# Advanced Calculus I: syllabus

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This course proposes an in-depth and rigorous discussion of the fundamental tools of real analysis and calculus, such as limits, sequences, continuity and differentiability of functions. The aim is to establish in a precise way the main notions, and to make the students familiar with mathematical reasoning (analyzing definitions, understanding and constructing proofs, etc...).

The course is mainly based upon the following textbook:

E.D. Gaughan, *Introduction to Analysis*, 5th edition, Brooks/Cole Publishing Co. ISBN: 0-534-35177-8; ISBN-13: 9780534351779, (2009).

**Personal work:** Our section meets twice a week, namely on Tuesday and Thursday, from 6:10 p.m. to 7.30 p.m. Prior to each lecture, the relevant sections in the textbook are suggested for reading, and the lecture emphasizes on the salient points of the topic.

A workshop session is organized by M. Balasubramanian, every Tuesday from 7.40 p.m. to 9.00 p.m. Students are asked to hand over a redaction of (a part of) their work during every session of the workshop.

Every Tuesday, a homework is assigned, focusing essentially on the material of the previous week's lectures. The homework is collected during the lecture on the next Tuesday.

*No late homework will be accepted, whatever the reason invoked.*

**Grading policy:** The final grade for this course is based upon points; the maximum number of points is 600 and the breakdown is as follows:

- 100 points for each of the two midterm exams,
- 200 points for the final exam,
- 100 points for the workshop sessions' work,
- 100 points for the homeworks.

From this number of points, a letter is eventually derived, by means of a cut-off yet to be decided.

## Tentative schedule of the lectures

### I. Revisions from the course Math 300; preliminaries

**Lecture 1:** Presentation of the course; introduction of the basic notions around *sets*: union, intersection, inclusion, proving an equality between two sets.

**Lecture 2:** Basic notions around *relations* and *functions*: definitions, composition, inverse, image of a relation / a function.

**Lecture 3:** The *induction principle* and several of its variants; applications on several examples.

**Lecture 4:** *Equivalence* between two sets, and the concept of *countable sets*; operations between countable sets (Cartesian product, union, etc...).

**Lecture 5:** Some facts around real numbers; lower and upper bound principles and density of rational numbers among real numbers.

### II. The fundamental objects of real analysis: sequences

**Lecture 6:** The notions of *sequence* and of *convergence* of a sequence: definitions, examples; unicity of the limit of a sequence.

**Lecture 7:** *Cauchy sequences*: definition, and connections with convergent sequences; definition of the notions of neighborhoods, accumulation points and the Bolzano-Weierstrass theorem.

**Lecture 8:** Operations on sequences: sum, product, etc... and consequences on the limits; passing to the limit in inequalities; examples.

**Lecture 9:** *Subsequences* and *monotone sequences*: definitions, properties, examples

### III. Limits of functions

**Lecture 10:** Definition of the limit of a function at a point; examples.

**Lecture 11:** No lecture! First midterm exam!

**Lecture 12:** Connections and characterizations of the limits of functions with limits of sequences.

**Lecture 13:** Behavior of the limits of functions with respect to operations: sums, products, etc... Handling limits of functions in inequalities.

**Lecture 14:** Limits of monotone functions.

### IV. Continuity of functions

**Lecture 15:** Notion of *continuity* of a function at a point; characterization in terms of limits; examples.

**Lecture 16:** Operations over continuous functions: sum, product, composition, etc...

**Lecture** 17**:** Topological considerations: *open*, *closed* and *compact* sets; the Heine-Borel theorem.

**Lecture** 18**:** *Uniform continuity* of functions; the Heine theorem.

**Lecture** 19**:** Further properties of continuous functions: connections with open, closed, compact sets; the Bolzano theorem, and the intermediate-value theorem (I).

**Lecture** 20**:** Further properties of continuous functions: connections with open, closed, compact sets; the Bolzano theorem, and the intermediate-value theorem (II).

## V. Differentiability of functions

**Lecture** 21**:** Definition of the *derivative* of a function at a point; examples.

**Lecture** 22**:** No lecture! Second midterm exam!

**Lecture** 23**:** Behavior of the derivative with respect to operations on functions: sums, products, etc...

**Lecture** 24**:** Rolle's theorem and the Mean-Value theorem (I): applications and examples.

**Lecture** 25**:** Rolle's theorem and the Mean-Value theorem (II): applications and examples.

**Lecture** 26**:** Further applications of the previous concepts: L'Hospital's rule and the Inverse-Function theorem.