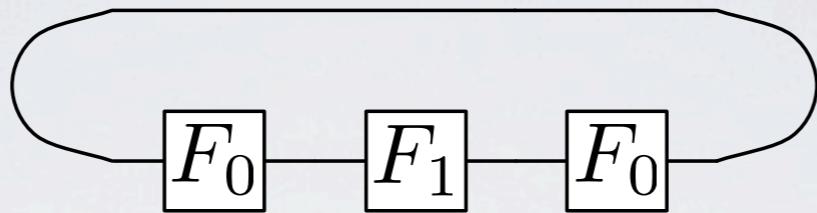
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$$\frac{F_0 \xrightarrow[0]{x} X \quad F_1 \xrightarrow[1]{0} F_0}{F_0; F_1 \xrightarrow[1]{x} X; F_0} \text{ (CUT)}$$

EXAMPLE CTD

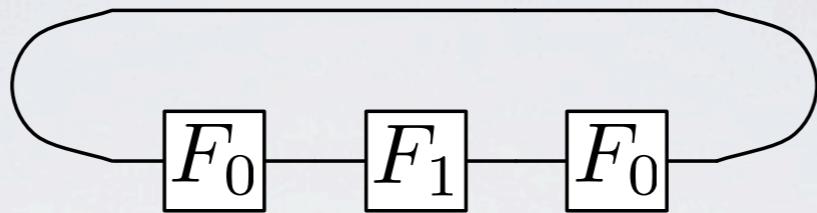


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$$\frac{F_0;F_1 \xrightarrow[1]{x} X;F_0 \quad F_0 \xrightarrow[0]{1} F_1}{F_0;F_1;F_0 \xrightarrow[0]{x} X;F_0;F_1} \text{ (CUT)}$$

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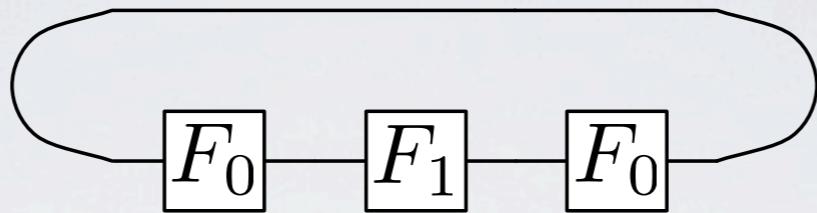
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$$\frac{F_0;F_1 \xrightarrow[1]{x} X;F_0 \quad F_0 \xrightarrow[0]{1} F_1}{F_0;F_1;F_0 \xrightarrow[0]{x} X;F_0;F_1} \text{ (CUT)}$$

$$\frac{}{I \otimes (F_0;F_1;F_0) \xrightarrow[w0]{wx} I \otimes (X;F_0;F_1)} \text{ (⊗)}$$

EXAMPLE CTD



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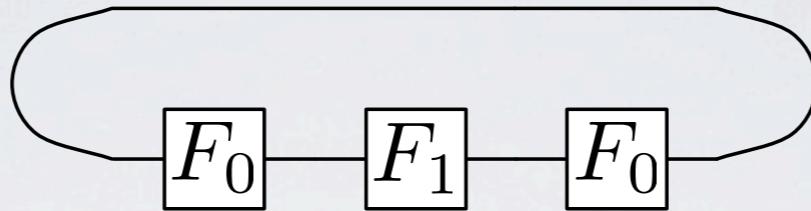
$$\frac{F_0 \xrightarrow[0]{x} X \quad F_1 \xrightarrow[1]{0} F_0}{F_0;F_1 \xrightarrow[1]{x} X;F_0} \text{ (CUT)}$$

$$\frac{F_0;F_1 \xrightarrow[1]{x} X;F_0 \quad F_0 \xrightarrow[0]{1} F_1}{F_0;F_1;F_0 \xrightarrow[0]{x} X;F_0;F_1} \text{ (CUT)}$$

$$\frac{}{I \otimes (F_0;F_1;F_0) \xrightarrow[w0]{wx} I \otimes (X;F_0;F_1)} (\otimes)$$

$$\frac{}{(I \otimes (F_0;F_1;F_0));\mathbf{e} \xrightarrow{0x} I \otimes (X;F_0;F_1);\mathbf{e}} \text{ (CUT)}$$

