AN INTRODUCTION TO SHAPE OPTIMIZATION AND ITS APPLICATIONS IN MECHANICS

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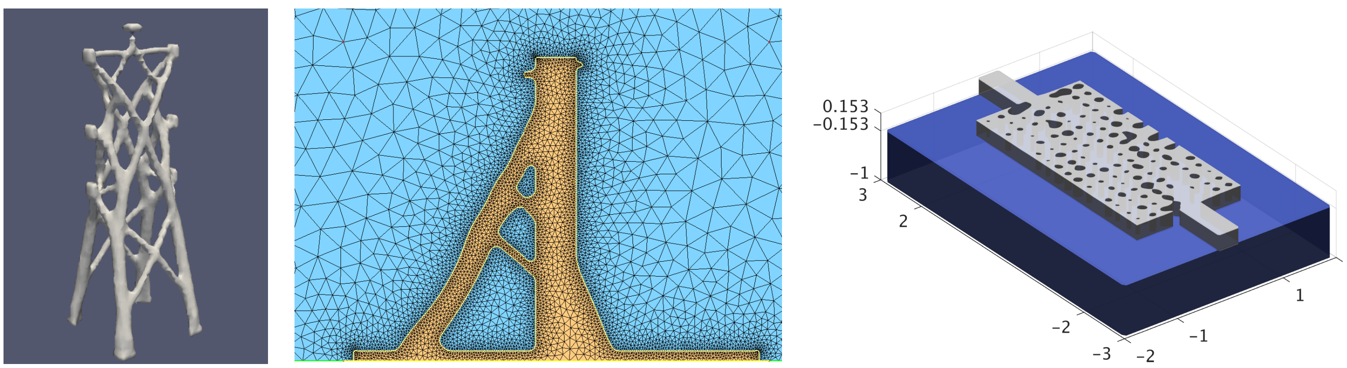
Fostered by its impressive technological and industrial achievements, the discipline of shape and topology optimization has aroused a growing enthusiasm among mathematicians, physicists and engineers since the seventies. Nowadays, problems pertaining to fields so diverse as mechanical engineering, fluid mechanics or biology, to name a few, are currently tackled with such techniques, and constantly raise new, challenging issues.



(Left) Optimized design of an electric mast; (middle) optimized shape of an obstacle to oppose an incoming flow and (right) optimized shape of a nanophotonic TE-TM mode converter.

A shape optimization problem typically shows up under the general form

$$\min_{\Omega} J(\Omega) \text{ s.t. } C(\Omega) \leq 0,$$

where $J(\Omega)$ is a performance criterion and $C(\Omega)$ is a contraint, both functions depending on the (variable) domain $\Omega \subset \mathbb{R}^d$ ($d = 2, 3$ in applications). In this course, we are particularly interested in the situation where the optimization problem emanates from mechanical or physical considerations. Then, $J(\Omega)$ and $C(\Omega)$ depend on Ω via a *state* u_Ω, solution to a Partial Differential Equation (PDE) characterizing the physical setting. For instance,

- $u_\Omega : \Omega \to \mathbb{R}$ is the temperature field, solution to the heat equation, when conduction problems are considered;
- $u_\Omega : \Omega \to \mathbb{R}^d$ stands for the displacement of Ω, solution to the linearized elasticity system when the design of mechanical structures is investigated;
- $u_\Omega : \Omega \to \mathbb{R}^d$ is the velocity field when the optimized domain Ω is filled with a fluid, satisfying the Stokes or the Navier-Stokes equations.

Part of the material of this course has been prepared in collaboration with Pascal Frey (ISCD & Laboratoire Jacques-Louis Lions, Université Paris VI) and Yannick Privat (CNRS & Laboratoire Jacques-Louis Lions), whose help is warmly acknowledged!

This type of shape and topology optimization problems raises fascinating theoretical issues - related e.g. to the existence of an optimal solution, or to its regularity - as well as thorny numerical concerns, both ranges of questions being the target of active ongoing research.

The purpose of this course is to discuss the main aspects related to the numerical resolution and the practical implementation of such problems, and to present state-of-the-art elements of response. In particular, the following issues will be addressed:

- How to define a 'good' notion of derivative for a function $\Omega \mapsto J(\Omega)$ depending on the domain;
- How to calculate the *shape derivative* of a function $\Omega \mapsto J(\Omega)$ which depends on the domain via the solution u_Ω of a PDE posed on Ω;
- How to devise efficient first-order algorithms (e.g. steepest-descent algorithms) based on the notion of shape derivative;
- How to numerically represent shapes Ω so that it is at the same time convenient to perform Finite Element computations on them, and to deal with their evolution in the course of the optimization process.

Program

The course is divided into a theoretical part ($\approx$ 6h), and a hands-on session ($\approx$ 6h). A basic knowledge in the mathematical analysis of PDE and numerical programming is assumed from the attendants.

The hands-on session will make extensive use of `FreeFem++` - a convenient programming environment for solving PDE within a few lines of command - which the attendants are required to have installed prior to the beginning of the course. The latter may be freely downloaded from the address:

`http://www.freefem.org`

The material covered during the course will be subject to the reactions of the audience, but it will be based on the following (tentative) outline, insofar as possible:

(1) Introduction: a historical perspective of shape optimization problems, a presentation of their general mathematical formulation, and a discussion of several typical applications.
(2) Presentation of several physical models - the Laplace equation, the linearized elasticity system and the Stokes equations - and of their mathematical stakes.
(3) The Hadamard method for calculating shape derivatives, and first-order shape gradient algorithms.
(4) Numerical methods for shape optimization:
    - Generalities about algorithms for shape optimization;
    - The model 'mesh-displacement / steepest-descent' algorithm;
    - The level set method for shape optimization:
    - A glimpse to density-based methods.
(5) Some current challenges in shape and topology optimization.

References

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