Elementary partial differential equations: syllabus

Charles Dapogny,
webpage: http://www.math.rutgers.edu/~cd581/
mail: cd581@math.rutgers.edu.

This course is meant as an introduction to the vast topic of partial differential equations, and covers qualitative issues about PDE (what are PDE in general ? What about their general behaviors ?), as well as some more advanced material (e.g. Fourier series and harmonic functions).

It is mainly based upon the following textbook:

W. Strauss, *Introduction to Partial Differential Equations*, 2^{nd} edition, John Wiley & Sons, (2008),

and some parts of the course will also rely on the more exhaustive book:

R. Haberman, *Applied Partial Differential Equations, with Fourier Series and Boundary Value Problems*, 4^{th} edition, Pearson Education, (2004).

Here is a tentative syllabus, to be updated in the course of the Spring semester.

I. First encounter with PDEs: first examples and basic principles

Lecture 1: Presentation of the course; introduction of the basic vocabulary: linear / nonlinear, stationary / unsteady, homogeneous / inhomogeneous PDE, and first general remarks.

Lecture 2: First encounter with some PDE, the first order linear equations. Introduction of the method of characteristics, and the method of change of coordinates.

Lecture 3: Some examples of PDE arising from physics: transport, diffusion, wave equations, etc... presentation of how they are derived from physical principles, as well as of some of their qualitative properties.

Lecture 4: The importance of *initial* and *boundary conditions* when studying PDE; discussion of the 'natural' conditions to be imposed on the various models of Lecture 3.

Lecture 5: About the notion of *well-posed* problem.

Lecture 6: Generalities about second-order equations: elliptic, parabolic, and hyperbolic equations, and their properties.

II. Two very different regimes: closer study of waves and diffusion

Lecture 7: The wave equation (1): the general solution, and some more specific situations.

Lecture 8: The wave equation (2): the principles of causality, and energy conservation.

Lecture 9: The heat equation (1): specific properties: the maximum's principle, uniqueness of the Dirichlet's problem, and stability of the problem.

Lecture 10: The heat equation (2): resolution of the equation in one space dimension, on the whole real line.

Lecture 11: No Lecture ! First midterm exam.

Lecture 12: Comparison between the regimes of waves and diffusion.

III. The method of separation of variables for solving PDE; a first taste of Fourier series

Lecture 13: Resolution of the heat equation by the method of separation of variables, in the case of Dirichlet boundary conditions.

Lecture 14: Resolution of the heat equation by the method of separation of variables, in the case of Neumann boundary conditions.

Lecture 15: Resolution of the heat equation by the method of separation of variables, in the case of Robin boundary conditions.

IV. Fourier series, and applications to PDE

Lecture 16: Presentation of the three types of Fourier series: sine, cosine, and full series; computations of Fourier coefficients, and the orthogonality principle.

Lecture 17: How to choose between the three kinds of Fourier series ? Even and odd functions.

Lecture 18: Complex Fourier series.

Lecture 19: Convergence of Fourier series in three different mathematical settings: pointwise convergence, uniform convergence, and L^2 convergence. The Gibbs phenomenon.

Lecture 20: Differentiation and integration of Fourier series.

Lecture 21: No Lecture ! Second midterm exam.

Lecture 22: Dealing with inhomogeneous boundary conditions in PDE, using Fourier series.

V. A closer look at the Laplace equation and harmonic functions

Lecture 23: Properties of the Laplace equation: the maximum's principle, invariance in two and three dimensions, and a study on well-posedness.

Lecture 24: Laplace equation in a disk.

Lecture 25: Harmonic functions (1): definition, and main properties.

Lecture 26: Harmonic functions (2): More properties.