Linear Algebra 2: Parallel programming tools for exact linear algebra

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Introduction

Back in the times, when everything was sequential

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Introduction :

Parallel architecture: heterogeneity

- multicore [>8 cores], ccNUMA
- network [mostly infiniband]
- GPU, separate address space
- Intel MIC
- FPGA
- ...

Main characteristics:

- complexity: memory hierarchy, number of cores
- changing hardware: Net. on Chip, Integration CPU/GPU...

Challenge

How to programm heterogeneous architectures ?

Criteria

- good performances
- portability across architectures
- abstraction for simplicity

Challenging key point: scheduling as a plugin

- Program: description of the parallelism
  e.g. which code portions are tasks
- Runtime: scheduling, mapping decision

3 main programming models:

1. Parallel loop [data parallelism]
2. Fork-Join (independent tasks) [task parallelism]
3. Dependent tasks with data flow dependencies [task parallelism]

Outline

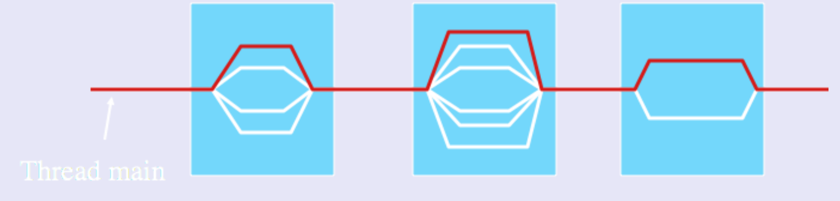
- Parallel programming models
  - Parallel loop model
  - Fork-join model
  - Data flow Tasks model
  - Existing solutions
- Comparison Fork-Join vs Data flow
  - Overhead of task management

Parallel loop model

$$\forall i \in [0, n[\text{ do } f(i),$$

- where $i \neq j \Rightarrow f(i)$ and $f(j)$ are independent,
- i.e. result is independent of the execution order of $f(i)$ and $f(j)$.

Reference software: OpenMP 1.0



OMP

```
for (int step = 0; step < 2; ++step){
#pragma omp parallel for
    for (int i = 0; i < count; ++i)
        A[i] = (B[i+1] + B[i-1] + 2.0*B[i])*0.25;
}
```

Cilk

```
for (int step = 0; step < 2; ++step){
  cilk_for (int i = 0; i < count; ++i)
    A[i] = (B[i+1] + B[i-1] + 2.0*B[i]) * 0.25;
}
```

Kaapi

```
for (int step = 0; step < 2; ++step){
#pragma kaapi parallel loop
  for (int i = 0; i < count; ++i)
    A[i] = (B[i+1] + B[i-1] + 2.0*B[i]) * 0.25;
}
```

Fork join model

- Task based program: **spawn** + **sync**
- Especially suited for recursive programs
- Naive canonical example: recursive Fibonacci computation

OMP

```
void fibonacci(long* result, long n) {
  if (n < 2)
    *result = n;
  else {
    long x,y;
#pragma omp task
    fibonacci( &x, n-1 );
    fibonacci( &y, n-2 );
#pragma omp taskwait
    *result = x + y;
  }
}
```

Fork join model

- Task based program: **spawn** + **sync**
- Especially suited for recursive programs
- Naive canonical example: recursive Fibonacci computation

Cilk+

```
long fibonacci(long n) {
  if (n < 2)
    return (n);
  else {
    long x, y;
    x = cilk_spawn fibonacci(n - 1);
    y = fibonacci(n - 2);
    cilk_sync;
    return (x + y);
  }
}
```

Fork join model

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Kaapi

```
void fibonacci(long* result, long n) {
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    fibonacci( &y, n-2 );
#pragma kaapi sync
    *result = x + y;
  }
}
```
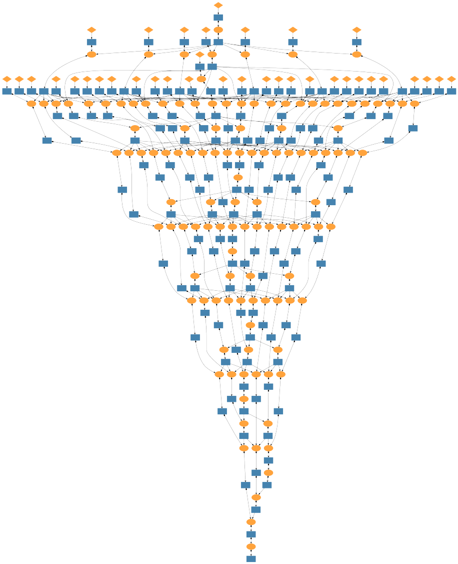
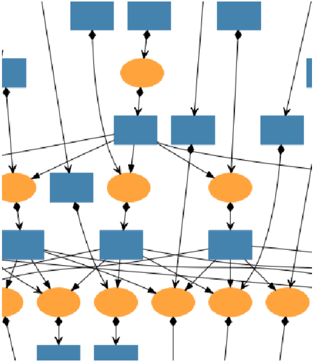