EXAMPLE CTD



$$\frac{F_0 \xrightarrow[z]{x} X \quad F_1 \xrightarrow[y]{z} Y}{F_0;F_1 \xrightarrow[y]{x} X;Y} \text{ (CUT)} \quad \begin{matrix} y = 1 \\ z = 0 \\ Y = F_0 \end{matrix}$$

$$\frac{F_0 \xrightarrow[0]{x} X \quad F_1 \xrightarrow[1]{0} F_0}{F_0;F_1 \xrightarrow[1]{x} X;F_0} \text{ (CUT)}$$

$$\frac{F_0;F_1 \xrightarrow[1]{x} X;F_0 \quad F_0 \xrightarrow[0]{1} F_1}{F_0;F_1;F_0 \xrightarrow[0]{x} X;F_0;F_1} \text{ (CUT)}$$

$$\frac{}{I \otimes (F_0;F_1;F_0) \xrightarrow[w0]{wx} I \otimes (X;F_0;F_1)} \text{ (}\otimes\text{)}$$

$$\frac{}{(I \otimes (F_0;F_1;F_0));\mathbf{e} \xrightarrow{0x} I \otimes (X;F_0;F_1);\mathbf{e}} \text{ (CUT)}$$

$$\frac{}{\mathbf{d};(I \otimes (F_0;F_1;F_0));\mathbf{e} \rightarrow \mathbf{d};I \otimes (F_0;F_0;F_1);\mathbf{e}} \text{ (CUT)}$$

ALGEBRA

- what is the resulting algebra?
 - objects = natural numbers; arrows = equivalence classes wrt bisimilarity
 - a strictly associative monoidal category
 - d and e yield compact closed structure
- easy to define a directed variant of calculus
- See “*A non-interleaving process calculus for multi-party synchronisation*” in Proc. ICE ’09

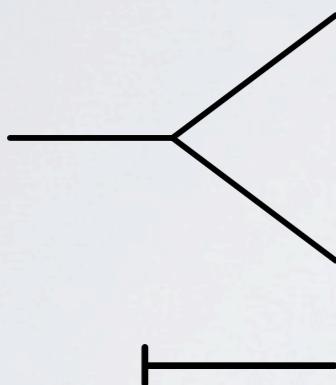
RELATED WORK

- **related work** (incomplete!):
 - RFC Walters et al:
 - $\text{Span}(\mathbf{Graph})$ - similar algebra, no SOS, dynamics not first class members of language
 - Gadducci Montanari et al:
 - tiles - similar algebra, same SOS for tensor and composition, dynamics not first class, less structure (more general) wrt weak issues
 - Abramsky et al:
 - interaction categories - similar SOS for tensor and composition, much more involved type structure
 - Arbab
 - Reo, etc: similar modelling style but has semantic issues, “user-defined” dynamics
 - Stefanescu
 - network algebra - monolithic, hard to tell what is primitive

CONCLUSION

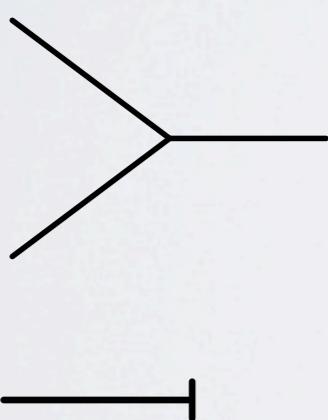
- Wire-calculus: a process calculus that is fundamentally different from existing calculi yet shares many of their features
- mathematically interesting algebra
- future work
 - expressivity issues wrt to traditional calculi
 - categorical structure?
 - continue Selinger's work on axioms for asynchrony

OTHER WIRES



$$\Delta \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x}{\lambda x \lambda x} Y$$

$$\frac{}{\Delta \xrightarrow{aa} \Delta} (\Delta)$$



$$\top \stackrel{\text{def}}{=} \mu Y. \frac{}{\lambda x} Y$$

$$\frac{}{\top \xrightarrow{a} \top} (\top)$$

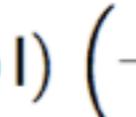
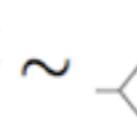
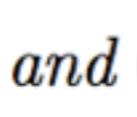
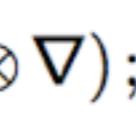
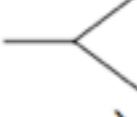
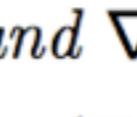
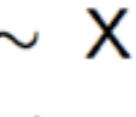
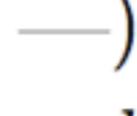
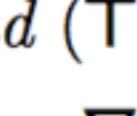
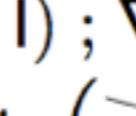
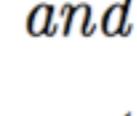
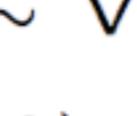
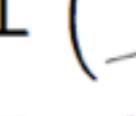
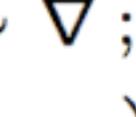
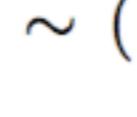
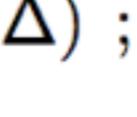
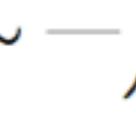
$$\nabla \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x \lambda x}{\lambda x} Y$$

