# Numerical analysis of the magnetization switching of a multilayered device driven by a current


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## Introduction

A spintronics subject :

- Spintronics, an emerging electronical technology, exploits the spin of electrons and its associated magnetic moment, instead of its charge (as it is the case in numerous components).
- The underlying physics studies interactions between local moments and spin accumulation of conduction electrons.

A recent discovered by Slonczewski [4] and Berger [1] :

- In 1996, both Slonczewski and Berger introduced the concept of switching the orientation of a magnetic layer of a multilayered structure by the current perpendicular to the layers.
- The main idea is of a spin transfer from a polarized current to the magnetization of the layer.

Numerous applications :

- Magnetic memories,
- Fast magnetic logic,
- Microwave frequency devices in telecommunication...

## I Physical device

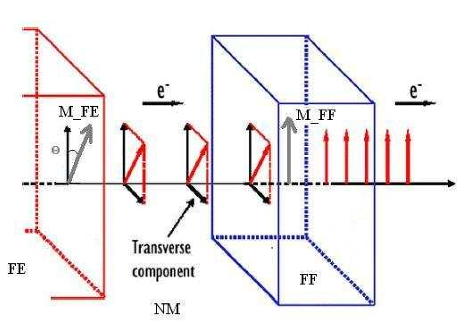


Fig.1 - A multilayered ferromagnetic device.

### A multilayered device :

- A large ferromagnetic layer FE : thickness L≈ 100 nm
  magnetization $\vec{M}_{FE}=(0,-\sin\theta,\cos\theta)$
- A thin ferromagnetic layer FF : thickness l≈ 1-5 nm
  initial magnetization $\vec{M}_{FF}$=(0,0,1)
- A non ferromagnetic layer NM : it avoids exchanges between FE and FF
  it will be replaced by interface conditions

### Mechanism of the spin transfer :

1- We introduced an electrical current in the device, perpendicularly to layers
2- FE polarizes the spin density $\vec{m}$ in the direction of $\vec{M}_{FE}$
3- Because of $\theta$ ($\approx 30°$), $\vec{m}$ reaches FF with a transverse component $\vec{m}_\perp$
4- A torque is created between $\vec{m}_\perp$ and $\vec{M}_{FF}$ (spin transfer)
5- If the spin transfer is sufficiently strong, $\vec{M}_{FF}$ can move (or even can switch)

## II Mathematical equations

### Modeling proposed by Zhang, Levy and Fert [3]

System of two coupled equations :

- Spin density $\vec{m}$ solution to a diffusive equation :

$$\frac{\partial \vec{m}}{\partial t} - 2D_0\frac{\partial^2 \vec{m}}{\partial x^2} + \frac{J}{\hbar}(\vec{m}\times\vec{M}) = -\frac{\vec{m}}{\tau_{sf}}$$

where - J quantifies interactions between $\vec{m}$ and $\vec{M}$ (0.1-0.4 eV),
- $\tau_{sf}\approx 10^{-12}s$ is the relaxation time of spin switching,
- $D_0\approx 10^{-3}m^2.s^{-1}$ is the diffusive constant of the metal,
- $\hbar=\frac{h}{2\pi}$ with h the Plank constant : $h=6,62.10^{-34}J.s.$

- Magnetization $\vec{M}$ solution to a Landau-Lifshitz equation :

$$\frac{d\vec{M}}{dt} = -\gamma\vec{M}\times(\vec{H}_e + J\vec{m}) + \alpha\vec{M}\times\frac{d\vec{M}}{dt}$$

where - $\gamma>0$ and $\alpha>0$ are two constants,
- $\vec{H}_e$ is the magnetic field. In our study, we work with

$$\vec{H}_e = -c(\vec{M}.\vec{u})\vec{u} + \nu\frac{\partial^2\vec{M}}{\partial x^2}$$

where - $\vec{u}$ is a unit vector which gives the anisotropy direction,
- c and $\nu$ are two constants in the order of 1.

### Dimensionless equations

To treat different spatial and temporal scales, we define the small parameter $\epsilon = l/L$. Finally, we obtain equations in ]-1;0[ (FE) and in ]0;1[ (FF)

$$\begin{cases} \epsilon^2\frac{\partial\vec{m}}{\partial t} - \frac{\partial^2\vec{m}}{\partial x^2} + \frac{(\vec{m}\times\vec{M})}{\epsilon^2} = -\vec{m} \\ \frac{d\vec{M}}{dt} = -\vec{M}\times\left(c(\vec{M}.\vec{u})\vec{u} + \frac{\vec{m}}{\epsilon} + \nu\frac{\partial^2\vec{M}}{\partial x^2}\right) + \alpha\vec{M}\times\frac{d\vec{M}}{dt} \end{cases} \quad \text{in } ]-1,0[$$

