Université Joseph Fourier – Grenoble I

L2 : Licence Sciences et Technologies

UE STA230 : midterm exam November 6^{th}, 2012

with answers

Duration 2h. Autorized documents: statistics tables and one A4 sheet handwriten (two pages). Calculators are authorized.

Exercise 1. (4pts) A sample of mice has been observed for two years: 75% were fed genetically modified corn, the others had another type of food. Among those having had genetically modified corn, 64% grew massive tumors, whereas 56% of mice having had the other type of food, still grew tumors. For a mouse chosen at random, let G and T be the following events:

G: the mouse was fed genetically modified corn

T: the mouse grew tumors

1. Give the values of $\mathbb{P}[G]$, $\mathbb{P}[T \,|\, G]$ and $\mathbb{P}[\overline{T} \,|\, \overline{G}]$. $\boxed{0.75, 0.64, 0.44}$
2. Compute $\mathbb{P}[T \text{ and } G]$, and $\mathbb{P}[T \text{ and } \overline{G}]$. $\boxed{0.48, 0.14}$
3. What proportion of mice grew tumors? $\boxed{0.62}$
4. Knowing that a mouse grew tumors, what are the chances it had been fed genetically modified corn? $\boxed{0.7742}$

Exercise 2. (7pts) It is a known experimental fact that among mice of a certain strain, 40% grow tumors in their lifetime.

1. A sample of 6 mice of that strain is observed. Among them, a random number X grow tumors.

   1a) What probability distribution do you propose for X? Give its parameters. $\boxed{\mathcal{B}(6, 0.4)}$

   1b) Compute the probability that in the sample of 6, at least 2 grow tumors. $\boxed{0.7667}$

2. A sample of 60 mice of the same strain is observed. Let Y be the random variable equal to the number of mice growing tumors in that sample.

   2a) What probability distribution do you propose for Y? Give its parameters. $\boxed{\mathcal{B}(60, 0.4)}$

   2b) The probability distribution of Y is approximated by a normal distribution. What theoretical result justifies this approximation? $\boxed{\text{Central Limit Theorem}}$

   2c) Give the parameters of the approximating normal distribution. $\boxed{\mathcal{N}(24, 14.4)}$

2d) Using the normal approximation, compute the probability that at least 20 mice grow tumors. $\boxed{0.8541}$

2e) Using the normal approximation, find which number m is such that, with probability larger than 0.95, less than m of the 60 mice grow tumors. $\boxed{m \geqslant 31}$

Exercise 3. (9pts) The focus of a human eye is measured in Diopters. Nearsighted (myopic) persons have a negative focus, farsighted (hyperopic) persons have a positive focus. For a given person, the focus is modelled by a random variable with normal distribution $\mathcal{N}(\mu, \sigma^2)$.

1. Under the age of 30, the expectation μ is 0 D. and the standard deviation σ is 1.9 D.

   1a) What proportion of persons under 30 have a focus higher than 2.5 D.? $\boxed{0.0941}$

   1b) It is known that 16% of persons under 30 need eye correction for nearsightedness, and 9% for farsightedness. Between which values is correction not necessary? $\boxed{[-1.89\ ;\ 2.55]}$

2. At the age of 40, the interval between -2 D. and 7 D. contains 98% of eye focus measurements.

   2a) Give the expectation μ and the standard deviation σ for age focus at the age of 40. $\boxed{\mu = 2.5, \sigma = 1.934}$

   2b) If two persons, one under 30 and the other 40 years old, are taken at random, what is the probability that the older person has a higher focus than the younger? $\boxed{0.8217}$

3. The following array gives the eye focus measurements in diopters of 6 persons above the age of 50.

D	0.4	5.1	5.4	3.9	6.1	7.4

   3a) Compute the empirical mean and standard deviation of that sample. $\boxed{\overline{x} = 4.72, \sqrt{s^2} = 2.20}$

   3b) Give unbiased estimates for the expectation μ and variance σ^2 of eye focus measurements above 50. $\boxed{4.72, 5.81}$

   3c) Using the above estimates, give a 98% confidence interval for the expectation μ. $\boxed{[1.4066\ ;\ 8.0267]}$

   3d) The standard deviation σ above 50 years old is the same as in younger persons. Assuming $\sigma = 1.9$ is known, give a 98% confidence interval for the expectation μ. $\boxed{[2.9121\ ;\ 6.5212]}$

   3e) With the same estimate of μ and $\sigma = 1.9$, what is the confidence level of the interval [3.21 ; 6.22]? $\boxed{0.9477}$