Control by state feedback of COVID-19 disease in France

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Epidemic models can bring huge panic and disaster to mankind once out of control. Therefore, it is crucial to have a better understanding of the evolution of diseases and to act effectively on them. One possible control policy in epidemic models is the vaccination. This is the one considered in this work.

An adapted version of the well-known SIR model of Kermack and McKendrick [1] is used. To be consistent with the features of the COVID-19 disease, an SIRD model is considered. In this model the population is divided in four distinct groups: the group S of susceptible individuals who can catch the disease, the group I of infected individuals who can transmit the disease, the group R of recovered indivdiuals who are assumed to have permanent immunity and the group D of dead individuals. Moreover, in this work, the importance of the age, denoted by a, of individuals is taken into account since several factors in epidemic models are age-dependent, vaccination being one of them.

The evolution of the disease propagation is described by a set of $4n$ nonlinear ordinary differential equations (n is the number of classes of age available in the data),

$$
\begin{aligned}
\frac{dS_k(t)}{dt} &= -\frac{\beta_k S_k(t) I(t)}{N_k} - \theta_k(t) S_k(t) \\
\frac{dI_k(t)}{dt} &= \frac{\beta_k S_k(t) I(t)}{N_k} - \left(\gamma_{R_k} + \gamma_{D_k}\right) I_k(t) \\
\frac{dR_k(t)}{dt} &= \gamma_{R_k} I_k(t) + \theta_k(t) S_k(t) \\
\frac{dD_k(t)}{dt} &= \gamma_{D_k} I_k(t)
\end{aligned}
$$

under non-negative initial conditions $S_k(0) = S_{k_0}$, $I_k(0) = I_{k_0}$, $R_k(0) = 0$, $D_k(0) = 0$, where I denotes the total number of infected individuals and $N_k = S_k(t) + I_k(t) + R_k(t) + D_k(t)$ is constant. The mode of transmission of the disease is assumed to be by contact between S-individuals and I-individuals. The transmission coefficient is given by β_k for the age class k. In addition, the I-individuals recover at a rate denoted by γ_{R_k} and die at a rate γ_{D_k} for people in the kth class of age. Finally, the term $\theta_k(t)$ is the input variable representing the rate of S-individuals in the class age k being vaccinated at time t. The vaccination is assumed to work perfectly: once vaccinated, an individual never catches the disease again.

The aim of this work is to design a feedback control-law of vaccination $\Theta_k(t)$ that improves the disease-eradication.

By applying Isidori's theory developed in [2, chap.5] it is shown that, by an appropriate choice of the control tuning parameters, the state feedback law $(\theta_1(t)...\theta_n(t))^T$ implies the exponential convergence towards zero of the infected population $I_k(t)$, for $k = 1,...,n$, as time tends to infinity. Moreover, conditions on the feedback design parameters are found to imply non-negativity of the feedback law in order to be consistent with its physical meaning.

The second part of this work is devoted to the application of those results on real data. In particular on Covid-19 data for France. In order to do this, a preliminary work is to estimate the model parameters. This is done by using the Poisson likelihood statistics, where we assumed that the observed data follow a Poisson distribution, and by minimizing the negative log-likelihood function,

$$
-\log \mathscr{L} = \sum (-k_i \log \lambda_i + \lambda_i)
$$

where k_i represents the observed data and λ_i the values predicted by the model. This is an ongoing research and we hope to be able to report on the progress made so far on this issue by the time of this talk.

Finally, since the designed state-feedback control law is not applicable in practice because it requires the knowledge of all the state variables, we aim also to develop a state observer.

References

[1] W. Kermack and A. Mckendrick. Contributions to the mathematical theory of epidemics—II. The problem of endemicity. *Bulletin of Mathematical Biology*, 53(1-2):57–87, 1991.

[2] A. Isidori. *Nonlinear control systems*. Springer, 3rd edition, 1995.