$$\begin{cases} \epsilon^4\frac{\partial\vec{m}}{\partial t} - \frac{\partial^2\vec{m}}{\partial x^2} + (\vec{m}\times\vec{M}) = -\epsilon^2\vec{m} \\ \frac{d\vec{M}}{dt} = -\vec{M}\times\left(c(\vec{M}.\vec{u})\vec{u} + \frac{\vec{m}}{\epsilon} + \frac{\nu}{\epsilon^2}\frac{\partial^2\vec{M}}{\partial x^2}\right) + \alpha\vec{M}\times\frac{d\vec{M}}{dt} \end{cases} \quad \text{in } ]0,1[$$

In FE, $\frac{\vec{m}\times\vec{M}}{\epsilon^2}$ $\Rightarrow$ Polarization of $\vec{m}$ in the direction of $\vec{M}$

In FF, $\frac{\nu}{\epsilon^2}\frac{\partial^2\vec{M}}{\partial x^2}$ $\Rightarrow$ A spatial homogeneous magnetization

### Boundary conditions

For Landau-Lifshitz, we choose homogeneous Neumann conditions

$$\begin{cases} \partial_x\vec{M}_{FE}(-1,t)=0 \ , \ \partial_x\vec{M}_{FE}(0^-,t)=0 \\ \partial_x\vec{M}_{FF}(0^+,t)=0 \ , \ \partial_x\vec{M}_{FF}(1,t)=0 \end{cases} \quad \forall t\in[0,T]$$

For the diffusive equation, we choose Dirichlet for x=-1. The value corresponds to the injected current.

$$\vec{m}_{FE}(-1,t) = \vec{m}_L \quad \forall t\in[0,T]$$

Then, for x=0, we preserve the continuity.

$$\begin{cases} \vec{m}_{FE}(0^-,t) = \vec{m}_{FF}(0^+,t) \\ \epsilon\frac{\partial\vec{m}_{FE}(0^-,t)}{\partial x} = \frac{\partial\vec{m}_{FF}(0^+,t)}{\partial x} \end{cases} \quad \forall t\in[0,T]$$

Finally, for x=1, we want a free evolution for remaining quantities. So, we build a Fourier condition.

$$\frac{\partial\vec{m}_{FF}(1,t)}{\partial x} = -A\vec{m}_{FF}(1,t) \quad \forall t\in[0,T]$$

where A is a positive matrix.

## III Discretization of the coupled system

### Diffusive equation discretization

To discretize the diffusive equation, we use an implicit method of finite differences. For example in ]0,1[, the discretized equation is

$$\epsilon^4\frac{\vec{m}^{n+1}(x)-\vec{m}^n(x)}{\Delta t} - \frac{\vec{m}^{n+1}(x+h_{FF}) - 2\vec{m}^{n+1}(x) + \vec{m}^{n+1}(x-h_{FF})}{h_{FF}^2} + (\vec{m}^{n+1}(x)\times\vec{M}(x)) = -\epsilon^2\vec{m}^{n+1}(x)$$

where $\Delta t$ is our time step and $h_{FF}$ our space step.

### Landau-Lifshitz equation discretization

To discretize the Landau-Lifshitz equation, two points are essential :

- The magnetization norm is preserved in time. To keep this property, we use a Crank-Nicholson scheme [5]

$$\frac{\vec{M}^{n+1}-\vec{M}^n}{\Delta t} = -\frac{\vec{M}^n+\vec{M}^{n+1}}{2}\times f(\vec{M}^{n+1},\vec{m}) + \alpha\frac{\vec{M}^n+\vec{M}^{n+1}}{2}\times\frac{\vec{M}^{n+1}-\vec{M}^n}{\Delta t}$$

- To solve the implicit scheme, we used two different methods : a Newton method [2] and a Gauss-Seidel projection method [6].

### A prediction-correction method to couple equations

To avoid a very small time step, we implement a prediction-correction method

$$\vec{m}^n, \vec{M}^n \Rightarrow \widetilde{M}(\vec{m}^n,\vec{M}^n) \Rightarrow \vec{m}^{n+1}(\widetilde{M},\vec{m}^n) \Rightarrow \vec{M}^{n+1}(\vec{m}^{n+1},\widetilde{M})$$

## IV Numerical results

### Observation of a magnetization switching

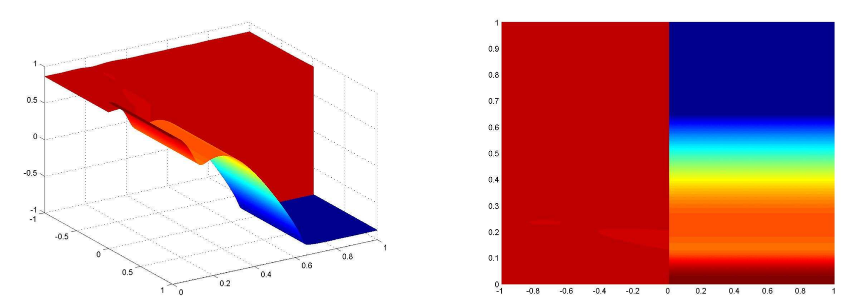

Fig.2 - Evolution of the component $M_z$ during the time (3D view and projection).