Data flow task model

- Task based model
- Basic definition:
  - A task is ready for execution when all its inputs variables are ready
  - A variable is ready when it was written (...)
- Old languages: ID, SISAL...
- New languages/libraries: Athapascan [96], Kaapi [06], StarSs [07], StarPU [08], Quark [10]...

Data flow graph: Cholesky factorization





SmpSS

```
#pragma smpss task write(array)
extern void compute( double* array, int count);
#pragma smpss task read(array)
extern void print( double* array, int count);
int main() {
#pragma smpss start
    compute( array, count);
    print( array, count);     // Read after write dependency
#pragma smpss sync
#pragma smpss finish
}
```

Kaapi

```
int main() {
#pragma kaapi parallel
  {
#  pragma kaapi task write(array[0..count])
    compute( array, count);
#  pragma kaapi task read(array[0..count])
    print( array, count);      // Read after write dependency
  } // implicit barrier at the end of Kaapi parallel region
}
```

Existing solutions

	// prog model	Architecture	Target app.
Cilk[96]	Fork-join	Multi-CPUs	Divide&Conquer
OMP 1.0 [97]	Parallel loop	Multi-CPUs	ForEach
+ 3.0 [08]	+ Fork-join	Multi-CPUs	+ Divide&Conquer
Athapascan[98]	Rec. Data flow	Clusters+multi-CPU	D&C, LinAlg
TBB[06]	Parallel loop	Multi-CPU	D&C, Linalg
	Fork-join		
Kaapi[06-12]	Rec. Data flow	Multi-CPUs & GPUs	D&Q, LinAlg
	Parallel loop		ForEach,
StarSs [07]	Flat data flow	multi-CPUs (SMPSs)	LinAlg
	Flat data flow	multi-CPUs (SMPSs)	LinAlg
	Flat data flow	Cell (CellSs)	LinAlg
	Flat data flow	Grid (GridSs)	LinAlg
StarPU [09]	Flat data flow	multi-CPUs&GPUs	LinAlg
Quark[10]	Flat data flow	Multi-CPUs	LinAlg

Outline

Parallel programming models

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- Fork-join model
- Data flow Tasks model
- Existing solutions

Comparison Fork-Join vs Data flow

- Overhead of task management

Comparison Fork-Join vs Data flow

Fork-Join: OpenMP-3.0

Data flow: Kaapi

Goal: how excessive synchronizations affect performances

By studying

- impact on performances on Cholesky/LU matrix factorization
- cost of task creation and scheduling (micro benchmark: Fibonacci)

Fork-Join vs Data flow

Strong synchronizations in Fork-Join model:

- if task T_1 depend on task T_0 e.g. task T_0 produces value for task T_1
- spawn T0; sync; spawn T1; spawn T2; ...
- synchronization point at “sync”: barrier that waits for all previous spawned tasks, even if concurrency exists with some tasks after the barrier

Fork-Join vs Data flow

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- synchronization point at “sync”: barrier that waits for all previous spawned tasks, even if concurrency exists with some tasks after the barrier

Data Flow model:

data flow tasks to express such dependencies

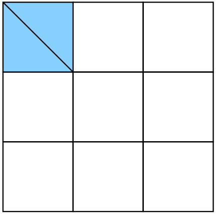
- program : creates tasks
- runtime : schedule tasks according to the real dependencies

