Impact of Metallic Interface Description on Sub-wavelength Cavity Mode Computations

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Abstract

We present a theoretical study of electromagnetic reflection and cavity modes of 1D-sub-wavelength rectangular metallic gratings exposed to TE-polarised light. Computations are made using the modal development proposed by Sheng [1] and are a generalisation of those used by Barbara et al. [2]. In particular we study the influence of the choice of boundary conditions on the metallic surfaces on the mode coefficients.

Introduction

We consider the reflection of an electromagnetic wave on the grating described on Figure 1. This grating is supposed to be infinite in the z-direction and periodic in the x-direction, with period d. The grating consists of grooves of width w and depth h. The width (and in practice the period d, see [2]) is supposed to be smaller than the wavelength, i.e. $w \leq \lambda = 2\pi/k_0$, where $k_0 = |\mathbf{k}_0|$.

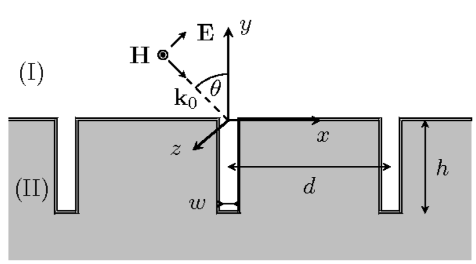

Figure 1: Reflecting grating in TE-polarisation

Given a monochromatic incident wave, we want to compute the reflected modes in the domain (I) and the cavity modes in the cavities (II). To this aim we can make different hypothesis for the treatment of the surface which is covered with gold. Computations can be easily made treating all the gold surfaces with a perfectly conducting boundary condition (perfect metal). In [2], they treat all vertical surfaces with such a condition and horizontal ones with a surface impedance condition (real metal). Our aim here is to compare these computations with the completely real metal one and to analyse the impact of the surface condition choice on the computed mode coefficients.

1 Problem setting

1.1 Reflected and cavity modes

Classically the reflected waves in domain (I) are only those which stem from constructive interferences and are given with a Rayleigh expansion

$$H_z^I(x,y) = e^{ik_0(\gamma_0 x - \beta_0 y)} + \sum_{m\in\mathbb{Z}} R_m e^{ik_0(\gamma_m x + \beta_m y)}. \quad (1)$$

The incident wave has a wave vector of modulus k_0 and angle θ with the vertical direction. The only reflected waves have the same wave vector modulus and correspond to $\gamma_m = \sin\theta + m\lambda/d$ and $\beta_m = \sqrt{1-\gamma_m^2}$. Only a finite number of these waves are propagative (β_m real). One of the goals of the computation is to find the reflection coefficients R_m, $m \in \mathbb{Z}$. The second goal is to find the cavity modes. We seek them in the Sheng [1] mode decomposition, i.e. as a superposition of waves of the type

$$H_z^{II}(x,y) = (Ae^{ik_1 x} + Be^{-ik_1 x})(Ce^{ik_2 y} + De^{-ik_2 y}) \quad (2)$$

where k_1 and k_2 are solution to $k_1^2 + k_2^2 = k_0^2$. The boundary conditions will give the possible discrete values for k_1 and k_2 as well as relations between the coefficients A, B, C, and D and their relation with the reflection coefficients.

1.2 Boundary conditions

Impedance conditions are used to model the interface between a dielectric medium and a metal. Indeed we do not want to model the electromagnetic wave which penetrates in the metal within the skin depth, but replace it (for the TE-polarisation) by the condition

$$\partial_n H_z + ik_0 Z H_z = 0,$$

where n is the exterior normal of the metallic surface and $Z = \sqrt{\varepsilon_1/\varepsilon}$ is the relative surface impedance, computed from the dielectric constants ε_1 and ε of the dielectric medium and the metal respectively. For a real metal the dielectric constant is complex and consists of a large negative real part and a smaller imaginary part.

Considering a perfect metal interface consists in letting $\varepsilon \to -\infty$, which yields $Z = 0$ and a Neumann condition $\partial_n H_z = 0$.

The different assumptions on the metallic surfaces that we compare are the following

1. perfect metal (Neumann condition) everywhere,
2. perfect metal on vertical surfaces and real metal on horizontal surfaces (mixed case of [2]),
3. real metal (impedance condition) everywhere.

In cases 2. and 3. we can also choose to neglect or not the imaginary part of ε.

2 Results

2.1 Method

We follow the computations in [2] which consist in three steps:

1. take into account the condition on the vertical walls. This defines a relation between A and B in (2), and discrete values for k_1 which we denote by ν_n, $n \in \mathbb{N}$. The corresponding values of k_2 follow immediately.
2. take into account the condition on the bottom of the cavity. This defines a relation between C and D in (2).
3. write a linear combination of the cavity modes corresponding to the contributions for the different values of μ_n and take into account the conditions on the surface $y = 0$ using also (1). The impedance condition yields each reflection coefficient R_m, $m \in \mathbb{Z}$ in terms of the amplitudes A_n, $n \in \mathbb{N}$, of each cavity mode. The continuity of the fields above the cavity yields each cavity mode amplitude in terms of the reflection coefficients. It leads to a linear system relating the cavity mode amplitudes, which we can solve, and from which we can deduce the values of the reflection coefficients.

The system is *a priori* infinite. To solve it an assumption has to be made on the number of relevant modes, which leads to a usual finite dimensional system. For narrow cavities ($w < \pi/k_0$) only a single mode can be kept in the cavity.

2.2 Comparison of the different cases

The mixed case contains in fact the fully perfect metal case by just having $Z \to 0$. In this case the first step leads to the very simple result $B = \exp(ik_1 w)A$ and $\mu_n = n\pi/w$. On the contrary the fully real case leads to the equation to $B = \mp A$ and

$$\exp(i\mu_n w) = \mp \frac{\mu_n + k_0 Z}{\mu_n - k_0 Z}.$$

The values of μ_n are therefore not explicit any more but only solution to this equation. They are real if ε is real and complex otherwise. The computations of the second step are the same in all the cases.

For the third step, the case of fully real metal is once more a little more complicated. In the mixed case, the impedance condition and the continuity of the fields are projected onto two bases (periodic L^2 functions on $[-d/2, d/2]$ and L^2 functions on $[-w/2, w/2]$ with vanishing derivatives on the border) which stem from the previous computations. In the fully real metal and the impedance condition on the walls, this second basis has no equivalent any more. We have however found a mean to perform the calculations, which do not yield each cavity mode amplitude in terms of the reflection coefficients, but a family of linear combinations of the cavity mode amplitude in terms of the reflection coefficients. Although more complex, this leads to essentially the same type of linear systems to solve as in the mixed case. Solving these systems, we find that the cavity coefficients μ_n and the amplitudes of the reflection and cavity modes are strongly dependent on the choice of the metallic surface conditions.

References

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