In Fig.2, abscissa axis corresponds to space with FE in ]-1;0[ and FF in ]0;1[ and ordinate axis represents time. We observe a switching of the vertical component $M_z$. Horizontal components $M_x$ and $M_y$ are less significant.

During a magnetization switching, we find 3 steps :
1- $\vec{M}_{FF}$ stays at an initial position (0,0,1)
2- It goes down rotating around the unit sphere
3- It stabilizes around a point close to the inferior pole

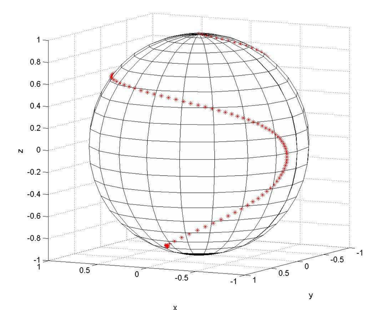

Fig.3 - Evolution of $\vec{M}$ during the time around the unit sphere.

### Impact of the injected current

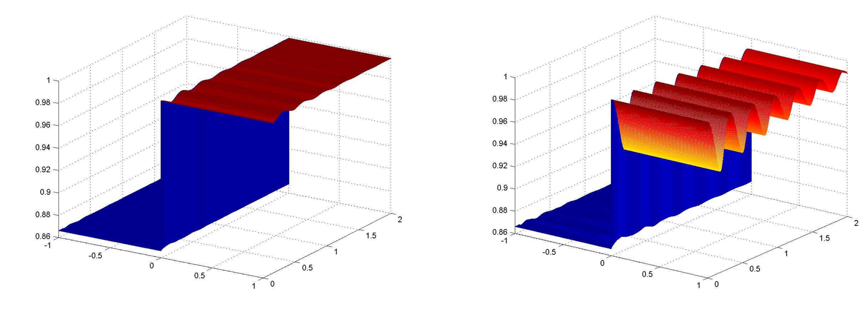

Fig.4 - Projection of the component $M_z$ in the case $\|\vec{m}(-1)\| = 0.5$ and 1.2.

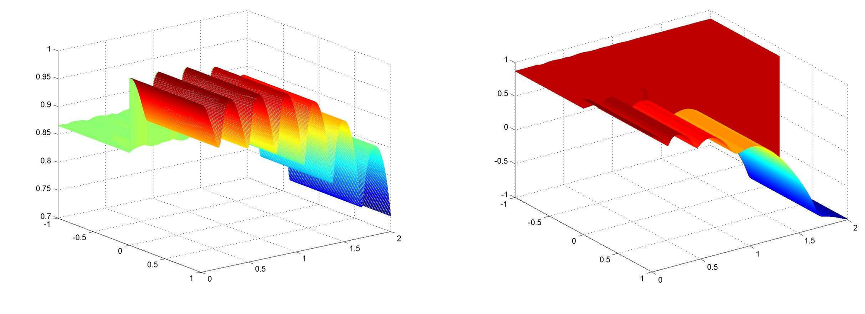

Fig.5 - Projection of the component $M_z$ in the case $\|\vec{m}(-1)\| = 1.4$ and 2.

To observe the impact of the injected current, we change the value of the Dirichlet condition $m_L$.

- $m_L=0.5$, $\vec{M}_{FF}$ does not move
- $m_L=1.2$, $\vec{M}_{FF}$ oscillates and then comes back to the initial position
- $m_L=1.4$, oscillations are not absorbed and we obtain a switching
- $m_L=2$, the switching occurs in a shorter time without too many oscillations

It exists a switching threshold. A sufficiently strong current is needed to create important interactions between the spin density and the magnetization.

## IV Perspectives

This model allows to observe magnetization switching and it is in accordance with physical experiments (in particular, with the notion of threshold concerning the injected current).

Future work :
- Study of the real model proposed by Zhang, Levy and Fert
  which couples the spin density with the charge density,
- Building of an asymptotic expansion (when $\epsilon$ approaches zero),
- 3D study of such devices...

### References


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