**Université Joseph Fourier – Grenoble I**

L2: Licence Sciences et Technologies

UE STA230: final exam, December 18th, 2012

**Duration 2h. Autorized documents: statistics tables and two handwritten A4 sheets (four pages). Calculators are authorized.**

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**Exercise 1 (12 points) :** In a widescale study of 1972, the prevalence of myopia in the US population aged 12–54, was found to be 25% (percentage of shortsighted people). On a sample of 120 persons in 2009, 39 were found to be shortsighted.

1. Give a confidence interval, with confidence level 0.95, for the proportion of shortsighted people in 2009. Is that interval compatible with the 1972 proportion? $\boxed{[0.241\,;\,0.409]\,,\ \text{yes}}$

2. The goodness-of-fit between the observed 2009 distribution of shortsighted and non-shortsighted people, with that of 1972 (taken as theoretical), is assessed by the chi-squared test.

   (a) Compute the test statistic. $\boxed{3.6}$

   (b) To which distribution should that value be compared? What is your decision at threshold 5%? $\boxed{\chi^2(1),\ \text{accept goodness-of-fit}}$

   (c) Give an interval containing the p-value. $\boxed{[0.05\,,\ 0.1]}$

3. In this question, the goal is to decide whether myopia has significantly increased between 1972 and 2009. The outcome of the 2009 study is considered as a large binary sample, the result of the 1972 study is taken as fixed.

   (a) What hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are you testing?
   $\boxed{\mathcal{H}_0\ :\ p = 0.25\,,\ \mathcal{H}_1\ :\ p > 0.25}$

   (b) Compute the value of the test statistic. $\boxed{1.897}$

   (c) Give the corresponding p-value. What is your decision at threshold 5%?
   $\boxed{0.0289\,,\ \text{significant increase}}$

4. On a sample of 200 persons in the 1972 study, 50 were shortsighted. In this question, the chi-squared test of independence is used to decide whether the observed distributions of 1972 and 2009 are significantly different.

   (a) Write the contingency table corresponding to the data.

   (b) Compute the test statistic. $\boxed{2.101}$

   (c) To which distribution should that value be compared? What is your decision at threshold 5%? $\boxed{\chi^2(1),\ \text{accept independence}}$

5. The two samples of 1972 and 2009 are considered as two large binary samples $X$ and $Y$, the means of which (50/200 and 39/120) are to be compared. The question is: has the proportion of shorsighted people increased?

   (a) Compute the value of the test statistic. $\boxed{1.426}$

   (b) Give the corresponding p-value. What is your decision, at threshold 5%? $\boxed{0.077\,,\ \text{no significant increase}}$

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**Exercise 2 (8 points) :** Age-related farsightedness or presbyopia comes gradually. A study has been made to assess how the focus of the human eye depends on age. The focus in diopters (focus: variable $Y$) of 40 persons aged 25–50 (age: variable $X$) has been recorded. The following data are given:

$$\overline{x} = 38.45\,,\ s_x^2 = 71.51\,,\ \overline{y} = 6.02\,,\ s_y^2 = 10.37\,,\ c_{xy} = -19.88\,.$$

1. Compute the correlation coefficient of $X$ and $Y$. $\boxed{r_{xy} = -0.73}$

2. Give the equation of the regression line of $Y$ onto $X$. What is the average focus of 50 years old people? $\boxed{y = -0.278\,x + 16.71\,,\ 2.81}$

3. Test the pertinence of the regression at threshold 1%.
   $\boxed{T = -6.5858 < -2.429\,;\ \mathcal{H}_0 : a = 0 \text{ rejected, pertinence accepted}}$

4. Give a confidence interval with level 0.99, for the average focus of 50 years old persons. $\boxed{[1.17; 4.45]}$

5. Mr. O. is 51 years old. Give a prediction interval with level 0.99 for his focus.
   $\boxed{[-3.83; 8.90]}$

6. Your ophtalmologist tells you that between 25 and 50, the focus decreases by 1 diopter in 3 years, on average. In the linear regression, which value of the slope does this correspond to? Which hypotheses are you testing to confirm your ophtalmologist's assertion?
   $\boxed{\mathcal{H}_0\ :\ a = -1/3 \text{ against } \mathcal{H}_0\ :\ a < -1/3}$

7. Give the value of the test statistic, and an approximation of the p-value. What is your conclusion?
   $\boxed{T = 1.31\,;\ \text{p-value} \simeq 0.1\,;\ \text{confirm}}$

8. A friend of yours, age 26, has a focus of 3.7. Is this unusual?
   $\boxed{T = -2.46 < -2.429 : \text{reject } \mathcal{H}_0 \text{ at } 1\%, \text{ yes it is unusual}}$

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