$$\frac{}{\nabla \xrightarrow{aa} \nabla} (\nabla)$$

$$\perp \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x}{\lambda x} Y$$

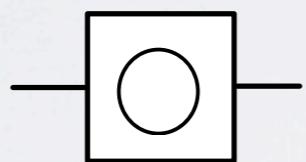
$$\frac{}{\perp \xrightarrow{a} \perp} (\perp)$$

TRIVIAL BISIMULATIONS

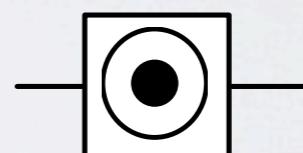
- (i) $\Delta ; (\mathbb{I} \otimes \Delta) \sim \Delta ; (\Delta \otimes \mathbb{I})$ ( \sim ) and $(\mathbb{I} \otimes \nabla) ; \nabla \sim (\nabla \otimes \mathbb{I}) ; \nabla$ ( \sim )
- (ii) $\Delta ; X \sim \Delta$ ( \sim ) and $\nabla \sim X ; \nabla$ ( \sim )
- (iii) $\Delta ; (\perp \otimes \mathbb{I}) \sim \mathbb{I}$ ( \sim ) and $(\top \otimes \mathbb{I}) ; \nabla \sim \mathbb{I}$ ( \sim )
- (iv) $\top ; \Delta \sim d$ ( \sim ) and $e \sim \nabla ; \perp$ ( \sim )
- (v) $(\Delta \otimes \mathbb{I}) ; (\mathbb{I} \otimes \nabla) \sim \nabla ; \Delta \sim (\mathbb{I} \otimes \Delta) ; (\nabla \otimes \mathbb{I})$ ( \sim  \sim ) and $\Delta ; \nabla \sim \mathbb{I}$ ( \sim )

PETRI NET PLACES

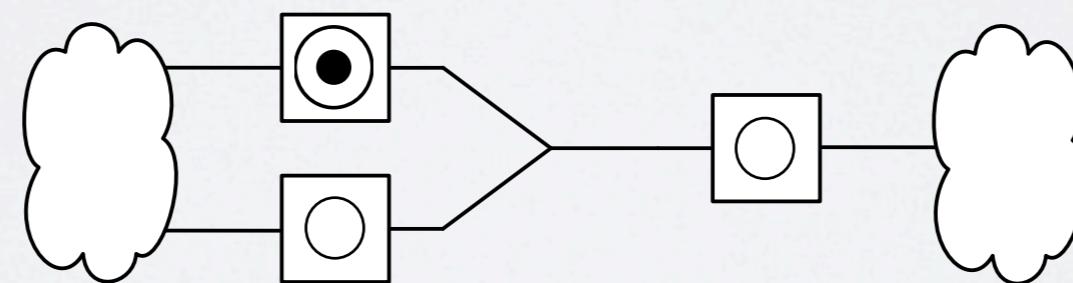
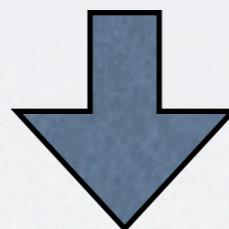
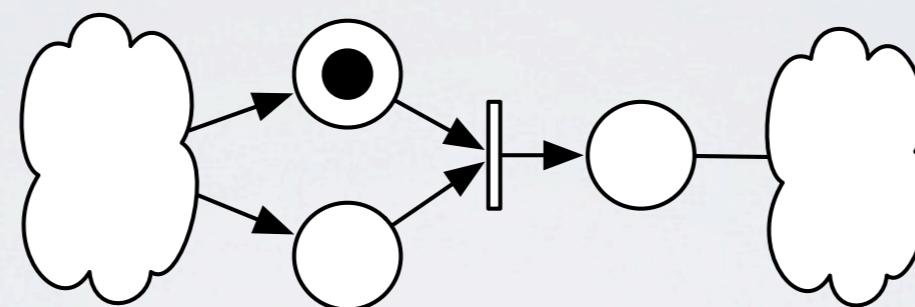
$$\textcircled{0} \stackrel{\text{def}}{=} \mu Y. \frac{\bullet}{\iota} \frac{\iota}{\bullet} Y$$



