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onglet Courses

Variance Based Global Sensitivity Analysis

**Preliminaries :**

```
library(sensitivity)
source('~/UTIL/')
```

# 1 Study of the designs of experiments for sensitivity analysis

Sobol' approach requires two designs $A$ and $B$

For *sobolEff*,*sobolroalhs* (package *sensitivity*) we observe:

- the number of model evaluations,
- the way the points of the designs fill the 2D space?

```
Ns <- 10
# Construction of the two designs A and B (uniform distribution)
A <- data.frame(matrix(runif(4 * Ns), nrow = Ns))
B <- data.frame(matrix(runif(4 * Ns), nrow = Ns))
```

```
# Methode sobolEff
# ---------------------------
res_SobolEff<-sobolEff(model=NULL,A,B)

# Components of the object
names(res_SobolEff)
str(res_SobolEff)

# Size of the design of experiments
dim(Res_SobolEff$X)

## 2D representations of the points in the samplings

# we freeze the first direction
plot(res_SobolEff$X[1:Ns,1:2],xlim=c(0,1),ylim=c(0,1))
points(res_SobolEff$X[(Ns+1):(2*Ns),1:2],col='blue',pch=3)
```

```
# ----------------------------
# Methode sobolroalhs
# ----------------------------

res_Sobolroalhs <- sobolroalhs(model=NULL, factors=4, levels=Ns, order=1)

# Size of the design of experiments
dim(res_SobolEff$X)

## 2D representations of the points in the samplings


plot(res_Sobolroalhs$X[1:Ns,1:2],xlim=c(0,1),ylim=c(0,1))
points(res_Sobolroalhs$X[(Ns+1):(2*Ns),1:2],col='blue',pch=3)
```

sobolEff requires $(d+1)N_s$ evaluations for all first-order Sobol' indices whereas sobolroalhs requires only $2N_s$.

# 2 Sensitivity analysis for additive and/or multiplicative models

## 2.1 Additive model

Perform the sensitivity analysis of the following model:

$$Y = X1 + X2$$

### 2.1.1 Case 1

$$X_1 \sim U(-1, 1)$$
$$X_2 \sim U(-1, 1)$$

What are the theoretical values of Sobol' indices?

```
# Size of both designs of experiments (DoE)
n<-1000
```

```
# construction of both DoE
A <- data.frame(matrix(runif(2*n,-1,1),nrow=n))
B <- data.frame(matrix(runif(2*n,-1,1),nrow=n))

# first-order sensitivity indices
res_sobolEff <- sobolEff(model=somme2, A, B ,nboot=100)
print(res_sobolEff)
plot(res_sobolEff)

# total indices
res_sobolTotal <- sobolEff(model = somme2, A, B, nboot = 100, order = 0)
print(res_sobolTotal)
plot(res_sobolTotal)
```

#### 2.1.2 Case 2

$$X_1 \sim U(0,2)$$
$$X_2 \sim U(-1,1)$$

What are the theoretical values of Sobol' indices?

#### 2.1.3 Case 3

$$X_1 \sim U(-1,1)$$
$$X_2 \sim U(-2,2)$$

What are the theoretical values of Sobol' indices?

#### 2.1.4 Case 4

$$X_1 \sim U(-1,1)$$
$$X_2 \sim \mathcal{N}\left(0, \frac{2}{\sqrt{3}}\right)$$

What are the theoretical values of Sobol' indices?

## 2.2 Multiplicative model

$$Y = X1 * X2$$

### 2.2.1 Case 1

$$X_1 \sim U(-1, 1)$$
$$X_2 \sim U(-1, 1)$$

What are the theoretical values of first-order Sobol' indices?

```
# Size of both DoE
n<-1000

# Construction of these DoE
A <- data.frame(matrix(runif(2*n,-1,1),nrow=n))
B <- data.frame(matrix(runif(2*n,-1,1),nrow=n))

# first-order sensitivity indices
res_sobolEff <- sobolEff(model=produit2, A, B ,nboot=100)
print(res_sobolEff)
plot(res_sobolEff$X)
plot(res_sobolEff)
```

```
# total indices
res_sobolTotal <- sobolEff(model = produit2, A, B, nboot = 100, order = 0)
print(res_sobolTotal)
plot(res_sobolTotal$X)
plot(res_sobolTotal)
```

### 2.2.2 Case 2

$$X_1 \sim U(0,2)$$
$$X_2 \sim U(-1,1)$$

What are the theoretical values of Sobol' indices?

### 2.2.3 Case 3

$$X_1 \sim U(-1,1)$$
$$X_2 \sim \mathcal{N}\left(0, \frac{1}{\sqrt{3}}\right)$$

What are the theoretical values of Sobol' indices?

## 3 Study for the g-Sobol function

We consider the *g-function* introduced by Sobol'.

$(X_i)$ $i = 1, \ldots, p$: p independent variables, uniformly distributed on $[0,1]$. The *g-function* is defined as:

$$f(X_1, \ldots, X_p) = \prod_{i=1}^{p} g_i(X_i)$$

with

$$g_i(X_i) = \frac{|4X_i - 2| + a_i}{1 + a_i}$$

We consider the following cases:

**Case 1** : g-function, dimension 2 with $(a_1, a_2) = (99, 1)$

**Case 2** : g-function, dimension 10 with $(a_1, a_2, .., a_10) = (1, 2, .., 10)$

**Case 3** : g-function, dimension 10 with $(a_1, a_2, .., a_10) = (1, 2^2, .., 10^2)$