Simulation of an epidemiological model structured in age of infection and age of immunity

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COVID-19 pandemic has once more demonstrated the need in modeling the dynamic spread of a specific epidemic. The compartmental models remain the dominant tool to respond to this need. The most famous one is the SIR model introduced by Kermack and McKendrick in 1927 [1]. In their article, they proposed a deterministic compartmental model structured in age of infection. However, they did not take into account the impact of vaccine nor the immunity loss over time. In this work, we consider the effect of the progressive immunity loss (see [2] and references therein). We propose a local deterministic compartmental model structured in age of infection and age of immunity. More precisely, the population is separated in seven categories: the group S of susceptible individuals, the group E of exposed individuals who are infected but not infectious yet, the group I of infectious individuals, the group R of recovered individuals, the group H of hospitalized individuals, the group D of deceased individuals and the group V of recently vaccinated individuals. The group I is structured in age of infection a and the groups R and V are structured in age of immunity α. The evolution of the system is described by the following equations,

$$
\begin{aligned}
&\frac{dS}{dt}(t) = -S(t)\left(\int_0^\infty \beta^*(t,a)I(t,a)da + \theta_S(t)\right) + \left(\int_0^\infty (\sigma_R^*(\alpha)R(t,\alpha) + \sigma_V^*(\alpha)V(t,\alpha))d\alpha\right)\\
&\frac{dE}{dt}(t) = S(t)\left(\int_0^\infty \beta^*(t,a)I(t,a)da\right) - \delta E(t)\\
&\frac{\partial I}{\partial t}(t,a) + \frac{\partial I}{\partial a}(t,a) = -\gamma_I^*(a)I(t,a) - \tau_A\theta_A(t)I(t,a) \qquad \forall\, a > 0\\
&I(t,0) = \delta E(t)\\
&\frac{\partial R}{\partial t}(t,\alpha) + \frac{\partial R}{\partial \alpha}(t,\alpha) = -\theta_R^*(t,\alpha)R(t,\alpha) - \sigma_R^*(\alpha)R(t,\alpha) \qquad \forall\, \alpha > 0\\
&R(t,0) = \int_0^\infty \gamma_I^*(a)(1-\tau_H)I(t,a)da + \gamma_H(1-\tau_D)H(t)\\
&\frac{dH}{dt}(t) = \int_0^\infty \gamma_I^*(a)\tau_H I(t,a)da - \gamma_H H(t)\\
&\frac{dD}{dt}(t) = \gamma_H\tau_D H(t)\\
&\frac{\partial V}{\partial t}(t,\alpha) + \frac{\partial V}{\partial \alpha}(t,\alpha) = -\sigma_V^*(\alpha)V(t,\alpha) \qquad \forall\, \alpha > 0\\
&V(t,0) = \int_0^\infty \theta_A(t)\tau_A I(t,a)da + \int_0^\infty \theta_R^*(t,\alpha)R(t,\alpha)d\alpha + \theta_S(t)S(t)
\end{aligned}
$$

with the non-negative initial conditions $S(0) = S_0$, $E(0) = E_0$, $I(0,a) = 0\ \forall a$, $R(0,\alpha) = 0\ \forall\alpha$, $H(0) = 0$, $D(0) = 0$ and $V(0,\alpha) = 0\ \forall\alpha$ and the quantity $S(t) + E(t) + \int_0^\infty I(t,a)da + \int_0^\infty (R(t,\alpha) + V(t,\alpha))d\alpha + H(t) + D(t)$ is supposed constant equal to 1. For the numerical simulations we propose a RBF-FD scheme (see [3] for details on the method, [4] and [5], for applications) to approximate the $\frac{\partial}{\partial a}$ and $\frac{\partial}{\partial \alpha}$ operators. Integrals are computed with a Gauss-Legendre quadrature (see [6]).

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