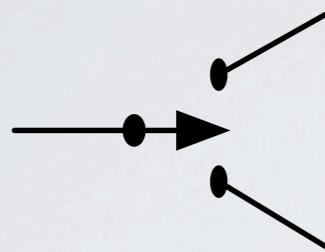
$$\textcircled{•} \stackrel{\text{def}}{=} \frac{\iota}{\bullet} \textcircled{0}$$



ENCODING A TRANSITION



OTHER WIRES



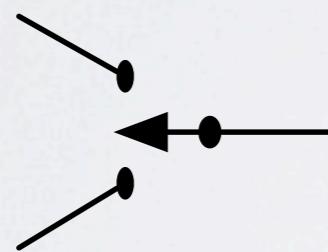
$$\Lambda \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x}{\lambda x \iota} Y + \frac{\lambda x}{\iota \lambda x} Y$$

$$\frac{}{\Lambda \xrightarrow{a \iota} \Lambda} (\Lambda L)$$



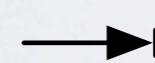
$$\uparrow \stackrel{\text{def}}{=} \mu Y : (0, 1). Y$$

$$\frac{}{\uparrow \xrightarrow{\iota} \uparrow} (\uparrow)$$



$$\vee \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x \iota}{\lambda x} Y + \frac{\iota \lambda x}{\lambda x} Y$$

$$\frac{}{\vee \xrightarrow{a \iota} \vee} (\vee L)$$



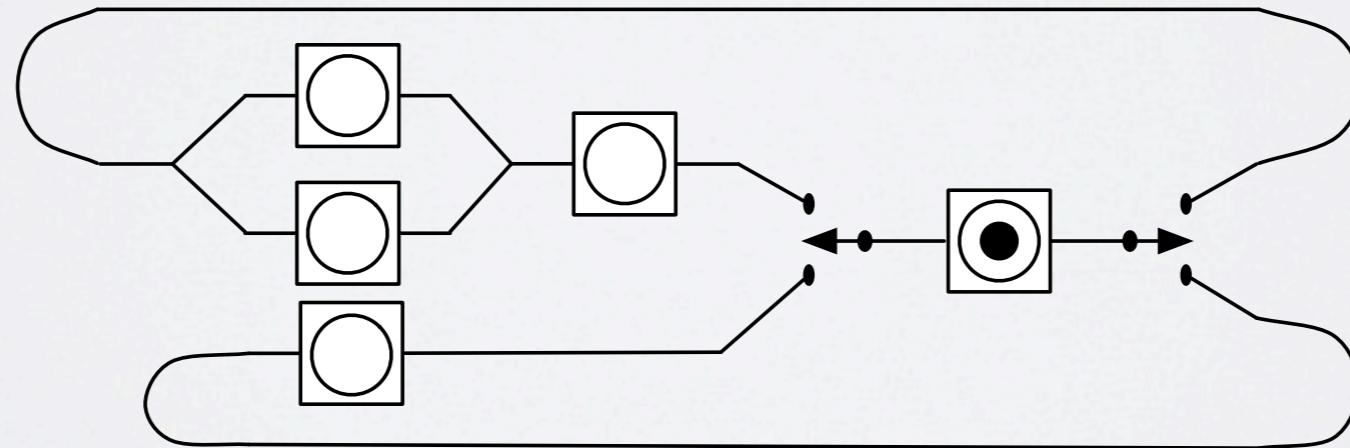
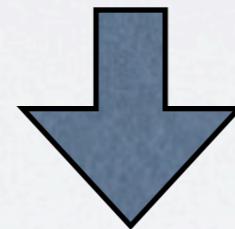
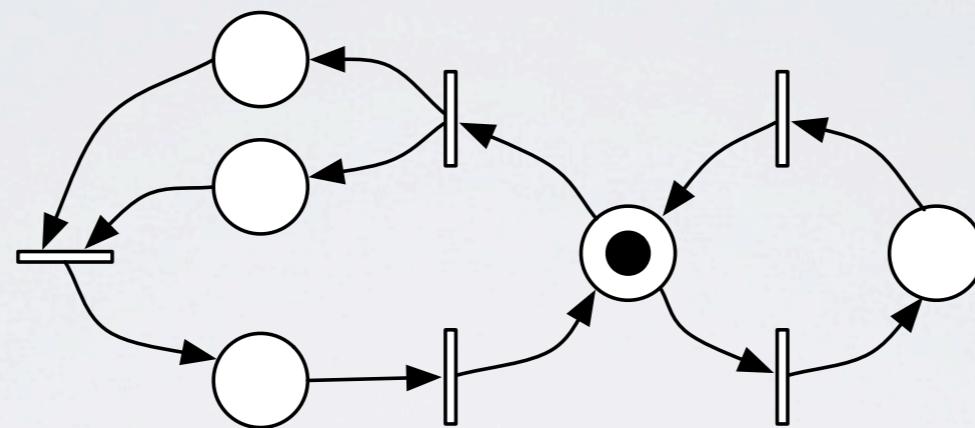
$$\downarrow \stackrel{\text{def}}{=} \mu Y : (1, 0). Y$$

