

## Advanced Calculus I: Workshop 10

### Exercise 1

Let  $C \subset \mathbb{R}$  be a connected set, and let  $f : C \rightarrow \mathbb{R}$  be a continuous, one-to-one function. Show that  $f$  is monotone.

### Exercise 2

Let  $D$  be a subset of  $\mathbb{R}$ , and  $x_0 \in D$  be an accumulation point of  $D$ . Let  $f, g, h : D \rightarrow \mathbb{R}$  be three functions such that  $f(x_0) = g(x_0) = h(x_0)$ , and:

$$\forall x \in D, \quad f(x) \leq h(x) \leq g(x).$$

- (1) Show that, if  $f$  and  $g$  are differentiable at  $x_0$ , then so is  $h$ , and:

$$f'(x_0) = g'(x_0) = h'(x_0).$$

- (2) *Application:* Let  $h : [0, +\infty] \rightarrow \mathbb{R}$  be the function defined by:

$$h(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x > 0, \\ 0 & \text{if } x = 0 \end{cases}.$$

Show that  $h$  is differentiable at 0 and that  $h'(0) = 0$ .