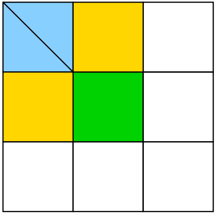
Illustration: Cholesky factorization

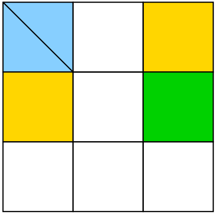
```
void Cholesky( double* A, int N, size_t NB ) {

  for (size_t k=0; k < N; k += NB)
  {
    clapack_dpotrf( CblasRowMajor, CblasLower, NB, &A[k*N+k], N );

    for (size_t m=k+ NB; m < N; m += NB)
    {

      cblas_dtrsm ( CblasRowMajor, CblasLeft, CblasLower, CblasNoTrans, CblasUnit,
        NB, NB, 1., &A[k*N+k], N, &A[m*N+k], N );
    }

    for (size_t m=k+ NB; m < N; m += NB)
    {

      cblas_dsyrk ( CblasRowMajor, CblasLower, CblasNoTrans,
        NB, NB, -1.0, &A[m*N+k], N, 1.0, &A[m*N+m], N );

      for (size_t n=k+NB; n < m; n += NB)
      {

        cblas_dgemm ( CblasRowMajor, CblasNoTrans, CblasTrans,
          NB, NB, NB, -1.0, &A[m*N+k], N, &A[n*N+k], N, 1.0, &A[m*N+n], N );
      }
    }

  }
}
```

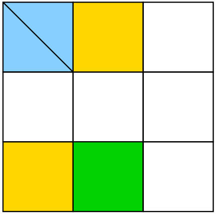
Illustration: Cholesky factorization

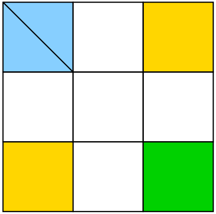
```
void Cholesky( double* A, int N, size_t NB ) {
#pragma omp parallel
#pragma omp single nowait
  for (size_t k=0; k < N; k += NB)
  {
    clapack_dpotrf( CblasRowMajor, CblasLower, NB, &A[k*N+k], N );

    for (size_t m=k+ NB; m < N; m += NB)
    {
#pragma omp task firstprivate(k, m) shared(A)
      cblas_dtrsm ( CblasRowMajor, CblasLeft, CblasLower, CblasNoTrans, CblasUnit,
        NB, NB, 1., &A[k*N+k], N, &A[m*N+k], N );
    }
#pragma omp taskwait // Barrier: no concurrency with next tasks
    for (size_t m=k+ NB; m < N; m += NB)
    {
#pragma omp task firstprivate(k, m) shared(A)
      cblas_dsyrk ( CblasRowMajor, CblasLower, CblasNoTrans,
        NB, NB, -1.0, &A[m*N+k], N, 1.0, &A[m*N+m], N );

      for (size_t n=k+NB; n < m; n += NB)
      {
#pragma omp task firstprivate(k, m) shared(A)
        cblas_dgemm ( CblasRowMajor, CblasNoTrans, CblasTrans,
          NB, NB, NB, -1.0, &A[m*N+k], N, &A[n*N+k], N, 1.0, &A[m*N+n], N );
      }
    }
#pragma omp taskwait // Barrier: no concurrency with tasks at iteration k+1
  }
}
```









SYNC.

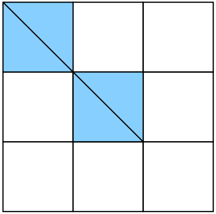
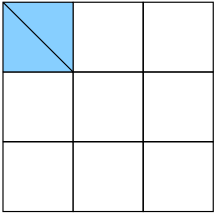
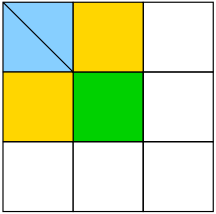
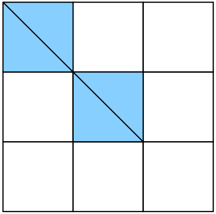


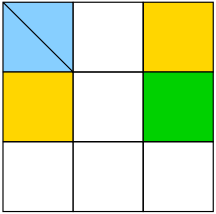
Illustration: Cholesky factorization

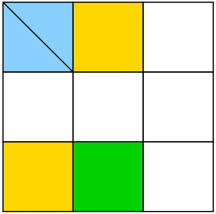
```
void Cholesky( double* A, int N, size_t NB ){
#pragma kaapi parallel
  for (size_t k=0; k < N; k += NB)
  {
#pragma kaapi task readwrite(&A[k*N+k]{ld=N; [NB][NB]})
    clapack_dpotrf( CblasRowMajor, CblasLower, NB, &A[k*N+k], N );

    for (size_t m=k+ NB; m < N; m += NB)
    {
#pragma kaapi task read(&A[k*N+k]{ld=N;[NB][NB]}) readwrite(&A[m*N+k]{ld=N; [NB][NB]})
      cblas_dtrsm ( CblasRowMajor, CblasLeft, CblasLower, CblasNoTrans, CblasUnit,
        NB, NB, 1., &A[k*N+k], N, &A[m*N+k], N );
    }

    for (size_t m=k+ NB; m < N; m += NB)
    {
#pragma kaapi task read(&A[m*N+k]{ld=N;[NB][NB]}) readwrite(&A[m*N+m]{ld=N; [NB][NB]})
      cblas_dsyrk ( CblasRowMajor, CblasLower, CblasNoTrans,
        NB, NB, -1.0, &A[m*N+k], N, 1.0, &A[m*N+m], N );

      for (size_t n=k+NB; n < m; n += NB)
      {
#pragma kaapi task read(&A[m*N+k]{ld=N; [NB][NB]}, &A[n*N+k]{ld=N; [NB][NB]})\
                          readwrite(&A[m*N+n]{ld=N; [NB][NB]})
        cblas_dgemm ( CblasRowMajor, CblasNoTrans, CblasTrans,
          NB, NB, NB, -1.0, &A[m*N+k], N, &A[n*N+k], N, 1.0, &A[m*N+n], N );
      }
    }
  }
  // Implicit barrier only at the end of Kaapi parallel region
}
```