$$\frac{}{\downarrow \xrightarrow{\iota} \downarrow} (\downarrow)$$

TRIVIAL BISIMULATIONS

- (i) $\Lambda; (\mathbf{I} \otimes \Lambda) \sim \Lambda; (\Lambda \otimes \mathbf{I}) \left(\text{---} \xrightarrow{\quad} \text{---} \sim \text{---} \xrightarrow{\quad} \text{---} \right)$ and $(\mathbf{I} \otimes \mathbf{V}); \mathbf{V} \sim (\mathbf{V} \otimes \mathbf{I}); \mathbf{V} \left(\text{---} \xrightarrow{\quad} \text{---} \sim \text{---} \xrightarrow{\quad} \text{---} \right)$;
- (ii) $\Lambda; \mathbf{X} \sim \Lambda \left(\text{---} \xrightarrow{\quad} \text{---} \sim \text{---} \xrightarrow{\quad} \text{---} \right)$ and $\mathbf{V} \sim \mathbf{X}; \mathbf{V} \left(\text{---} \xrightarrow{\quad} \text{---} \sim \text{---} \xrightarrow{\quad} \text{---} \right)$;
- (iii) $\Lambda; (\downarrow \otimes \mathbf{I}) \left(\text{---} \xrightarrow{\quad} \text{---} \sim \text{---} \right)$ and $(\uparrow \otimes \mathbf{I}); \mathbf{V} \sim \mathbf{I} \left(\text{---} \xrightarrow{\quad} \text{---} \sim \text{---} \right)$
- (iv) $\Lambda; \mathbf{V} \sim \mathbf{I} \left(\text{---} \xrightarrow{\quad} \text{---} \sim \text{---} \right)$.

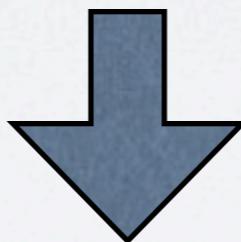
TRANSLATING NETS



FULL ABSTRACTION

$$M \xrightarrow{e} M' \quad \longrightarrow \quad \llbracket M \rrbracket \rightarrow \llbracket M' \rrbracket$$

$$\llbracket M \rrbracket \rightarrow P$$



$$\exists M'. \llbracket M' \rrbracket = P \ \& \ (M' = M \ \vee \ \exists e_1, \dots, e_m. \xrightarrow{e_1} \dots \xrightarrow{e_m} M')$$

BUFFERS & QUEUES

(inspired by Selinger's Axioms for Asynchrony)

Unbounded buffer

$$\begin{aligned} B_1 &\stackrel{\text{def}}{=} \frac{\lambda x}{\iota} \frac{\iota}{\lambda x} 0 \\ \mathcal{B} &\stackrel{\text{def}}{=} \mu Y. B_1 \mid Y \end{aligned}$$

$$\boxed{\mathcal{B}} \quad \stackrel{\text{def}}{=} \quad \begin{array}{c} B_1 \\ \downarrow \\ \boxed{\mathcal{B}} \end{array}$$

$$\overline{C[X]} \xrightarrow{\iota} C[X]$$

$$\frac{\sigma \in \Sigma}{C[X] \xrightarrow[\ell]{\sigma} C[X+\sigma]}$$

$$C[X+\sigma] \xrightarrow[\sigma]{\iota} C[X]$$

Queue

$$\mathcal{Q} \stackrel{\text{def}}{=} \mu Y. \frac{\lambda x}{\iota}(Y ; \frac{\iota}{\lambda x} I)$$