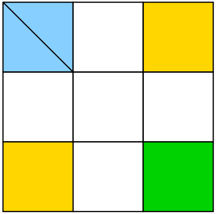













Benchmarks

Sparse version of the above: Kaapi vs OMP codes.

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Sparse version of the above: Kaapi vs OMP codes.

Also confirmed by other versions of data-flow tasks:

- PLASMA [Dongarra& Al.]
- SMPSs [Badia & Al.]

Challenges proper to exact linear algebra

Slicing dimensions

- Uniform block slicing leads to unbalanced load
- Varying block sizes set statically
- Dynamically adapted block sizes (work-stealing)

Rank deficient matrices

- block sizes revealed during execution

Overhead of task management

Algorithm: naive recursive Fibonacci computation

Fork-join model:

- OpenMP : gcc-4.6.2
- Cilk+ / Intel : icc-12.1.2
- TBB 4.0

Data flow model: Kaapi-1.0.2

AMD Opteron 4×12 cores

OpenMP

```
void fibonacci(long* result, const long n){
  if (n<2) *result = n;
  else
  {
    long x,y;
#pragma omp task
    fibonacci( &x, n-1 );
    fibonacci( &y, n-2 );
#pragma omp taskwait
    *result = x + y;
  }
}
```

Kaapi

```
void fibonacci(long* result,const long n){
  if (n<2) *result = n;
  else
  {
    long x,y;
#pragma kaapi task write(x)
    fibonacci( &x, n-1 );
    fibonacci( &y, n-2 );
#pragma kaapi sync
    *result = x + y;
  }
}
```

Cilk +

```
long fibonacci(long n){
  if (n < 2) return (n);
  else {
  long x, y;
  x = cilk_spawn fibonacci(n - 1);
  y = fibonacci(n - 2);
  cilk_sync;
  return (x + y);
  }
}
```

Intel TBB

```
struct FibContinuation: public tbb::task {
    long* const sum; long x, y;
    FibContinuation(long* sum_):sum(sum_){}
    tbb::task* execute() {*sum = x+y; return NULL;}
};
struct FibTask: public tbb::task {
    long n; long * sum;
    FibTask(const long n_,long*const sum_):
        n(n_), sum(sum_) {}
    tbb::task* execute() {if( n<2){*sum = n;return N
        } else {
            FibContinuation& c = *new(allocate_conti
            FibTask& b = *new( c.allocate_child() )
            recycle_as_child_of(c);
            n -= 2;
            sum = &c.x;
            c.set_ref_count(2);
            c.spawn( b );
```

Results

Sequential	Cilk+	TBB-4.0	OpenMP	Kaapi
0.0904s	1.063s	2.356s	2.429s	0.728s
Slowdown ($\frac{T_1}{\text{Sequential}}$)	$\times 11.7$	$\times 26$	$\times 27$	$\times 8$

# cores	Cilk+	TBB-4.0	Kaapi	OpenMP
1	1.063	2.356	0.728	2.43
8	0.127	0.293	0.094	51.06
16	0.065	0.146	0.047	104.14
32	0.035	0.072	0.024	No time
48	0.028	0.049	0.017	No time

Conclusion

Difficult choice of the parallel programming language:

- POSIX threads: set the scheduling at programming time
- OpenMP:
  - Parallel loops
  - Fork-join Tasks
  - But still no data flow capabilities
- Cilk, TBB, Kaapi:
  - Parallel loop
  - Data flow tasks model (recursive or flat)
  - annotation, library, or proper compiler

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Towards fully adaptive parallelism

- Work-stealing but in a fixed set of tasks (created at start-up time)
- Aim at *on-the-fly tasks creations* (extraction of parallelism from sequential code)